

Combined Effect of Slip Velocity and Roughness on the Jenkins Model Based Ferrofluid Lubrication of a Curved Rough Annular Squeeze Film

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ABSTRACT

This paper theoretically analyzes the combined effect of slip velocity and surface roughness on the performance of Jenkins model based ferrofluid squeeze film in curved annular plates. The effect of slip velocity has been studied resorting to the slip model of Beavers and Joseph. The stochastically averaging method of Christensen and Tonders has been deployed for studying the effect of surface roughness. The pressure distribution is derived by solving the associated stochastically averaged Reynolds type equation with suitable boundary conditions, leading to the computation of load carrying capacity. The graphical representations reveal that the transverse surface roughness adversely affects the bearing performance. However, Jenkins model based ferrofluid lubrication offers some scopes in minimizing this adverse effect when the slip parameter is kept at minimum. Of course, an appropriate choice of curvature parameters adds to this positive effect in the case of negatively skewed roughness. Moreover, it is established that this type of bearing system supports certain amount of load; even when there is no flow which does not happen in the case of conventional lubricant based bearing system.

Keywords: Jenkins model; Slip velocity; Magnetic fluid; Annular plates; Roughness.

1. INTRODUCTION

For centuries, many fascinating materials have been attracting the scientists and researchers due to their extraordinary physical properties and technological usage. Ferrofluid is one of such smart materials, which are not available free state in nature, but are to be synthesized. These fluids have variety of applications in the field of sciences and engineering, which are being commercialized.

Owing to the wide application of the ferrofluid, many authors have used magnetic fluids as a lubricant in different geometry of bearing systems. Tipei (1982) dealt with the theory of lubrication using ferrofluid and applied it in short bearings. Agrawal (1986) investigated the performance of porous inclined slider bearing using a ferrofluid and found that the magnetization of magnetic particles in the lubricant increased the load carrying capacity. Sinha *et al.* (1993) analyzed the effect of ferrofluid lubrication on cylindrical rollers. Ram and Verma (1999) worked on the performance of porous inclined slider bearing using ferrofluid lubrication. Osman et al. (2001) examined the static and dynamic characteristics of magnetized journal bearings lubricated with ferrofluid. Shah and Bhat (2005) discussed the effect of ferrofluid lubrication on a squeeze film between curved annular plates considering rotation of magnetic particles. Deheri et al. (2006) analyzed the performance of circular step bearings under the presence of a magnetic fluid. Ahmad and Singh (2007) studied the effect of porouspivoted slider bearing with slip velocity using ferrofluid. Urreta et al. (2009) investigated the effect of hydrodynamic bearing lubricated with magnetic fluids. Patel et al. (2010) studied the performance of a short hydrodynamic slider bearing in the presence of magnetic fluids. Patel et al. (2012) evaluated the performance of hydrodynamic short journal bearings lubricated with magnetic fluids. All the above investigations have established that the performance of the bearing system gets enhanced due to magnetization.

The application of squeeze film is commonly found in gears, aircraft engines, automotive engines, gyroscopes and the mechanics of synovial joints in human being and animals. The squeeze film behaviour arises from the incident of two lubricated surfaces approaching each other with a normal viscosity. Many representative researches concerning squeeze films have been discussed for the parallel surfaces by Gould (1967), in curved annular plates by Gupta and Vora (1980), in annular disks by Lin (2001) and a sphere and plane surface by Chou *et al.* (2003).

Since the late nineteenth century, it has usually been accepted that the correct boundary condition between fluid and solid surface is the noslip boundary condition. This boundary condition means that there is no relative velocity between the fluid and the surface, i.e., if the solid surface is at rest, the fluid immediately adjacent to the surface has zero velocity, whereas if the surface is moving, the fluid immediately adjacent has a velocity equal to the surfaces velocity. This boundary condition has been extensively verified by experiment and by kinetic theory, and it is one of the basic principles upon which the Reynolds equation is based. The only exceptions, until recently, have been the flows of rarefied gases and high molecular weight polymer melts. Beavers and Joseph (1967) studied the interface between a porous medium and fluid layer in an experimental study and introduced a slip boundary condition at the interface. Several investigations concerned with slip velocity have been presented; for the circular disks by Patel (1980), the slider bearing by Salant and Fortier (2004), Wu et al. (2006), Ahmad and Singh (2007), Patel and Deheri (2011), the radial sleeve bearing by Wang et al. (2012) and infinitely long bearing by Patel and Deheri (2014c). In all of the above study, it was found that effect of slip was important for modifying the bearing performance.

