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Similarity Solution for a Cylindrical Shock Wave in a Self-gravitating, Rotating Axisymmetric Dusty Gas with Heat Conduction and Radiation Heat Flux

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ABSTRACT

Similarity solutions are obtained for unsteady adiabatic propagation of a cylindrical shock wave in a self gravitating, rotating, axisymmetric dusty gas with heat conduction and radiation heat flux in which variable energy input is continuously supplied by the piston. The dusty gas is taken to be a mixture of non-ideal gas and small solid particles. Azimuthal fluid velocity and axial fluid velocity in the ambient medium are taken to be variable. The equilibrium flow conditions are assumed to be maintained. The initial density is assumed to be constant. The heat conduction is expressed in terms of Fourier's law and the radiation is taken to be of the diffusion type for an optically thick grey gas model. The thermal conductivity and the absorption coefficient are assumed to vary with temperature and density. The effects of the variation of the gravitational parameter and the heat transfer parameters on the shock strength and the flow variables such as radial velocity, azimuthal velocity, axial velocity, density, pressure, total heat flux, mass behind the shock front, azimuthal vorticity vector, axial vorticity vector, isothermal speed of sound and adiabatic compressibility are studied. It is found that the presence of gravitation effect in the medium modify the radiation and conduction effect on the flow variables.

Keywords: Shock wave; Self similar solution; Dusty gas; Self gravitation; Conduction and radiation heat flux.

1. INTRODUCTION

The investigation of the most important celestial phenomena must be centred on the problems of motion of gaseous masses with shock waves in gravitational field. The gravitational force has considerable effect on many astrophysical problems. The unsteady motion of a large mass of gas followed by a sudden release of energy results in flareups in novae and supernovae. A qualitative behaviour of the motion of gaseous mass may be discussed with the help of equations of motion and equilibrium, taking gravitational forces into account. There are two ways to formulate the problem of shock waves in gravitational field. The first, in which, the gravitating effect of the gas itself is considered (i.e. self gravitating). The second, in which, the gravitational effect of the gas, around nucleus having large mass m, can be neglected compared with the at-traction of the heavy nucleus (i.e. Roche model). Carrus *et al.* (1951) have studied the propagation of shock waves in a gas under the gravitational attraction of a central body of fixed mass (Roche model) and obtained the similarity solutions by numerical method. Rogers (1957) has discussed a method for obtaining analytical solution of the same problem. Ojha *et al.* (1998) have discussed the dynamical behaviour of an unsteady magnetic star by employing the concepts of the Roche model in an electrically conducting atmosphere.

Singh (1988) obtained analytical solution of magnetogasdynamic cylindrical shock waves under the influence of self gravitation and rotation in perfect gas. Singh and Singh (1995) studied cylindrical blast wave with radiation heat flux in self gravitating perfect gas. Vishwakarma and Nath(2012) has obtained similarity solution of spherical shock wave propagating in a dusty gas under gravitational field. In all of the works mentioned above, study of cylindrical shock wave in dusty gas under gravitational field is not considered. Vishwakarma and Singh (2009) obtained similarity solution of spherical shock wave in an ideal gas with heat conduction and radiation heat flux and with or without self gravitational effects. Nath (2012) obtained similarity solution of a rotating dusty gas behind the spherical shock wave with increasing energy, conduction and radiation heat flux. Vishwakarma and Nath (2012) obtained similarity solution for a cylindrical shock wave in a rotational axisymmetric dusty gas with heat conduction and radiation heat flux.

Singh and Vishwakarma (2012) obtained similarity solution of a spherical shock wave in dusty gas in presence of heat conduction, radiation heat flux and a gravitational field. In all of the work mentioned above, the influence of gravitational field on the cylindrical shock wave with heat conduction and radiation heat flux is not considered.

In the present work, we have, therefore, obtained similarity solution for a cylindrical shock wave in a self gravitating, rotating, axisymmetric dusty gas with heat conduction and radiation heat flux in which variable energy input is continuously supplied by the piston. The dusty gas is taken to be a mixture of non-ideal gas and small solid particles. Azimuthal fluid velocity and axial fluid velocity in the ambient medium are taken to be variable. The equilibrium flow conditions are assumed to be maintained. The initial density is assumed to be constant. The heat conduction is expressed in terms of Fourier's law and the radiation is taken to be of the diffusion type for an optically thick grey gas model. The thermal conductivity and the absorption coefficient of the gas are assumed to be proportional to appropriate powers of temperature and density Ghoniem *et al.* (1982). The viscosity terms are negligible. Also, it is assumed that the dusty gas is grey and opaque and the shock is isothermal. Radiation pressure and radiation energy are neglected.

The effects of the variation of the gravitational parameter and the heat transfer parameters on the shock strength and the flow variables such as radial velocity, azimuthal velocity, axial velocity, density, pressure, total heat flux, mass behind the shock front, azimuthal vorticity vector, axial vorticity vector, isothermal speed of sound and adiabatic compressibility are studied. Also, the effect of gravitational parameter on all the three component of velocity is studied which was not done earlier. It is found that the presence of gravitation effect in the medium modify the radiation and conduction effect on the flow variables

2. FUNDAMENTAL EQUATIONS ANDBOUNDARY CONDITIONS

he conservation equations governing the un-steady, adiabatic, self gravitating, axisymmetric, rotational flow of dusty gas with heat conduction and radiation heat flux taken into ac-count can be written as (c.f. Chaturani (1970), Ghoniem *et al.* (1982), Levin and Skopina (2004), Nath (2010), Vishwakarma and Nath (2010))

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\frac{\partial u}{\partial r}+\frac{u\rho}{r}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}-\frac{v^2}{r}+\frac{\overline{G}m}{r}=0, \tag{2}$$

$$\frac{\partial U_m}{\partial t}+u\frac{\partial U_m}{\partial r}-\frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)+\frac{1}{r\rho}\frac{\partial (rF)}{\partial r}=0, \tag{3}$$

$$\frac{\partial m}{\partial r}=2\pi\rho r, \tag{4}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{uv}{r}=0, \tag{5}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}=0, \tag{6}$$

where r and t are independent space and time co-ordinates, u, v and w are the radial, azimuthal and axial components of the fluid velocity $\vec{q}$ in the cylindrical coordinates (r,θ,z^*), $\bar{G}$ is gravitational constant and F is the heat flux.

We consider the medium to be dusty gas which is rotating about an axis of symmetry. The equation of state of the dusty gas which is a mixture of non-ideal gas and small solid particles is taken to be (c.f. Vishwakarma and Nath(2009), Vishwakarma and Nath(2010))

$$p=\frac{(1-K_p)}{(1-Z)}\left[1+b\rho(1-K_p)\right]\rho R^* T, \tag{7}$$

where p and ρ are the pressure and the density of the mmixture, $Z=\frac{Vsp}{V}$ is the volume fraction and $Kp=\frac{msp}{m}$ is the mass fraction of the solid particle in the mixture, m_{sp} and Vsp being respectively the total mass and the volumetric extensions of the solid particles in a volume 'V ' and mass 'm' of the mixture.

