Modified Model for Binary Nanofluid Convection with Initial Constant Nanoparticle Volume Fraction

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ABSTRACT

A modified model considering effects of density as well as conductivity of nanoparticles is used to investigate the instability of a binary nanofluid layer. It is assumed that volume fraction of nanoparticles is small and remains constant at the initial state which leads to very interesting and useful results. The perturbed equations so found are analyzed using normal modes and weighted residual method. It is found that oscillatory motions are not possible and instability is invariably through stationary mode. After solving the problem analytically, numerical solutions are found for metallic (aluminium, copper, silver, iron) and non-metallic (alumina, silica, titanium oxide, copper oxide) nanoparticles using the software Mathematica. The effects of size of nanoparticles, difference in solute concentration, volume fraction of nanoparticles, difference in temperature, conductivity and density of nanoparticles are studied on the onset of convection. The increase in density of nanoparticles destabilizes the fluid layer system where as increase in conductivity stabilizes the same. Lower density of aluminium makes it more stable than other nanoparticles in spite of having its lower conductivity. Metals are largely more stable than non-metals.

NOMENCLATURE

c	nanofluid specific heat		p	pressure
C	solute concentration		s	growth rate
C_0	solute concentration at the upper layer		t	time
C_1	solute concentration at the lower layer		T	temperature
			T_0	temperature at the upper layer
d	depth of the layer		T_1	temperature at the lower layer
d_p	nanoparticle's diameter		v	fluid velocity defined as
D_B	Brownian diffusion coefficient			$v = (u_1, u_2, u_3)$
D_{CT}	diffusivity of Soret type			
D_S	solutal diffusivity		*Subscripts*	
D_T	thermophoretic diffusion coefficient		i	initial solution
D_{TC}	diffusivity of Dufour type		p	nanoparticle
g	acceleration due to gravity		**Superscripts**	
k	thermal conductivity		$*$	dimensional variable
k_B	Boltzmann's constant		$\sim$	perturbed variable
k_x	wave number in x-direction		**Greek symbols**	
k_y	wave number in y-direction		α	wave number
			α_f	thermal diffusivity of fluid

β_C	solutal volumetric coefficient	L_n	nanofluid Lewis number
β_T	thermal volumetric coefficient	N_B	particle density increment
μ	viscosity of the nanofluid	N_A	diffusivity ratio
ρ	density of the nanofluid	N_{CT}	Soret parameter
ρ_0	fluid density at reference temperature T_0	N_{TC}	Dufour parameter
ρc	heat capacity	P_r	Prandtl number
ϕ	nanoparticle volume fraction	R_m	basic-density Rayleigh number
ϕ_b	nanoparticle volume fraction at the initial state	R_n	nanoparticle Rayleigh number
		R_s	solute Rayleigh number
ω	dimensionless frequency	R_A	thermal Rayleigh number

Non-dimensional parameters

L_e solute Lewis number

1. INTRODUCTION

Low thermal conductivity of fluids limit the compactness and effectiveness of heat exchange equipments. Many techniques are used to enhance the heat transfer in fluids which can be classified into two categories: passive and active. Passive techniques employ fluid additives and special surface geometries by using coated, rough and extended surfaces, displaced insert devices, swirl flow for enhancement. Active techniques require external power such as electric or acoustic fields and surface vibration. An innovative way for improving the thermal conductivity of fluids is to suspend the nanoparticles was revealed by Choi (1995) and this visualized the concept of nanofluids. To analyze the convective heat transfer in nanofluids, partial differential equations based on conservation laws for nanofluids were given by Buongiorno (2006). Using this model, Nield and Kuznetsov (2009, 2010) studied the thermal convection problem in a nanofluid layer assuming that the nanoparticle volume fraction is constant along the walls. Their work is carried forward by many researchers. Gupta *et al*. (2013) and Agarwal *et al*. (2011) studied nanofluid convection analytically as well numerically with additional parameter of magnetic field and rotation respectively. Comparative numerical study for bio-nanofluid transport phenomena was conducted by Anwar Beg *et al*. (2014) employing a single-phase model and three different two-phase models, using the finite volume method. Garoosi *et al*. (2015a) analyzed numerically the steady state mixed convection flow of nanofluids for the model of two-phase mixture using finite volume method. The effects of various parameters on flow structure were investigated. Natural convection of nanofluids in a cavity is studied using a Buongiorno model by Garoosi *et al*. (2015b) and found that the heat transfer rate increases by reducing the diameter of the nanoparticles. Seth *et al*. (2016) studied the convective flow of magnetohydrodynamic viscoelastic nanofluid under the presence of thermal and nanoparticles buoyancy forces. Relevant studies on various nanofluid convection problems have been investigated by Sheikholeslami *et al*. (2016), Yadav and Lee, (2016), and Seth and Mishra (2017).