All the above studies considered smooth bearing surfaces. But it is almost impossible because, the bearing surfaces could be rough after having some run-in and wear or through the manufacturing process and the impulsive damage. Several methods have been proposed to deal with the effect of surface roughness on the performance characteristics of squeeze film bearings, Christensen and Tonder (1970), Christensen and Tonder (1969a) and Christensen and

Tonder (1969b) developed the stochastic theory of Tzeng and Saibel (1967) to study the effect of surface roughness in general. A good number of research papers are abound dealing with the hydrodynamic lubrication of rough surfaces using stochastic approaches of Christensen and Tonder (1970), Christensen and Tonder (1969a) and Christensen and Tonder (1969b) such as the works on the porous annular disks by Ting (1975), the journal bearing by Guha (1993), Chiang et al. (2004), the spherical bearing by Gupta and Deheri (1996), Hydrodynamic slider bearing by Naduvinamani et al. (2003), the curved annular plates by Bujurke et al. (2007), Deheri et al. (2011) and Shimpi and Deheri (2012), the circular plates by Patel et al. (2009), Shimpi and Deheri (2010). All the above studies make it clear that roughness affects the performance significantly. Patel and Deheri (2013a) dealt with the effect of various porous structures on the performance of a Shliomis model based ferrofluid lubrication of a squeeze film in rotating rough porous curved circular plates. It was found that the adverse effect of transverse roughness could be overcome by the positive effect of ferrofluid lubrication in the case of negatively skewed roughness by suitably choosing curvature parameters and rotational inertia when Kozeny- Carmans model was used for porous structure. Patel and Deheri (2014d) theoretically analyzed the performance of Shliomis model based ferrofluid lubrication of a squeeze film between curved rough annular plates with comparison between two different porous structures. It was manifest that the effect of morphology parameter and volume concentration parameter increased the load carrying capacity. Patel et al. (2014) studied the performance of ferrofluid lubrication of squeeze film in curved circular plates with associated porous structures. It was established that the effect of porosity in the case of capillary fissures model is nominal. Recently, Patel and Deheri (2014a) discussed the effect of slip velocity and surface roughness on the performance of Jenkins model based magnetic squeeze film in curved rough circular plates. It was found that for enhancing the performance characteristics of the bearing system the slip parameter was required to be reduced even if variance (-ve) occurs and suitable magnetic strength was in force.

In the present study, it is proposed to deal with the performance of Jenkins model based ferrofluid lubrication of a curved rough annular squeeze film taking into consideration of the slip velocity.



Fig. 1. Configuration of the bearing system.

2. ANALYSIS

The squeeze film bearing configuration, presented below consists of two annular plates each of inside radius *b* and outside radius *a*. The upper plate and lower plate are taken as curved. Here *r* is the radial coordinate and h_0 is the central film thickness.

The bearing surfaces are considered transversely rough. In view of the stochastic theory of Christensen and Tonder (1970), Christensen and Tonder (1969a) and Christensen and Tonder (1969b), the thickness h of the lubricant film is assumed as

$$h = \bar{h} + h_s \tag{1}$$

where \bar{h} represents the mean film thickness and h_s stand for the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. h_s is governed by the probability density function

$$f(h_s) = \begin{cases} \frac{35}{32c^7} (c^2 - h_s^2)^3 & ; & -c \le h_s \le c \\ 0 & , & elsewhere, \end{cases}$$

wherein *c* symbolizes the maximum deviation from the mean film thickness. The mean α , the standard deviation σ and the parameter ε which is the measure of symmetry of the random variable h_s , are considered as the discussions of Christensen and Tonder (1970), Christensen and Tonder (1969a) and Christensen and Tonder (1969b). A simple model to express the flow of a magnetic fluid was established by Jenkins in 1972. According to theory of Jenkins, Jenkins model was not only a generalization of the Neuringer-Rosensweig model but also modified both the pressure and the velocity of the magnetic fluid.