The internal energy per unit mass of the mixture is given by (c.f. Vishwakarma and Nath(2009), Vishwakarma and Nath(2010))

$$U_m=\frac{p(1-Z)}{(\Gamma-1)\rho\left[1+b\rho(1-K_p)\right]} \tag{8}$$

The azimuthal component v of fluid velocity $\vec{q}$ is given by

$$v=Ar, \tag{9}$$

where A is the angular velocity of the medium at radial distance r from the axis of symmetry. In this case the vorticity vector $\vec{\zeta}=\frac{1}{2}\,\mathrm{curl}\vec{q}$, has the com-ponents,

$$\xi_r=0,\ \xi_\theta=-\frac{1}{2}\frac{\partial w}{\partial r},\ \xi_{z^*}=\frac{1}{2r}\frac{\partial (rv)}{r} \tag{10}$$

It is assumed that a cylindrical shock wave is propagating outwards from the axis of symmetry inthe undisturbed medium with constant density having zero radial velocity and variable azimuthal and axial velocities. The flow variables immediately ahead of the shock front are

$$u=0, \tag{11}$$

$$\rho=\rho_a=cons\tan t, \tag{12}$$

$$v_a=BR^{\lambda}, \tag{13}$$

$$m_a=\pi\rho_a R^2, \tag{14}$$

$$w_a = E^* R^{\mu}, \tag{15}$$

$$F = F_a = 0, Laumbach\ \ and\ \ \Pr obstein (1970) \tag{16}$$

where R is the shock radius, B, E^*, λ and μ are constants and the subscript *'a'* denotes the conditions immediately ahead of the shock.

The components of the vorticity vector ahead of the shock vary as

$$\xi_{r_a} = 0, \tag{17}$$

$$\xi_{\theta_a} = -\frac{E^* \mu R^{\mu-1}}{2}, \tag{18}$$

$$\xi_{z^*_a} = \frac{B(\lambda+1) R^{\lambda-1}}{2}. \tag{19}$$

From Eqs. (9) and (13), we find that the initial angular velocity vary as

$$A_a = BR^{\lambda-1}. \tag{20}$$

It decreases as the distance from the axis increases, if $\lambda - 1 < 0$, and is constant if $\lambda = 1$.

The total heat-flux F may be decomposed as

$$F = F_c + F_R, \tag{21}$$

where Fc = conduction heat flux and FR = radiation heat flux.

According to Fourier's law of heat conduction

$$F_c = -K \frac{\partial T}{\partial r}, \tag{22}$$

where 'K' is the coefficient of the thermal conductivity of the gas and 'T ' is the absolute temperature.

Assuming local thermodynamic equilibrium and using the radiative diffusion model for an optically thick grey gas Pomraning(1973), the radiative heat flux FR may be obtain from the differential approximation of the radiation transport equation in the diffusion limit as (c.f. Mukhopadhyay(2009), Babu *et al.* (2014))

$$F_R = -\frac{4}{3}\left(\frac{\partial}{\alpha_R}\right)\frac{\partial T^4}{\partial r}, \tag{23}$$

where σ is the Stefan-Boltzmann constant and α_R is the Rosseland mean absorption coefficient.

The thermal conductivity K and the absorption coefficient α_R of the medium are assumed to vary with temperature and density. These can be written in the form of power laws, namely Ghoniem *et al.* (1982), Vishwakarma *et al.* (2008), Vishwakarma and Nath(2008), Vishwakarma and Nath(2010)

$$K = K_0 \left(\frac{T}{T_0}\right)^{\beta_c} \left(\frac{\rho}{\rho_0}\right)^{\xi_c}, \alpha_R = \alpha_{R_0} \left(\frac{T}{T_0}\right)^{\beta_R} \left(\frac{\rho}{\rho_0}\right)^{\xi_R}, \tag{24}$$

where the subscript *'0'* denotes a reference state. The exponents in the above equations should satisfy the similarity requirements if a self similar solution is sought.

From Eqs. (2), (13) and (14), we have

$$p_a = \frac{B^2 \rho_a R^{2\lambda}}{2\lambda} - \frac{\overline{G}\pi \rho_a^2 R^2}{2}, \lambda \rangle 0. \tag{25}$$

The disturbance is headed by an isothermal shock (the formation of the isothermal shock is a result of the mathematical approximation in which the flux is taken to be proportional to the temperature gradient). This excludes the possibility of a temperature jump, see for example *Zel'dovich* and Raizer(1967), Rosenau and Frankenthal(1976), Vishwakarma *et al.* (2007), Vishwakarma *et al.* (2008) and hence, the conditions across it are

$$\rho_n \left(\dot{R} - u_n\right) = \rho_a \dot{R}, \tag{26}$$

$$p_n + \rho_n \left(\dot{R} - u_n\right)^2 = p_a + \rho_a \dot{R}^2, \tag{27}$$

$$U_{m_n} + \frac{p_n}{\rho_n} + \frac{\left(\dot{R} - u_n\right)^2}{2} - \frac{F_n}{\rho_a \dot{R}} = U_{m_a} + \frac{p_a}{\rho_a} + \frac{\dot{R}^2}{2} - \frac{F_a}{\rho_a \dot{R}}, \tag{28}$$

$$v_n = v_a, \tag{29}$$

$$m_n = m_a, \tag{30}$$

$$w_n = w_a, \tag{31}$$

$$\frac{Z_n}{\rho_n} = \frac{Z_a}{\rho_a}, \tag{32}$$

$$T_n = T_a, \tag{33}$$

where the subscript *'n'* denotes the conditions immediately behind the shock front and $R = \frac{dR}{dt}$ denotes the velocity of the shock front.

The expression for the initial volume fraction of the solid particles Z_a is given by

$$Z_a = \frac{V_{sp}}{V_a} = \frac{K_p}{\left(1 - K_p\right) G_a + K_p}, \tag{34}$$

where $G_a = \frac{\rho sp}{\rho ga}$ is the ratio of the species density of the solid particles to the initial species density of the gas ρ_{ga} in the mixture.

From Eqs. (26)-(33), we get

$$u_n = (1 - \beta) R, \tag{35}$$

$$\rho_n = \frac{\rho_a}{\beta}, \tag{36}$$

$$Z_n = \frac{Z_a}{\beta}, \tag{37}$$

$$p_n = \left[(1-\beta) + \frac{1}{\gamma M^2}\right]\rho_a R^2, \tag{38}$$

$$m_n = \pi \rho_a R^2, \tag{39}$$

$$F_n = (1-\beta)\left[\frac{Z_a + \bar{b}(1-K_p)}{\gamma M^2(\beta - Z_a)[1+\bar{b}(1-K_p)]} - \frac{1+\beta}{2}\right]\rho_a R^3, \tag{40}$$

$$v_n = BR^{\lambda}, \tag{41}$$

$$w_n = E^* R^{\mu}, \tag{42}$$

where $\bar{b} = b\rho_a$, non-idealness parameter and *M*, the shock-Mach number referred to the frozen speed of sound $(\frac{\gamma \rho a}{pa})^{1/2}$ in the perfect gas, is given by M = $(\frac{R2\rho a}{\gamma pa})^{1/2}$. The quantity $\beta(0 < \beta < 1)$ is obtained by the relation

$$\beta^3 - \beta^2\left[Z_a + 1 + \frac{1}{\gamma M^2}\right] + \beta\left[\frac{(1-K_p)Z_a\bar{b}(1+\gamma M^2) + Z_a\gamma M^2 + 1}{[1+\bar{b}(1-K_p)]\gamma M^2}\right] + \frac{(1-Z_a)\bar{b}(1-K_p)}{\gamma M^2[1+\bar{b}(1-K_p)]} = 0. \tag{43}$$

Following Levin and Skopina (2004) and Nath (2010), we obtained the jump conditions for the components of vorticity vector across the shock as

$$\xi\theta_n = \frac{\xi\theta_a}{\beta},$$

$$\xi z_n^* = \frac{\xi z_a^*}{\beta}$$

The total energy *'E'* of the flow field behind the shock is not constant, but assumed to be dependent on shock radius obeying a power law(Ranga Rao and Purohit(1972))

$$E = E(R) = R_a R^s; s \geq 0, \tag{44}$$

$$\eta = \frac{r}{R}, R = R(t),$$

where Ra and *'s'* are constants. The positive values of *'s'* corresponds to the class in which the total energy increases with shock radius.