Binary nanofluid convection in a fluid layer was considered by Nield and Kuznetsov (2011) which is heated and soluted from below. Their work is extended by Gupta *et al*. (2014) and Yadav *et al*. (2012) by investigating the convection in a binary nanofluid layer for alumina-water nanofluid. The influence of magnetic field and rotation on double diffusive nanofluid convection was studied in non-porous and porous medium by Gupta *et al*. (2015) and Sharma *et al*. (2016) respectively, using different nanofluid parameter values. The natural convection flow of a viscoelastic heat radiating nanofluid over a linearly stretching sheet in the presence of uniform transverse magnetic field with Dufour and Soret effects was investigated by Seth *et al*. (2017). Numerical results obtained were compared with earlier published results and were found to be in agreement.

Nield and Kuznetsov (2014a,b) introduced a revised model for nanofluid convection problem in which nanoparticle flux at the boundaries is assumed to be zero instead of constant nanoparticle volume fraction. It was shown that oscillatory motions can no longer occur and presence of nanoparticles destabilizes the fluid layer. Further, Chand and Rana (2015) and Agarwal (2014) studied the instability of binary nanofluid layer under the influence of magnetic field and rotation respectively, using revised model. The influence of nanofluid parameters on the onset of convection has been presented numerically. The models/revised model used so far are not able to show the effect of conductivity of nanoparticles present in the fluid layer on the variation of thermal Rayleigh number. The present model uses both the physical properties of the nanoparticles (density and conductivity) for metallic and non-metallic nanoparticles and the effect of solute on it. The nanoparticle volume fraction is small and it is assumed to be constant at the initial state instead of varying in vertical direction. After applying the small perturbations on the initial flow, eigenvalue equation is obtained using normal modes. The modified model introduced here very efficiently shows the behavior of metallic (aluminium, copper, silver, iron) and non-metallic (alumina, silica, titanium oxide, copper oxide) nanoparticles. Numerical computations are carried out to show how aluminium nanoparticles in spite of lower

conductivity as compared to silver and copper nanoparticles make the system more stable. The reason lies in its low density as compared to other two nanoparticles. Metallic nanofluids are generally found to be more stable with higher values of thermal Rayleigh number as compared to metal oxides except metals like iron with sufficiently low thermal conductivity. The earlier known models (Nield and Kuznetsov (2010, 2014b)) do not account for these behaviors as these models do not provide the influence of diffusivity ratio (conductivity of nanoparticles) on the instability of fluid. The problem under consideration is solved analytically using normal modes and one term Galerkin method The impact of size of nanoparticles, difference in solute concentration, volume fraction of nanoparticles, temperature in the layer and solute Lewis number are found numerically for metallic and non-metallic nanoparticles using the software Mathematica.