With Maugin's development, equations of the steady flow come out to be (Jenkins (1972) and Ram and Verma (1999))

$$\rho(\bar{q}\nabla)\bar{q} = -\nabla_p + \eta\nabla^2\bar{q} + \mu_0(\bar{M}\nabla)\bar{H} + \frac{\rho A^2}{2}\bar{M}_1(2)$$

where

$$ar{M_1} =
abla imes [rac{ar{M}}{M} imes \{ (
abla imes ar{q}) imes ar{M} \}]$$

together with

$$\nabla \bar{q} = 0, \nabla \times \bar{H} = 0, \bar{M} = \bar{\mu}\bar{H}, \nabla(\bar{H} + \bar{M}) = 0$$

(Bhat (2003)). where ρ stand for the fluid density, \bar{q} indicates the fluid velocity in the film region, \bar{H} is external magnetic field, $\bar{\mu}$ represents magnetic susceptibility of the magnetic fluid, pis the film pressure, η represents the fluid viscosity, μ_0 denotes the permeability of the free space, A being a material constant and \bar{M} indicates magnetization vector. From the above equation one concludes that Jenkins model is a generalization of Neuringer-Rosensweig model with the additional term

$$\frac{\rho A^2}{2} \nabla \times \left[\frac{\bar{M}}{M} \times \left\{ (\nabla \times \bar{q}) \times \bar{M} \right\} \right] = \frac{\rho A^2 \bar{\mu}}{2} \nabla \times \bar{H}_1(3)$$

where

$$ar{H_1} = rac{ar{H}}{H} imes \{ (
abla imes ar{q}) imes ar{H} \}$$

which modifies the velocity of the fluid. At this point one finds that Neuringer-Rosensweig model modifies the pressure while Jenkins model modifies both the pressure and velocity of the ferrofluid.

Consider (u, v, w) to be the velocity of the fluid at any point (r, θ, z) between two solid surfaces, with OZ as axis. Applying hydrodynamic lubrication theory and remembering that the flow is steady and axially symmetric, the equations of motion are

$$\left(1 - \frac{\rho A^2 \bar{\mu} H}{2\eta}\right) \frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta} \frac{d}{dr} \left(p - \frac{\mu_0 \bar{\mu}}{2} H^2\right) \tag{4}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \tag{5}$$

The solution of the above Eq. (4) under the boundary conditions, u = 0 when z = 0, h, takes the form

$$u = \frac{z(z-h)}{2\eta(1 - \frac{\rho A^2 \bar{\mu} H}{2\eta})} \frac{d}{dr} \left(p - \frac{\mu_0 \bar{\mu}}{2} H^2\right)$$
(6)

Substituting the value of u in Eq. (5) and integrating it with respect to z over the interval (0,h) one gets Reynolds type equation for film pressure as

$$\frac{1}{r}\frac{d}{dr}\left(\frac{h^{3}}{\left(1-\frac{pA^{2}\bar{\mu}H}{2\eta}\right)}r\frac{d}{dr}\left(p-\frac{\mu_{0}\bar{\mu}}{2}H^{2}\right)\right) = 12\eta\dot{h_{0}}$$
(7)

It is supposed that the upper plate lying along the surface resolute by (Bhat (2003), Abhangi and Deheri (2012), Patel and Deheri (2013b))

$$z_u = h_0 sec(\beta r^2); b \le r \le a$$

approaches with normal velocity $\dot{h_0}$ to the lower plate lying along the surface governed by

$$z_l = h_0[sec(\gamma r^2) - 1]; b \le r \le a$$

where β and γ represent the curvature parameter of the corresponding plates. The film thickness h(r) then is defined by (Bhat (2003), Abhangi and Deheri (2012), Patel and Deheri (2014b))