3. SIMILARITY SOLUTIONS

The system of partial differential Eqs. (1)-(6) of gas dynamics reduces to a system of ordinary differential equations in new unknown functions of the similarity variable $\eta = \frac{r}{R}$. For, we represent the solution of the partial differential Eqs. (1)-(6) in terms of products of scale functions and the new unknown functions of the similarity variable *η*,

$$u = RU(\eta), \tag{45}$$

$$v = RV(\eta), \tag{46}$$

$$w = RW(\eta), \tag{47}$$

$$\rho = \rho_a D(\eta), \tag{48}$$

$$p = \rho_a R^2 P(\eta), \tag{49}$$

$$m = \rho_a R^2 \Omega(\eta), \tag{50}$$

$$F = \rho_a R^3 \phi(\eta), \tag{51}$$

$$Z = Z_a D(\eta), \tag{52}$$

where *U, V , W , D, P,* Ω and *φ* are new dimensionless functions of the similarity variable *η*, in terms of which the differential equations are to be formulated.

For existence of similarity solutions, the shock Mach number *M* which occurs in the shock conditions (35)-(42) must be a constant parameter. Using

(12) and (25) into M =$(\frac{R2\rho a}{\gamma pa})^{1/2}$, we have

$$M^2 = \frac{R^2}{\gamma\left(\frac{B^2 R^{2\lambda}}{2\lambda} - \frac{\bar{G}\pi\rho_a R^2}{2}\right)}. \tag{53}$$

Therefore, *M* is constant for

$$R = QR \quad \text{and} \quad \lambda = 1, \tag{54}$$

where *'Q'* is a constant of dimension T^{-1}. Therefore, we obtain a relation for gravitation parameter

$$G_0 = \left(\frac{B}{Q}\right)^2 - \frac{2}{\gamma M^2}, \tag{55}$$

Where

$$G_0 = \frac{\bar{G}\pi\rho_a}{Q^2}.$$

The relation (55) is analogous to the relations (103) of Rogers(1957) and (20) of Singh(1982) in the case of perfect gas with variable initial density of the medium, Eq. (3.7) of Patel(2013) in thecase of mixture of small solid particles and perfect gas. The quantity G_0 is a parameter of gravitation analogous to the parameter l_1 of Rosenau(1977). In absence of gravitation field, the parameter $\frac{B}{Q}$ is similar to the parameter $\frac{B}{Q}$ in relation (66) of Vishwakarma and Nath (2012).

The total energy of the flow field between the piston and the cylindrical shock wave is given by

$$E = 2\pi\int_r^R \rho\left[\frac{1}{2}(u^2 + v^2 + w) + U_m - \bar{G}m\right] r dr, \tag{56}$$

where r is the radius of the piston or inner expanding surface . Now by using the similarity transformations (45) to (52) and the Eqs. (54) and

(55) in the relation (56), we get

$$E = 2\pi R^4 \rho_a Q^2 J, \tag{57}$$

Where

$$J = \int_{\eta}^{1} \left[\frac{D}{2}\left(U^2 + V^2 + W^2\right) + \frac{P(1 - Z_a D)}{(\Gamma - 1)\left[1 + \bar{b} D (1 - K_P)\right]} - \frac{\Omega}{\pi} G_0 \right] \eta \, d\eta, \tag{58}$$

η being the value of *'η'* at the piston or inner expanding surface.

From Eqs. (44) and (57), we get $s = 4$ which show that the total energy of the flow field behind the shock wave is not constant but proportional to the fourth power of the shock radius R. This in-crease can be achieved by the pressure exerted on the fluid by the inner expanding surface (a contact surface or a piston). It is also dependent on the gravitation parameter G_0.

Now by integrating Eq. (54) under the condition $R(t_0) = R_0$, we get

$$R = R_0 e^{Q(t - t_0)}, t \rangle t_0. \tag{59}$$

From Eqs. (57) and (59), we get

$$E = 2\pi \rho_a Q^2 J R_0^{\,4} e^{4Q(t - t_0)}, t \rangle t_0, \tag{60}$$

which show that for $\lambda = 1$, the total energy of the flow field behind the shock wave is time dependent and vary as an exponential law with time.

Then, the shock conditions (35)-(42) are trans-formed into

$$U(1) = (1 - \beta), \tag{61}$$

$$D(1) = \frac{1}{\beta}, \tag{62}$$

$$P(1) = \left[(1 - \beta) + \frac{1}{(\gamma M^2)} \right], \tag{63}$$

$$\phi(1) = (1 - \beta) \left[\frac{Z_a + \bar{b}(1 - K_p)}{\gamma M^2 (\beta - Z_a)\left[1 + \bar{b}(1 - K_p)\right]} - \frac{1 + \beta}{2} \right], \tag{64}$$

$$\Omega(1) = \pi, \tag{65}$$

$$V(1) = \left(\frac{2}{\gamma M^2} + G_0 \right)^{\frac{1}{2}}, \tag{66}$$

$$W(1) = \frac{E^*}{Q}, \tag{67}$$

where $\lambda = \mu = 1$.

The condition to be satisfied at the inner boundary surface is that the velocity of the fluid is equal to the velocity of inner boundary itself. This kinematic conditions can be written as

$$U(\eta_p) = \eta_p, \tag{68}$$

Using $\eta_p = \frac{Rp}{R}$ into Eq. (45).