2. GOVERNING EQUATIONS

A binary nanofluid layer is considered as shown in Fig.1. The conservation equations for binary nanofluid convection (refer: Buongiorno (2006), Nield and Kuznetsov (2011)) are
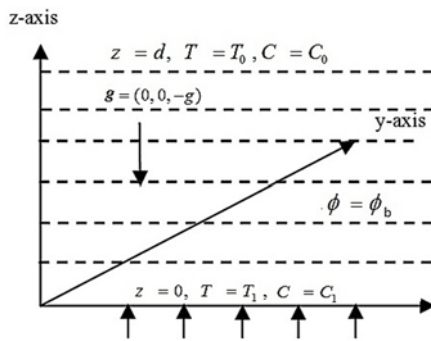

Fig. 1. Sketch of the physical system.

$$\nabla \cdot \boldsymbol{v} = 0 \tag{1}$$

$$\rho_0\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v}\right) = -\nabla p + \mu \nabla^2 \boldsymbol{v} +$$
$$\left[\phi \rho_p + (1-\phi)\left\{\rho_0\left(1-\beta_T(T-T_0)\right) - \beta_C(C-C_0)\right\}\right]\boldsymbol{g}, \tag{2}$$

$$\rho c\left[\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T\right] = k\nabla^2 T + \rho_p c_p$$
$$\left[D_B \nabla \phi \cdot \nabla T + \frac{D_T}{T_0}\nabla T \cdot \nabla T\right] + \rho c D_{TC} \nabla^2 C, \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \boldsymbol{v} \cdot \nabla \phi = D_B \nabla^2 \phi + D_T \nabla^2 \frac{T}{T_0}, \tag{4}$$

$$\frac{\partial C}{\partial t} + \boldsymbol{v} \cdot \nabla C = D_S \nabla^2 C + D_{CT} \nabla^2 T. \tag{5}$$

Let us write variables as

$$\left(x^*, y^*, z^*\right) = \frac{(x,y,z)}{d}, \quad t^* = \frac{t\alpha_f}{d^2}, \quad \boldsymbol{v}^* = \frac{\boldsymbol{v}\,d}{\alpha_f},$$

$$p^* = \frac{pd^2}{\mu\alpha_f}, \quad \phi^* = \frac{\phi}{\phi_b}, \quad T^* = \frac{T-T_0}{T_1-T_0}, \quad C^* = \frac{C-C_0}{C_1-C_0},$$

where $\alpha_f = \dfrac{k}{\rho C}$. $\tag{6}$

Using Eqs. (6), Eqs. (1)-(5) (after dropping the asterisks) are

$$\nabla \cdot \boldsymbol{v} = 0, \tag{7}$$

$$\frac{1}{Pr}\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v}\right) = -\nabla p + \nabla^2 \boldsymbol{v} - R_m \hat{k} + R_A T\hat{k} - R_n \phi \hat{k}$$
$$+\frac{R_s}{L_e}C\hat{k}, \tag{8}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T = \nabla^2 T + \frac{N_B}{L_n}\nabla\phi \cdot \nabla T$$
$$+\frac{N_A N_B}{L_n}\nabla T \cdot \nabla T + N_{TC}\nabla^2 C, \tag{9}$$

$$\frac{\partial C}{\partial t} + \boldsymbol{v} \cdot \nabla C = \frac{1}{L_e}\nabla^2 C + N_{CT}\nabla^2 T, \tag{10}$$

$$\frac{\partial \phi}{\partial t} + \boldsymbol{v} \cdot \nabla \phi = \frac{1}{L_n}\nabla^2 \phi + \frac{N_A}{L_n}\nabla^2 T, \tag{11}$$

where

$$Pr = \frac{\mu}{\rho\alpha_f}; \quad L_n = \frac{\alpha_f}{D_B}; \quad L_e = \frac{\alpha_f}{D_S};$$

$$R_A = \frac{\rho\,\boldsymbol{g}\,\beta_T d^3(T_1-T_0)}{\mu\alpha_f}; \quad R_s = \frac{\rho\,\boldsymbol{g}\,\beta_C d^3(C_1-C_0)}{\mu D_s};$$

$$R_n = \frac{\left(\rho_p - \rho\right)\phi_b\,\boldsymbol{g}\,d^3}{\mu\alpha_f}; \quad R_m = \frac{\rho\boldsymbol{g}d^3}{\mu\alpha_f};$$

$$N_A = \frac{D_T(T_1-T_0)}{D_B T_0 \phi_b}; \quad N_B = \frac{(\rho C)_P}{\rho C}\phi_b;$$

$$N_{TC} = \frac{D_{TC}(C_1-C_0)}{\alpha_f(T_1-T_0)}; \quad N_{CT} = \frac{D_{CT}(T_1-T_0)}{\alpha_f(C_1-C_0)}. \tag{12}$$