$$h(r) = h_0[sec(\beta r^2) - sec(\gamma r^2) + 1]; b \le r \le a$$

For the stochastic averaging of the differential Eq. (7), a method has been projected by Christensen and Tonder (1970), Christensen and Tonder (1969a) and Christensen and Tonder (1969b). Here an effort has been made to adopt this method, which on certain simplifications yields, under the usual assumptions of hydro-magnetic lubrication (Bhat (2003), Prajapati (1995), Patel *et al.* (2009)), the modified Reynolds type equation,

$$\frac{1}{r}\frac{d}{dr}\left(\frac{g(h)}{(1-\frac{\rho A^{2}\bar{\mu}H}{2\eta})}r\frac{d}{dr}\left(p-\frac{\mu_{0}\bar{\mu}}{2}H^{2}\right)\right) = 12\eta\dot{h_{0}}$$
(8)

where

$$H^{2} = K(r-b)(a-r), G = \left(\frac{4+sh}{2+sh}\right)$$
$$g(h) = (h^{3}+3h^{2}\alpha+3(\sigma^{2}+\alpha^{2})h+3\sigma^{2}\alpha+\alpha^{3}+\varepsilon)G$$

where K is a suitably chosen constant so as to produce a required magnetic strength (Bhat (2003)).

Introducing following non dimensional quantities

$$\bar{h} = \frac{h}{h_0}, R = \frac{r}{b}, k = \frac{a}{b},$$

$$P = -\frac{h_0^3 p}{\eta b^2 \dot{h_0}}, B = \beta b^2, C = \gamma b^2, \mu^* = -\frac{K \mu_0 \bar{\mu} h_0^3}{\eta \dot{h_0}},$$

$$\bar{A}^2 = \frac{\rho A^2 \bar{\mu} b \sqrt{K}}{2\eta}, \bar{\sigma} = \frac{\sigma}{h_0}, \bar{\alpha} = \frac{\alpha}{h_0}, \bar{\varepsilon} = \frac{\varepsilon}{h_0^3}, \bar{s} = sh_0$$
(9)

Making use of the Eq. (9) in Eq. (8) one gets

$$\frac{1}{R}\frac{d}{dR}(\frac{g(\bar{h})}{J_1}R\frac{d}{dR}(p-\frac{J}{2})) = -12$$
(10)

where

$$g(\bar{h}) = (\bar{h}^3 + 3\bar{h}^2\bar{\alpha} + 3(\bar{\sigma}^2 + \bar{\alpha}^2)\bar{h} + 3\bar{\sigma}^2\bar{\alpha} + \bar{\alpha}^3 + \bar{\epsilon})\bar{G}$$
$$\bar{G} = (\frac{4 + \bar{s}\bar{h}}{2 + \bar{s}\bar{h}}), J = \mu^*(R - 1)(k - R),$$
$$J_1 = (1 - \bar{A}^2\sqrt{(R - 1)(k - R)})$$

Solving above Eq. (10) under the boundary conditions

$$P(1) = P(k) = 0 \tag{11}$$

one obtains the expression for the dimensionless pressure distribution as

$$P = \frac{J}{2} - 6I_1 + 6\frac{I_2}{I_3}I_4 \tag{12}$$

where

$$I_1 = \int_1^R \frac{R}{g(\bar{h})} J_1 dR,$$

$$I_2 = \int_1^k \frac{R}{g(\bar{h})} J_1 dR,$$

$$I_3 = \int_1^k \frac{1}{Rg(\bar{h})} J_1 dR,$$

$$I_4 = \int_1^R \frac{1}{Rg(\bar{h})} J_1 dR.$$

The dimensionless load carrying capacity of the bearing system then, is calculated as

$$W = \frac{\mu^*}{24}(k^2 - 1)(k - 1)^2 + 3I_5 - 3\frac{(I_2)^2}{I_3}$$
(13)

where

$$W = -\frac{h_0^3 w}{2\pi\eta b^4 \dot{h_0}}, I_5 = \int_1^k \frac{R^3}{g(\bar{h})} J_1 dR$$