Using the transformations (45)-(52), the equations of motion (1)-(6) take the form

$$(U = \eta)\frac{dD}{D\eta} + D\frac{dU}{d\eta} + \frac{DU}{\eta} = 0, \tag{69}$$

$$(U = \eta)\frac{dU}{d\eta} + \frac{1}{D}\frac{dP}{d\eta} + U - \frac{V^2}{\eta} \tag{70}$$

$$+ \frac{G_0}{\pi \eta}\Omega(\eta) = 0, \tag{71}$$

$$(U - \eta)\frac{dP}{d\eta} + L\frac{dD}{d\eta} + S\frac{d\phi}{d\eta} + N + 2P = 0, \tag{72}$$

$$\frac{d\Omega}{d\eta} = 2\pi \eta D, \tag{73}$$

$$(U - \eta)\frac{dV}{d\eta} + V + \frac{UV}{\eta} = 0, \tag{74}$$

$$(U - \eta)\frac{dW}{d\eta} + W = 0, \tag{75}$$

Where

$$L = L(\eta) = \frac{\left[\bar{b} D (1 - K_p)(Z_a D - 2) - 1 - (\Gamma - 1)\left(1 + \bar{b} D (1 - K_p)\right)^2\right]}{D\left[1 + \bar{b} D (1 - K_p)\right]} \times \frac{p(u - \eta)}{(1 - Z_a D)},$$

$$S = S(\eta) = \frac{\left[1 + \bar{b} D (1 - K_p)\right](\Gamma - 1)}{(1 - Z_a D)},$$

$$N = N(\eta) = \frac{\phi(\Gamma - 1)\left[1 + \bar{b} D (1 - K_p)\right]}{\eta (1 - Z_a) D}.$$

From Eqs. (69)-(75), we have

$$\frac{dU}{d\eta} = -\frac{U - \eta}{D}\frac{dD}{d\eta} - \frac{U}{\eta}, \tag{76}$$

$$\frac{dP}{d\eta} = (U - \eta)^2 \frac{dD}{d\eta} + \frac{DU(U - \eta)}{\eta} - DU + \frac{V^2 D}{\eta} - \frac{G_0}{\pi \eta}\Omega D, \tag{77}$$

$$\frac{dV}{d\eta} = -\frac{V}{U - \eta} - \frac{UV}{(U - \eta)\eta}, \tag{78}$$

$$\frac{dW}{d\eta} = -\frac{W}{U - \eta}, \tag{79}$$

$$\frac{d\Omega}{d\eta} = 2\pi \eta D, \tag{80}$$

$$\left[-\frac{DU(U-\eta)^2}{\eta S}+\frac{DU(U-\eta)}{S}-\frac{V^2D(U-\eta)}{\eta S}\right.$$
$$+\frac{G_0}{\pi\eta}\frac{\Omega D(U-\eta)}{S}-\left[\frac{L+(U-\eta)^3}{S}\right] \tag{81}$$
$$\left.\times\frac{dD}{d\eta}-\frac{N+2P}{S}\right].$$

By using Eqs. (22), (23) and (24) in (21), we get

$$F=-\frac{K_0}{T_0^{\beta c}\rho_0^{\xi c}}T^{\beta_c}\rho^{\xi_c}\frac{\partial T}{\partial r}$$
$$-\frac{16\partial T_0^{\beta_R}\rho_0^{\xi_R}}{3\alpha_{R_0}}T^{3-\beta_R}\rho^{-\xi_R}\frac{\partial T}{\partial r}. \tag{82}$$

Using the Eqs. (7) and (45)-(52) in Eq. (82), we get

$$\phi=-\left[\frac{K_0\rho_a^{(\xi_c-1)}Q}{T_0^{\beta_c}\rho_0^{\xi_c}R^{*(1+\beta_c)}}\right.$$
$$(1-K_p)^{-(\beta_c+1)}D^{(\xi_c-\beta_c)}$$
$$\times\left\{\frac{P(1-Z_aD)}{[1+\bar{b}D(1-K_p)]}\right\}^{\beta_c}R^{(2\beta_c-2)}$$
$$+\frac{16\sigma T_0^{\beta_R}\rho_0^{\xi_R}Q}{3\alpha_{R_0}\rho_a^{(1+\xi_R)}R^{*(4-\beta_R)}}$$
$$(1-K_p)^{(\beta_R-4)}R^{(4-2\beta_R)}$$
$$\times\left\{\frac{P(1-Z_aD)}{1+\bar{b}D(1-K_p)}\right\}^{(3-\beta_R)}$$
$$\left.\times D^{(\beta_R-3-\xi_R)}\right]\frac{d}{d\eta}\left[\frac{P(1-Z_aD)}{\{1+\bar{b}D(1-K_p)\}D}\right]. \tag{83}$$

Equation (83) shows that the similarity solution of the present problem exists only when

$\beta_c = 1$ and $\beta_R = 2$.

Therefore Eq. (83) becomes

$$\phi=-X\left[\frac{(1-Z_aD)}{D\{1+\bar{b}D(1-K_p)\}}\frac{dP}{d\eta}\right.$$
$$\left.+\frac{P(1-K_p)\bar{b}D(Z_aD-2)}{D^2\{1+\bar{b}D(1-K_p)\}^2}\frac{dD}{d\eta}\right], \tag{84}$$

where

$$X=\left[\Gamma_cD^{(\xi_c-1)}+\Gamma_RD^{-(\xi_R+1)}\right](1-K_p)^{-2}$$
$$\times\left[\frac{P(1-Z_aD)}{\{1+\bar{b}D(1-K_p)\}}\right], \tag{85}$$

Γ_c and Γ_R are the conductive and radiative non-dimensional heat transfer parameters, respectively. The parameters Γ_C and Γ_R depend on the thermal conductivity K and the mean free path of radiation $1/\alpha_R$, respectively and also on the exponent λ, and they are given by

$$\Gamma_c=\frac{K_0\rho_a^{(\xi_c-1)}Q}{T_0\rho_0^{\xi_C}R^{*2}}\quad\text{and}\quad\Gamma_R=\frac{16\sigma T_0^2\rho_0^{\xi_R}Q}{3\alpha_{R_0}\rho_0^{(1+\xi_R)}R^{*2}}.$$

From Eq. (77) and (84), we obtain

$$\left(\frac{dD}{d\eta}\right)=\left[(1-Z_aD)\left\{U-\frac{U(U-\eta)}{\eta}-\frac{V^2}{\eta}\right.\right.$$
$$\left.\left.+\frac{G_0}{\pi\eta}\Omega\right\}-\frac{\phi\{1+\bar{b}D(1-K_p)\}}{X}\right] \tag{86}$$
$$\times\left\{\frac{P(1-K_p)\bar{b}D(Z_aD-2)-P+(1-Z_aD)}{(U-\eta)^2D(1+\bar{b}D(1-K_p))}\right\}^{-1}$$
$$D^2[1+\bar{b}D(1-K_p)]$$

Also, applying the similarity transformations (46),(47) and the non-dimensional components of thevorticity vector

$$l=\frac{\xi_r}{(R/R)},l_\theta=\frac{\xi_\theta}{(R/R)},l_{z^*}=\frac{\xi_{z^*}}{(R/R)},$$

inthe flow-field behind the shock in Eq. (10), we obtain

$$l_r=0, \tag{87}$$

$$l_\theta=\frac{W}{2(U-\eta)}, \tag{88}$$

$$l_{z^*}=\frac{V}{(U-\eta)}. \tag{89}$$

For an isentropic change of state of the mixture of non-ideal gas and small solid particles, under the thermodynamic equilibrium condition, we may calculate the equilibrium sound speed of the mixture, as follows

$$a_m=\left(\frac{\partial p}{\partial\rho}\right)_s^{\frac{1}{2}}=\left[\frac{\{\Gamma+(2\Gamma-Z)b\rho(1-K_p)\}P}{(1-Z)\rho\{1+b\rho(1-K_p)\}}\right]^{\frac{1}{2}}, \tag{90}$$

neglecting $b^2\rho^2$, where subscript *'S'* refers to the process of constant entropy.