3. **RESULT AND DISCUSSIONS**

It is well known that magnetization induces an increase in the viscosity of the lubricant resulting in increased pressure and hence leading to enhanced load carrying capacity. It is suggested from Eq. (12) that pressure gets increased by

$$\frac{\mu^*}{2}(R-1)(k-R)$$

while the load carrying capacity enhances by

$$(\mu^*/24)(k^2-1)(k-1)^2$$

as compared to the conventional lubricant based bearing system. Further, it is also clear from Eq. (13) that the expression involved is linear with respect to the magnetization parameter accordingly, the load carrying capacity increases with increase in the magnetization. The fact that the load carrying capacity rises sharply with increasing magnetization, is presented in Fig. 2-9. However, the combined effect of the material constant and the standard deviation on the variation of load carrying capacity with respect to the magnetization is almost negligible for initial values of both the parameters. The magnetization induces an increase in the viscosity of the lubricant leading to increased pressure and hence the load carrying capacity.



Fig. 2.Variation of load carrying capacity with respect to μ^* and \bar{A} .



Fig. 3. Variation of load carrying capacity with respect to μ^* and *B*.

The material constant parameter nominally decreases the load carrying capacity when considered with curvature parameters and aspect ratio while there is a heavy reduction of load carrying capacity when considered with all other parameters, which can be visualized



Fig. 4. Variation of load carrying capacity with respect to μ^* and C.



Fig. 5. Variation of load carrying capacity with respect to μ^* and k.



Fig. 6. Variation of load carrying capacity with respect to μ^* and $\bar{\sigma}$.



Fig. 7. Variation of load carrying capacity with respect to μ^* and $\bar{\epsilon}$.



Fig. 8. Variation of load carrying capacity with respect to μ^* and $\bar{\alpha}$.



Fig. 9. Variation of load carrying capacity with respect to μ^* and $1/\overline{s}$.



Fig. 10. Variation of load carrying capacity with respect to \overline{A} and B.



Fig. 11. Variation of load carrying capacity with respect to \overline{A} and C.



Fig. 12 Variation of load carrying capacity with respect to \overline{A} and k.



Fig. 13. Variation of load carrying capacity with respect to \bar{A} and $\bar{\sigma}$.



Fig. 14. Variation of load carrying capacity with respect to \bar{A} and $\bar{\epsilon}$.

from Fig. 10-16. Also it is observed that the decreased load due to material constant gets further reduced by the slip effect. The effect of material constant parameter modifies the velocity of the ferrofluid and consequently leads to decreased pressure resulting in reduced load carrying capacity.



Fig. 15. Variation of load carrying capacity with respect to \bar{A} and $\bar{\alpha}$.



Fig. 16. Variation of load carrying capacity with respect to \bar{A} and $1/\bar{s}$.

The fact that upper plates curvature parameter has a significant role in bettering the performance of the bearing system is displayed in Fig. 17-22. However, the combined effect of skewness and standard deviation on the load carrying capacity with respect to upper plates curvature parameter is negligible. Further, it is revealed that higher values of the ratio of curvature parameters may help in enhancing the performance characteristics.



Fig. 17. Variation of load carrying capacity with respect to *B* and *C*.



Fig. 18. Variation of load carrying capacity with respect to *B* and *k*.

Figures 23-27 establish that the load carrying capacity increases with increasing lower plates curvature parameter that means the trends of load carrying capacity with respect to both the curvature parameters are alike up to certain extent which is hardly manifest in other forms of film thickness (Deheri *et al.* (2011), Shimpi and Deheri (2012), Patel and Deheri (2013b)).



Fig. 19. Variation of load carrying capacity with respect to B and $\bar{\sigma}$.



Fig. 20. Variation of load carrying capacity with respect to *B* and $\bar{\epsilon}$.



Fig. 21. Variation of load carrying capacity with respect to *B* and $\bar{\alpha}$.



Fig. 22. Variation of load carrying capacity with respect to *B* and $1/\overline{s}$.



Fig. 23. Variation of load carrying capacity with respect to *C* and *k*.



Fig. 24. Variation of load carrying capacity with respect to C and $\bar{\sigma}$.



Fig. 25. Variation of load carrying capacity with respect to *C* and $\bar{\epsilon}$.



Fig. 26. Variation of load carrying capacity with respect to *C* and $\bar{\alpha}$.