The adiabatic compressibility of the mixture of non-ideal gas and small solid particles may be calculated as (c. f. *Moelwyn–Hughes*(1961))

$$C_{adi}=-\rho\left(\frac{\partial}{\partial p}\left(\frac{1}{\rho}\right)\right)_s=\frac{1}{\rho a_{m^2}}$$
$$=\frac{(1-Z)[1+b\rho(1-K_p)]}{[\Gamma+(2\Gamma-Z)b\rho(1-K_p)]P}. \tag{91}$$

Using Eqs. (48),(49) and (52) in (91), we get the expression for the adiabatic compressibility as

$$(C_{adi})pa=\frac{(1-Z_aD)[1+\bar{b}D(1-K_p)]}{[\Gamma+\bar{b}D(1-K_p)(2\Gamma-Z_aD)]\gamma M^2P}. \tag{92}$$

In addition, the isothermal speed of sound may also play a role, when thermal radiation is taken into account. The isothermal sound speed in the mixture is

$$a_{iso} = \left(\frac{\partial p}{\partial \rho}\right)_T^{\frac{1}{2}} = \left[\frac{\{1+(2-Z)b\rho(1-K_p)\}P}{(1-Z)\rho\{1+b\rho(1-K_p)\}}\right]^{\frac{1}{2}}, \quad (93)$$

where the subscript *'T '* refers to the process of constant temperature.

By using Eqs. (45) to (52) in (93), we get the expression for reduce isothermal speed of sound as

$$\frac{a_{iso}}{\dot{R}} == \left[\frac{\{1+(2-Z_a D)bD(1-K_p)\}P}{(1-Z_a D)D\{1+\bar{b}\rho(1-K_p)\}}\right]^{\frac{1}{2}}. \quad (94)$$

The ordinary differential Eqs. (76)-(81) and (80) with boundary conditions (61)-(67) can now be numerically integrated to obtain the solution for the flow behind the shock surface.

Normalizing the variables *u, v, w, ρ, p, m* and F with their respective values at the shock, we obtain

$$\frac{u}{u_n} = \frac{U(\eta)}{U(1)}, \frac{v}{v_n} = \frac{V(\eta)}{V(1)}, \frac{w}{w_n} = \frac{W(\eta)}{W(1)},$$

$$\frac{\rho}{\rho_n} = \frac{D(\eta)}{D(1)}, \frac{p}{p_n} = \frac{P(\eta)}{P(1)}, \frac{m}{m_n} = \frac{\Omega(\eta)}{\Omega(1)},$$

$$\frac{F}{F_n} = \frac{\phi(\eta)}{\phi(1)}.$$

4. RESULTS AND DISCUSSION

We get the following relations among the constants λ and μ due to similarity considerations $\lambda = \mu = 1$.

The distribution of the flow variables between the shock front ($\eta = 1$) and the inner expanding surface or piston ($\eta = \eta_p$) is obtained by numerical integration of Eqs. (76)-(81) and (86) with the boundary conditions (61) to (67). The values of the constant parameters are taken to be $\gamma = 1.4; K_p = 0.2; G_a = 50; \beta' = 1; \bar{b} = 0.02; \delta_c = 1, \delta_R = 2; \Gamma_R = 0.5, 10, 100, 1000, 5000, 10{,}000, 15{,}000, \infty; \Gamma_c = 0.5, 1, 10, \infty; M^2 = 25; \lambda = 1, G_0 = 0.01, 0.94, 24.94$ and $E^*/Q = 0.005$. The values $\gamma = 1.4$; $\beta' = 1$ correspond to the mixture of air and glass particles Miura and Glass(1985). The value $M = 5$ of the shock Mach number is appropriate, because we have treated the flow of a non-ideal gas and a pseudo-fluid (small solid particles) at a velocity and temperature equilibrium. The set of values $\Gamma_c = 1, \Gamma_R = 10$ is the representative of the case in which there is heat transfer by both the conduction and the radiative diffusion.

Values of the piston position η_p and shock strength $(1-\beta)$ are tabulated in Table 1 for different values of gravitation parameter G_0 *with* $K_p = 0.2, G_a = 50, \bar{b} = 0.02, \beta' = 1, \gamma = 1.4, M = 5, \lambda = 1, \delta_c = 1, \delta_R = 2, E^*/Q = 0.005, \Gamma_c = 0.5$ and $\Gamma_R = 10$. Fig. 1 show the variation of the reduced flow variables u/u_n, v/v_n, w/w_n, ρ/ρ_n, p/p_n, m/m_n, F/F_n, $a_{iso}/\dot{R}$ and the adiabatic compressibility $(C_{adi})p_a$ with η at various values of the parameters G_0 for $\Gamma_c = 0.5, \Gamma_R = 10$. Table 2 show the position of the inner expanding surface (piston) and shock strength for $K_p = 0.2, G_a = 50, \bar{b} = 0.02, G_0 = 0.01$ and $\Gamma_R = 10$ for different values of Γ_c. Table 3 show the position of the inner expanding surface (piston) and shock strength for $K_p = 0.2, G_a = 50, \bar{b} = 0.02, G_0 = 0.01$ and $\Gamma_c = 0.5$ fordifferent values of Γ_R. Fig. 2 show the variation of the reduced flow variables u/u_n, v/v_n, w/w_n, ρ/ρ_n, p/p_n, m/m_n, F/F_n, $a_{iso}/\dot{R}$ and the adiabatic compressibility $(C_{adi})p_a$ with η at various values of the parameters Γ_c and Γ_R for $G_0 = 0.01$. It is shown that, as we move from the inner contact surface towards the shock front, the radial compo-nent of fluid velocity u/u_n, the pressure p/p_n, the density ρ/ρ_n, the axial component of vorticity vector l_{z*}, the isothermal speed of sound $a_{iso}/\dot{R}$ de-crease and azimuthal component of fluid velocity v/v_n, the axial component of fluid velocity w/w_n, the total heat flux F/F_n, the mass m/m_n, the azimuthal component of vorticity vector l_θ and the adiabatic compressibility $(C_{adi})p_a$ increase. The behaviour of the heat flux are similar to those obtained by Elliot(1960), Ghoniem *et al.* (1982) and Vishwakarma *et al.* (2008).

Table 1 Variation of the position of the piston η_p for different values of G_0 with $K_p = 0.2$, $G_a = 50$, $\bar{b} = 1$, $\gamma = 1.4$, $M = 5$, $\lambda = 1$, $\delta_c = 1$, $\delta_R = 2$, $E^*/Q = 0.005$, $\Gamma_c = 0.5$ and $\Gamma_R = 10$

G_0	β	$1-\beta=\frac{u_n}{R}$	η_p
0.01	0.043774	0.956226	0.985197
0.94	0.043774	0.956226	0.985265
25.94	0.043774	0.956226	0.987635

Table 2 Variation of the position of the piston η_p for different values of Γ_c with $K_p = 0.2$, $G_a = 50$, $\bar{b} = 0.02$, $G_0 = 50$ and $\Gamma_R = 10$

Γ_c	β	$1-\beta=\frac{u_n}{R}$	η_p
0.5	0.043774	0.956226	0.985198
1	0.043774	0.956226	0.982475
10	0.043774	0.956226	0.982069
∞	0.043774	0.956226	0.981149

It is found that the effects of an increase in the value of the gravitational parameter G_0 on flow variables are

(i) increase in the flow variables u/u_n, ρ/ρ_n, p/p_n, $a_{iso}/\dot{R}$, l_{z*},

(ii) decrease in the flow variables v/vn, w/wn, F/Fn, m/mn, $l\theta$, $(C_{adi})p_a$,

(iii) decrease in the distance of the piston from the shock front (see Table 1),

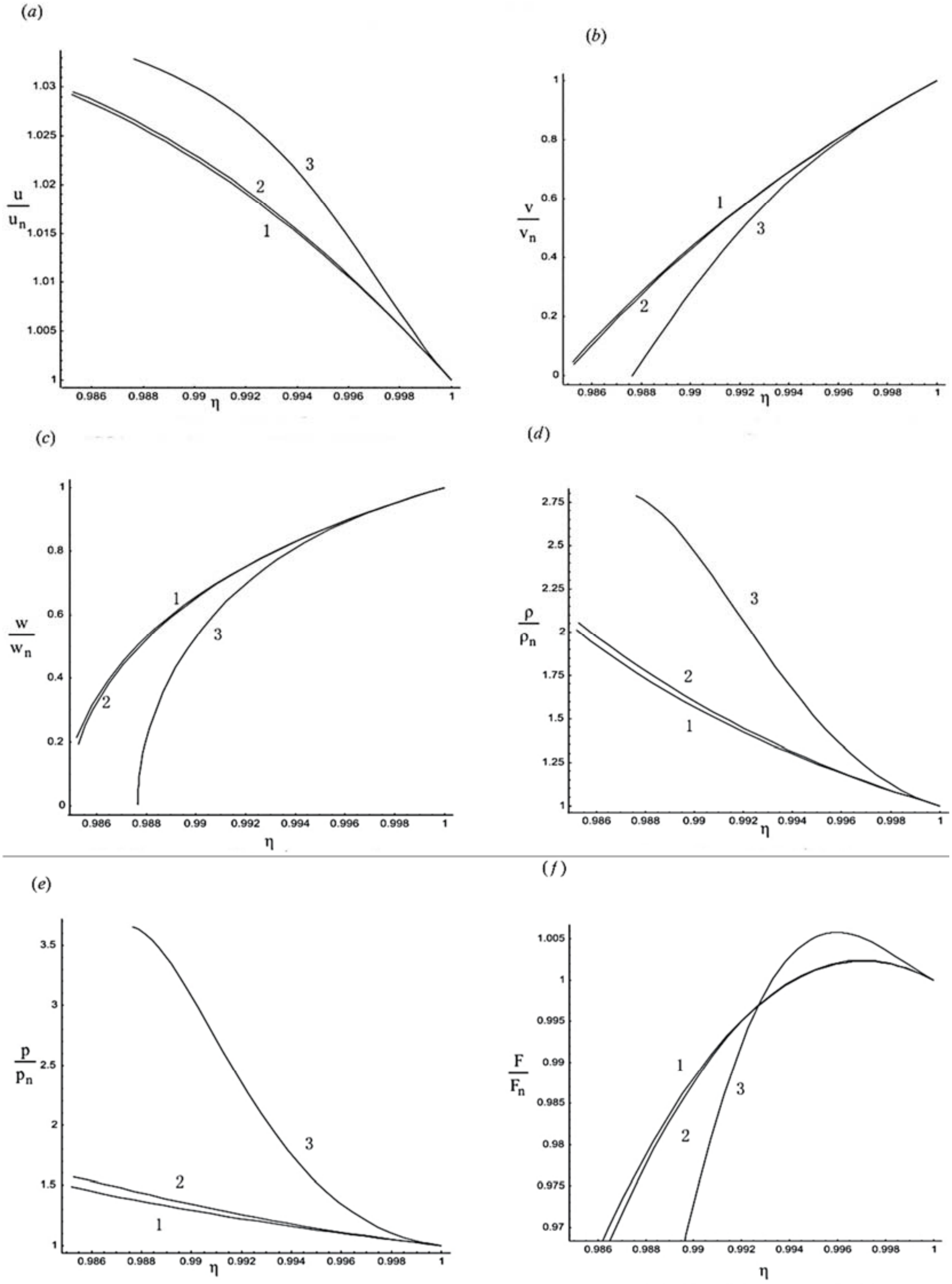

Fig. 1. Variation of the flow variables (a) radial component of fluid velocity (b) azimuthal component of fluid velocity (c) axial component of fluid velocity (d) density (e) pressure (f) total heat flux, in the region behind the shock front in case of K_p = 0.2, G_a = 50, = 0.002, Γ_c = 0.5, Γ_R = 10, $\frac{E*}{Q}$ = 0.005;1. G_0 = 0.01; 2. G_0 = 0.94; 3. G_0 = 24.94.

and the shock strength remain constant.

It is found that the effects of an increase in the value of conductive heat transfer parameter Γ_c on flow variables are

(i) decrease in the flow variables u/u_n, ρ/ρ_n, p/p_n, F/F_n, l_{z*} ,

(ii) increase in the flow variables v/v_n, w/w_n, m/m_n, l_θ ,$a_{iso}/R^{\cdot}$, $(C_{adi})p_a$,