Fig. 27. Variation of load carrying capacity with respect to *C* and $1/\bar{s}$.

From Fig. 28-31, it is noticed that while the effect of skewness on the variation of load carrying capacity with respect to aspect ratio is negligible, the effect of standard deviation on the distribution of load carrying capacity with respect to aspect ratio is nominal at the beginning.



Fig. 28. Variation of load carrying capacity with respect to k and $\bar{\sigma}$.



Fig. 29. Variation of load carrying capacity with respect to k and $\bar{\epsilon}$.



Fig. 30. Variation of load carrying capacity with respect to k and $\bar{\alpha}$.



Fig. 31. Variation of load carrying capacity with respect to k and $1/\overline{s}$.

The standard deviation has a considerable adverse effect on the performance of the magnetic squeeze film which can be seen from Fig. 32-34. Therefore, the combined effect of standard deviation and the slip is still adverse. Roughness retards the motion of the lubricant and hence causes reduced pressure resulting in decreased load carrying capacity.



Fig. 32. Variation of load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\epsilon}$.



Fig. 33. Variation of load carrying capacity with respect to $\bar{\sigma}$ and $\bar{\alpha}$.



Fig. 34. Variation of load carrying capacity with respect to $\bar{\sigma}$ and $1/\bar{s}$. [!hbp]

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		0.052			
		0.05 -			
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×	~	0.046 🕂			×
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-0.05	-0.03	-0.01	0.01	0.03	0.05
\rightarrow $\tilde{\alpha}$ = -0.05 \rightarrow $\tilde{\alpha}$ = -0.025 \rightarrow $\tilde{\alpha}$ = 0 \rightarrow $\tilde{\alpha}$ = 0.025 \rightarrow $\tilde{\alpha}$ = 0.05					

Fig. 35. Variation of load carrying capacity with respect to $\bar{\epsilon}$ and $\bar{\alpha}$.



Fig. 36. Variation of load carrying capacity with respect to $\bar{\epsilon}$ and $1/\bar{s}$.

As can be seen from Fig. 35 and 36, positive skewness causes load reduction while the load carrying capacity enhances due to negatively skewed roughness. The last graph asserts that the trends of load carrying capacity with respect to variance are almost similar to that of skewness. Therefore, the increase load carrying capacity due to skewness gets a further boost in case of variance(-ve) (Fig. 37).



Fig. 37. Variation of load carrying capacity with respect to $\bar{\alpha}$ and $1/\bar{s}$.

Some of the graphs tend to suggest that with a suitable choice of aspect ratio and the ratio of curvature parameters, the adverse effect of slip, standard deviation and material constant parameter can be overcome by the positive effect of magnetization at least in the case of negatively skewed roughness when variance negative is in place. If at all the bearing performance is to be improved slip parameter must be kept at minimum.

4. CONCLUSION

This investigation establishes that Jenkins model goes ahead of the Neuringer-Rosensweig model in reducing the adverse effect of roughness. However, from bearings life period point of view the roughness is required to be addressed judiciously while designing the bearing system. If at all the performance characteristics are to be kept at the reduced level. In spite of several factors reducing the load carrying capacity, this type of bearing system supports certain amount of load even when there is no flow, a thing that never happens in the case of conventional lubricant based bearing system. The combined effect of negatively skewed roughness and variance (-ve) goes a long way in mitigating the adverse effect of slip velocity and standard deviation when Jenkins model is considered for the magnetic fluid flow. The role of the ratio of curvature parameters is equally important from bearings life period point of view.