(iii) increase in the distance of the piston from the shock front (see Table 2),

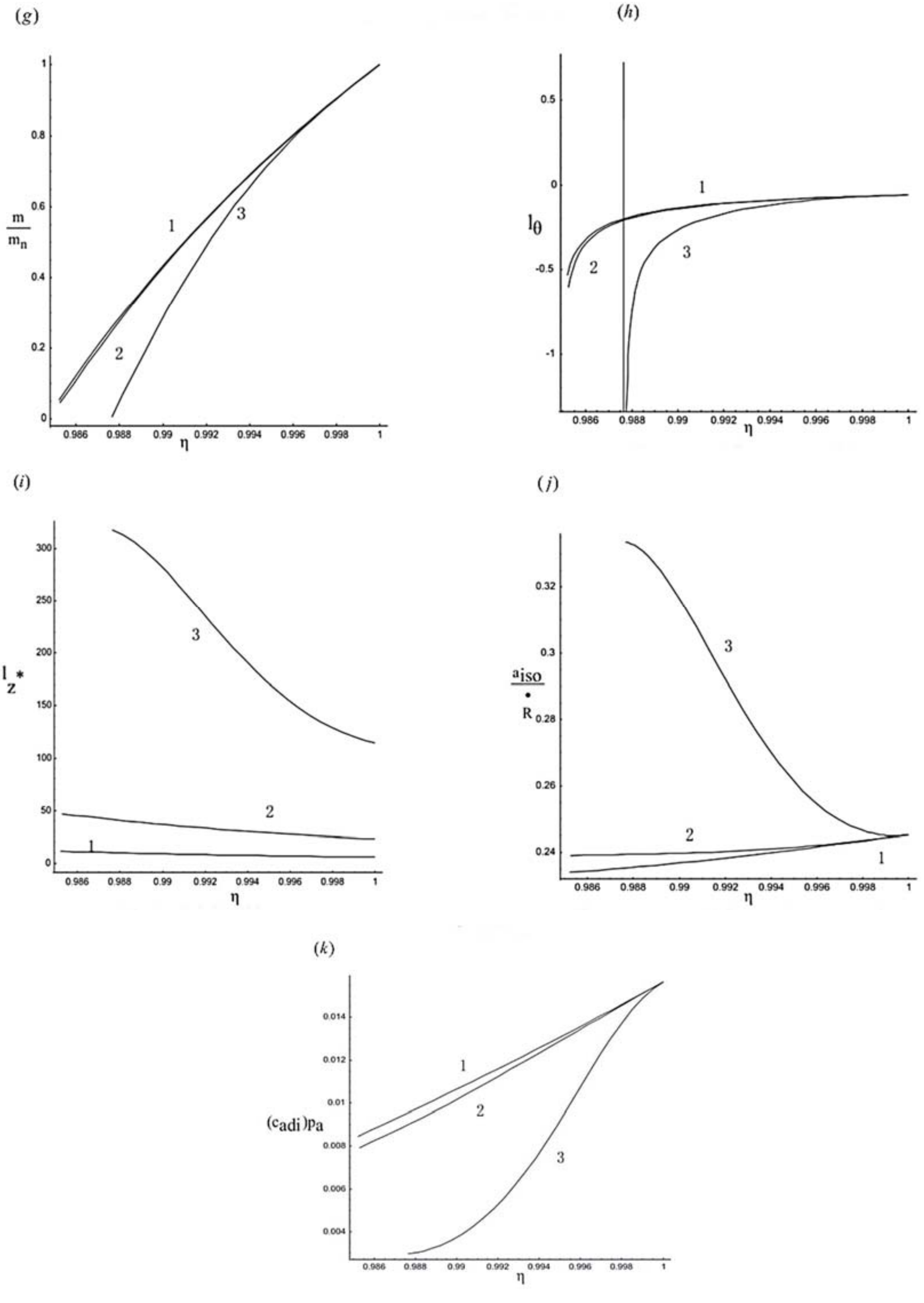

Fig. 1. Variation of the flow variables (g) mass (h) azimuthal component of vorticity vector (i) axial component of vorticity vector (j) isothermal speed of sound (k) adiabatic compressibility, in the regionbehind the shock front in case of $K_p = 0.2$, $G_a = 50$, $= 0.002$, $\Gamma_c = 0.5$, $\Gamma_R = 10$, $\frac{E_*}{Q} = 0.005$; 1. $G_0 = 0.01$; 2. $G_0 = 0.94$; 3. $G_0 = 24.94$.

and the shock strength remain constant.

It is found that the effects of an increase in the value of radiative heat transfer parameter Γ_R on flow variables are

(i) decrease in the flow variables u/u_n, ρ/ρ_n, p/p_n, F/F_n, l_{z*},

(ii) increase in the flow variables v/v_n, w/w_n, m/m_n, l_θ, $a_{iso}/\dot{R}$, $(C_{adi})p_a$,

(iii) increase in the distance of the piston from the shock front (see Table 3),

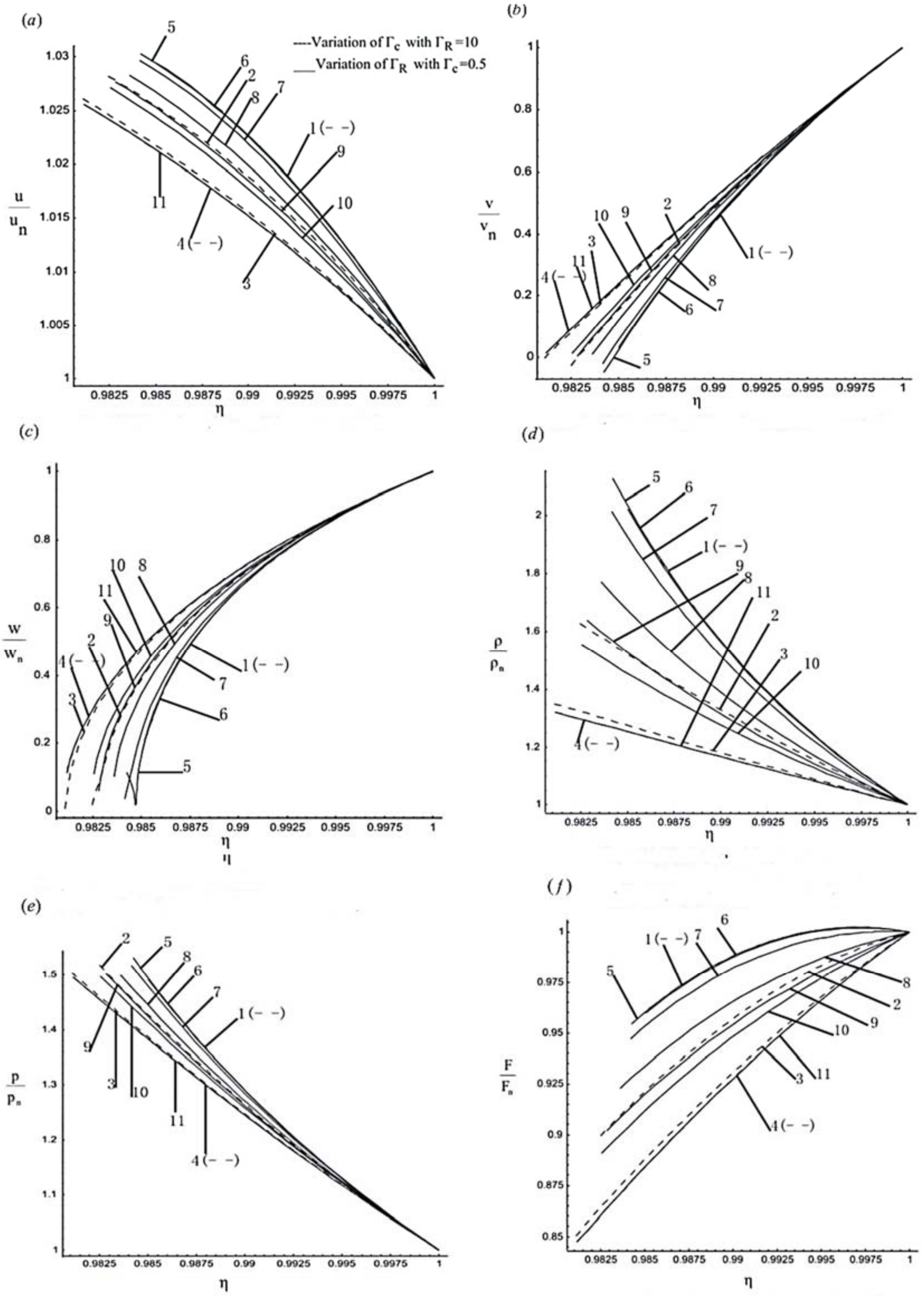

Fig. 2. Variation of the flow variables (a) radial component of fluid velocity (b) azimuthal component of fluid velocity (c) axial component of fluid velocity (d) density (e) pressure (f) total heat flux, in the region behind the shock front in case of $K_p = 0.2$, $G_a = 50$, $\bar{b} = 0.002$, $G_0 = 0.01$; 1. $\Gamma_c = 0.5$, $\Gamma_R = 10$; 2. $\Gamma_c = 1$, $\Gamma_R = 10$; 3. $\Gamma_c = 10$, $\Gamma_R = 10$; 4. $\Gamma_c = \infty$, $\Gamma_R = 10$; 5. $\Gamma_c = 0.5$, $\Gamma_R = 0.5$; 6. $\Gamma_c = 0.5$, $\Gamma_R = 100$; 7. $\Gamma_c = 0.5$, $\Gamma_R = 1000$; 8. $\Gamma_c = 0.5$, $\Gamma_R = 5000$; 9. $\Gamma_c = 0.5$, $\Gamma_R = 10000$; 10. $\Gamma_c = 0.5$, $\Gamma_R = 15000$; 11. $\Gamma_c = 0.5$, $\Gamma_R = \infty$.

and the shock strength remain constant.