REFERENCES

- Abhangi, N. and G. Deheri (2012). Numerical modeling of squeeze film performance between rotating transversely rough curved circular plates under the presence of a magnetic fluid lubricant. *ISRN Mechanical Engineering 2012*, Article ID 873481.
- Agrawal, V. (1986). Magnetic fluid based porous inclined slider bearing. *Wear 107*, 133–139.
- Ahmad, N. and J. P. Singh (2007). Magnetic fluid lubrication of porous-pivoted slider bearing with slip velocity. *Journal of Engineering Tribology 221*, 609–613.
- Beavers, G. and D. Joseph (1967). Boundary conditions at a naturally permeable wall. *Journal of Fluid Mechanics* 30, 197–207.
- Bhat, M. (2003). *Lubrication with a Magnetic fluid*. India: India: Team Spirit (India) Pvt. Ltd.
- Bujurke, N., N. Naduvinamani and D. Basti (2007). Effect of surface roughness on the squeeze film lubrication between curved annular plates. *Industrial Lubrication and Tribology* 59(4), 178–185.
- Chiang, H., C. Hsu and J. Lin (2004). Lubrication performance of finite journal bearings considering effects of couple stresses and surface roughness. *Tribology International* 37(4), 297–307.
- Chou, T. L., J. W. Lai and J. R. Lin (2003). Magneto-hydrodynamic squeeze film characteristics between a sphere and a plane surface. *Journal of marine science and technology* 11(3), 174–178.
- Christensen, H. and K. Tonder (1969a). Tribology of rough surfaces: paramet-

ric study and comparison of lubrication models. *SINTEF Report No.22*, 69–18.

- Christensen, H. and K. Tonder (1969b). Tribology of rough surfaces: stochastic models of hydrodynamic lubrication. *SINTEF Report No.10*(2), 69–18.
- Christensen, H. and K. Tonder (1970, October). The hydrodynamic lubrication of rough bearing surfaces of finite width. In *ASME-ASLE Lubrication Conference*, Cincinnati. OH.
- Deheri, G., R. Patel and N. Abhangi (2011). Magnetic fluid based squeeze film behavior between transversely rough curved annular plates: a comparative study. *Industrial Lubrication and Tribology* 63(4), 254–270.
- Deheri, G. M., H. Patel and R. Patel (2006). Performance of magnetic fluid based circular step bearings. *MECHANIKA* 57(1), 22–27.
- Gould, P. (1967). Parallel surface squeeze films. ASME J. Lubric. Technol. 89, 375– 380.
- Guha, S. (1993). Analysis of dynamic characteristics of hydrodynamic journal bearings with isotropic roughness effects. *Wear 167*(2), 173–179.
- Gupta, J. and G. Deheri (1996). Effect of roughness on the behavior of squeeze film in a spherical bearing. *Tribol Trans.* 39(1), 99–102.
- Gupta, J. and K. Vora (1980). Analysis of squeeze films between curved annular plates. ASME J. Lubric. Technol. 102, 48–50.
- Jenkins, J. (1972). A theory of magnetic fluids. Arch. Rational Mech. Anal. 46(1), 42–60.
- Lin, J. (2001). Magneto-hydrodynamic squeeze film characteristics between annular disks. *Indust. Lubric. Tribol.* 53(2), 66–71.
- Naduvinamani, N., S. Fathima and P. Hiremath (2003). Hydrodynamic lubrication rough slider bearings with couple stress fluids. *Tribology International* 36(12), 949–959.
- Osman, T., G. Nada and Z. Safar (2001). Static and dynamic characteristics of magnetized journal bearings lubricated with ferrofluid. *Tribology International* 34(6), 369–380.

- Patel, H., G. Deheri and R. Patel (2009). Magnetic fluid-based squeeze film between porous rotating rough circular plates. *Industrial Lubrication and Tribol*ogy 61(3), 140–145.
- Patel, J. and G. Deheri (2013a). Shliomis model based ferrofluid lubrication of squeeze film in rotating rough curved circular disks with assorted porous structures. *American Journal of Industrial Engineering* 1(3), 51–61.
- Patel, J. and G. Deheri (2013b). Shliomis model based magnetic fluid lubrication of a squeeze film in rotating rough curved circular plates. *Caribbean Journal of Science and Technology 1*, 138–150.
- Patel, J. and G. Deheri (2014a). Combined effect of surface roughness and slip velocity on jenkins model based magnetic squeeze film in curved rough circular plates. *International Journal of Computational Mathematics 2014*, Article ID 367618.
- Patel, J. and G. Deheri (2014b). Shliomis model-based magnetic squeeze film in rotating rough curved circular plates: a comparison of two different porous structures. *Int. J. of Computational Materials Science and Surface Engineering* 6(1), 29–49.
- Patel, J. and G. Deheri (2014c, October). Slip velocity and roughness effect on magnetic fluid based infinitely long bearings. In Proceedings of International Conference on Advances in Tribology and Engineering Systems Lecture Notes in Mechanical Engineering, Gujarat, India, pp. 97–109.
- Patel, J. and G. Deheri (2014d). Theoretical study of shliomis model based magnetic squeeze film in rough curved annular plates with assorted porous structures. *FME Transactions* 42(1), 56–66.
- Patel, K. (1980). The hydromagnetic squeeze film between porous circular disks with velocity slip. *Wear 58*(2), 275–281.
- Patel, N. and G. Deheri (2011). Effect of surface roughness on the performance of a magnetic fluid based parallel plate porous slider bearing with slip velocity. *J. Serb. Soc. Comput. Mech.* 5(1), 104– 118.
- Patel, N., G. Deheri and S. Mehta (2014, October). Ferrofluid lubrication of squeeze film in curved circular plates with assorted porous structures. In *Proceedings*