The effects of variation of gravitation parameter G_0 on all the three component of velocity of fluid velocity and other flow variables behind the shock front is studied (Fig. 1(a)-1(k)). It is found that the effect of conduction and radiation heat parameter on the flow variables is modified due to the presence of

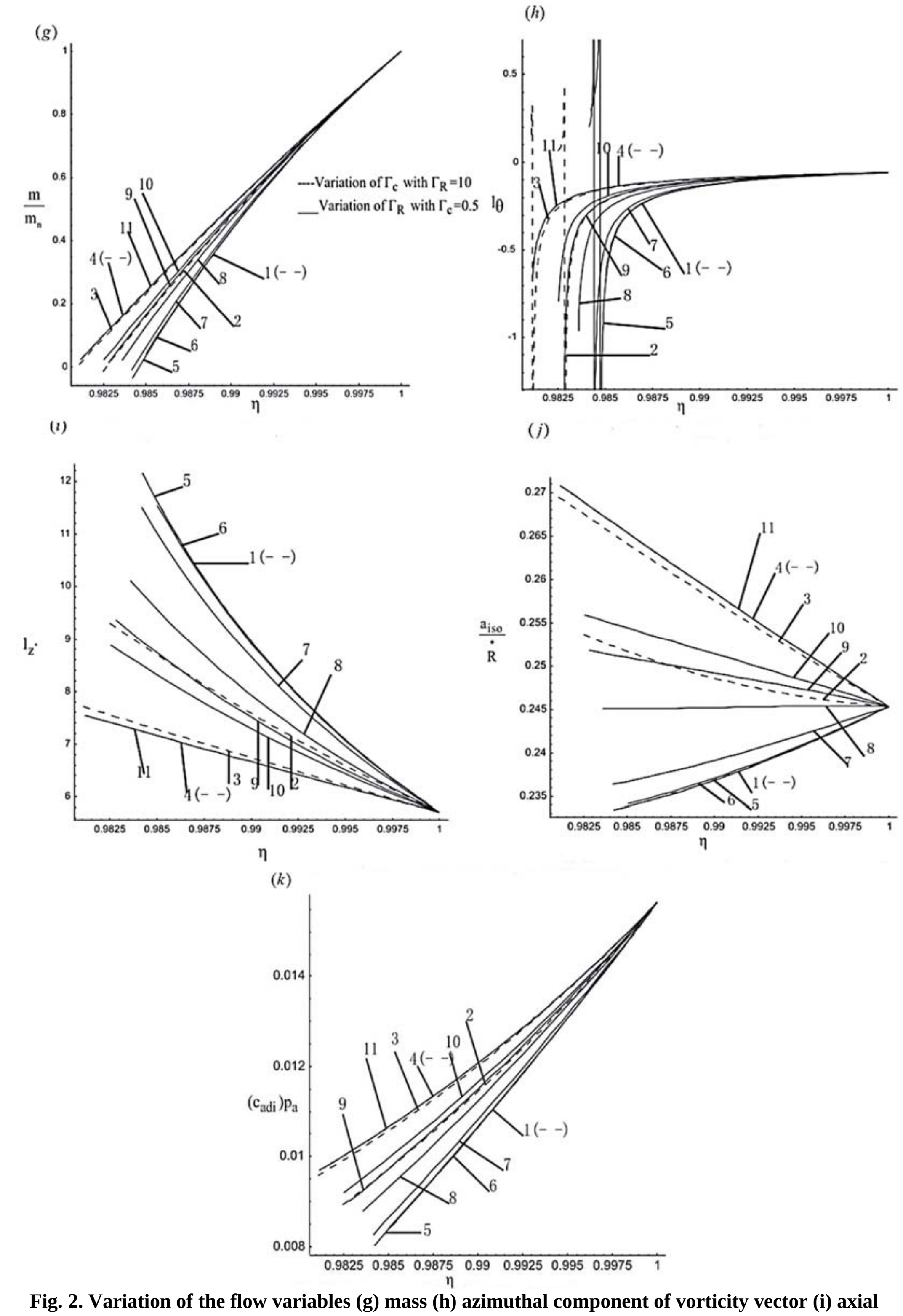

Fig. 2. Variation of the flow variables (g) mass (h) azimuthal component of vorticity vector (i) axial component of vorticity vector (j) isothermal speed of sound (k) adiabatic compressibility, in the region behind the shock front in case *of $K_p = 0.2$, $G_a = 50$, $\bar{b} = 0.002$, $G_0 = 0.01$; 1. $\Gamma_c = 0.5$, $\Gamma_R = 10$; 2. $\Gamma_c = 1$, $\Gamma_R = 10$; 3. $\Gamma_c = 10$, $\Gamma_R = 10$; 4. $\Gamma_c = \infty$, $\Gamma_R = 10$; 5. $\Gamma_c = 0.5$, $\Gamma_R = 0.5$; 6. $\Gamma_c = 0.5$, $\Gamma_R = 100$; 7. $\Gamma_c = 0.5$, $\Gamma_R = 1000$; 8. $\Gamma_c = 0.5$, $\Gamma_R = 5000$; 9. $\Gamma_c = 0.5$, $\Gamma_R = 10000$; 10. $\Gamma_c = 0.5$, $\Gamma_R = 15000$; 11. $\Gamma_c = 0.5$, $\Gamma_R = \infty$.*

gravitation parameter. This is justified by the complete change in the variation of l_θ and partial change in the variation of F/F_n, $(C_{adi})p_a$, l_{z*} and $a_{iso}/R^{\cdot}$ etc in comparison to the corresponding variation of the flow variables in Vishwakarma and Nath(2012).

Table 3 Variation of the position of the piston η_p for different values of Γ_R with K_p = 0.2, G_a = 50, $\bar{b}$= 0.02, G_0 = 0.01, and Γ_c = 0.5

Γ_R	β	$1-\beta=\frac{u_n}{R}$	η_p
0.5	0.043774	0.956226	0.985217
10	0.043774	0.956226	0.985198
100	0.043774	0.956226	0.985013
1000	0.043774	0.956226	0.984154
5000	0.043774	0.956226	0.98358
10000	0.043774	0.956226	0.982818
15000	0.043774	0.956226	0.982539
∞	0.043774	0.956226	0.981149

5. CONCLUSIONS

In this paper, similarity solutions are obtained for a cylindrical shock wave in a self gravitating, rotat-ing, axisymmetric dusty gas with heat conduction and radiation heat flux. Some of the important con-clusions are:

(i) The effect of gravitation parameter is studied on all the three components of fluid velocitywhich was not done earlier.

(ii) The presence of gravitational field modify the effect of heat conduction and radiation field on propagation of shock wave significantly.

(iii) The similarity solution of the present problem exists only when the initial angular velocity of the medium is constant (since $\lambda = 1$).

(iv) The total energy of the flow field behind the shock wave is not constant but varies as an ex-ponential law with time. It is also dependent on the shock radius and the gravitation param-eter.

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