of International Conference on Advances in Tribology and Engineering Systems Lecture Notes in Mechanical Engineering, Gujarat, India, pp. 223–230.

- Patel, N., D. Vakharia and G. Deheri (2012). A study on the performance of a magnetic-fluid-based hydrodynamic short journal bearing. *ISRN Mechanical Engineering 2012*, Article ID 603460.
- Patel, R., G. Deheri and P. Vadher (2010). Performance of a magnetic fluid-based short bearing. *Acta Polytechnica Hungarica* 7(3), 63–78.
- Prajapati, B. (1995). On Certain Theoretical Studies in Hydrodynamic and Electromagneto hydrodynamic Lubrication. Ph. D. thesis, S.P. University, Vallabh Vidya-Nagar, Gujarat, India.
- Ram, P. and P. Verma (1999). Ferrofliuid lubrication in porous inclined slider bearing. *Indian Journal of Pure and Applied Mathematics 30*(12), 1273–1281.
- Salant, R. and A. Fortier (2004). Numerical analysis of a slider bearing with a heterogeneous slip/no-slip surface. *Tribology Transactions* 47(3), 328–334.
- Shah, R. and M. Bhat (2005). Ferrofluid squeeze film between curved annular plates including rotation of magnetic particles. *Journal of Engineering Mathematics* 51, 317–324.
- Shimpi, M. and G. Deheri (2010). Surface roughness and elastic deformation effects on the behavior of the magnetic fluid based squeeze film between rotating porous circular plates with concentric circular pockets. *Tribology in Industry* 32(2), 21–30.
- Shimpi, M. and G. M. Deheri (2012). Magnetic fluid-based squeeze film behaviour in curved porous-rotating rough annular plates and elastic deformation effect. *Advances in Tribology 2012*, Article ID 148281.
- Sinha, P., P. Chandra and D. Kumar (1993). Ferrofluid lubrication of cylindrical rollers with cavitation. *Acta Mechanica* 98, 27–38.
- Ting, L. (1975). Engagement behavior of lubricated porous annular disks. part i: Squeeze film phase – surface roughness and elastic deformation effects. *Wear 34*(2), 159–172.

- Tipei, N. (1982). Theory of lubrication with ferrofluids: Application to short bearings. *Transactions of ASME 104*, 510– 515.
- Tzeng, S. and E. Saibel (1967). Surface roughness effect on slider bearing lubrication. *ASLE Trans.* 10(3), 334–348.
- Urreta, H., Z. Leicht, A. Sanchez, A. Agirre, P. Kuzhir and G. Magnac (2009). Hydrodynamic bearing lubricated with magnetic fluids. *Journal of Physics: Conference Series 149*(1), Article ID 012113.
- Wang, L., C. Lu, M. Wang and W. Fu (2012). The numerical analysis of the radial sleeve bearing with combined surface slip. *Tribology International* 47, 100–104.
- Wu, C., G. Ma, P. Zhou and C. Wu (2006). Low friction and high load support capacity of slider bearing with a mixed slip surface. *Journal of Tribology 128*(4), 904–907.