3. PERTURBATIONS ON INITIAL FLOW

Initially, the fluid layer is at rest with constant nanoparticle volume fraction so the quantities: pressure, temperature and concentration of solute vary along z-axis only. We get initial solution of Eqs. (7)-(11) as

$$\boldsymbol{v}_i = 0, \ \phi_i = 1, \ T_i = C_i = 1-z. \tag{13}$$

At the initial state pressure can be found from Eq. (8) using Eqs. (13). Here, it is necessary to mention that to find initial solution of the problem we have used

the fact that for nanofluids, Lewis number is very large as compared to diffusivity ratio and particle density increment (refer Buongiorno (2006)). Let us write

$$(\mathbf{v}, p, T, C, \phi) = (\mathbf{v}_i + \tilde{\mathbf{v}}, p_i + \tilde{p}, T_i + \tilde{T}, C_i + \tilde{C}, \phi_i + \tilde{\phi}). \tag{14}$$

After using Eq. (14) in Eqs. (7)-(11), we get perturbation equations as

$$\nabla \cdot \tilde{\mathbf{v}} = 0, \tag{15}$$

$$\frac{1}{Pr}\frac{\partial \tilde{\mathbf{v}}}{\partial t} = -\nabla \tilde{p} + \nabla^2 \tilde{\mathbf{v}} + R_A \tilde{T}\hat{k} + \left(\frac{R_s}{L_e}\right)\tilde{C}\hat{k} - R_n\tilde{\phi}\hat{k} \tag{16}$$

$$\frac{\partial \tilde{T}}{\partial t} - \tilde{u}_3 = \nabla^2 \tilde{T} - \frac{N_B}{L_n}\frac{\partial \tilde{\phi}}{\partial z} - 2\frac{N_A N_B}{L_n}\frac{\partial \tilde{T}}{\partial z} + N_{TC}\nabla^2 \tilde{C}, \tag{17}$$

$$\frac{\partial \tilde{C}}{\partial t} - \tilde{u}_3 = \frac{1}{L_e}\nabla^2 \tilde{C} + N_{CT}\nabla^2 \tilde{T}, \tag{18}$$

$$\frac{\partial \tilde{\phi}}{\partial t} = \frac{1}{L_n}\nabla^2 \tilde{\phi} + \frac{N_A}{L_n}\nabla^2 \tilde{T}. \tag{19}$$

Making use of the identity $curlcurl \equiv graddiv - \nabla^2$ on Eq. (16) together with Eq. (15), we get

$$\frac{1}{Pr}\frac{\partial}{\partial t}\left(\nabla^2 \tilde{u}_3\right) - \nabla^4 \tilde{u}_3 = R_A \nabla_H^2 \tilde{T} + \left(\frac{R_s}{L_e}\right)\nabla_H^2 \tilde{C} - R_n \nabla_H^2 \tilde{\phi}, \tag{20}$$

where $\nabla_H^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$. Note that the seven variables $\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{p}, \tilde{T}, \tilde{C}, \tilde{\phi}$ are now reduced to four $\tilde{u}_3, \tilde{T}, \tilde{C}, \tilde{\phi}$.

4. EIGENVALUE EQUATION

Equations (17)-(20) are solved using method of normal modes. Let

$$\left(\tilde{u}_3, \tilde{T}, \tilde{C}, \tilde{\phi}\right) = \left(W(z), T(z), \Psi(z), \Phi(z)\right)$$
$$\exp(ik_x x + ik_y y + st), \tag{21}$$

Using Eq. (21) in the above mentioned set of equations, we get

$$\left(\left(D^2 - \alpha^2\right)^2 - \frac{s}{Pr}\left(D^2 - \alpha^2\right)\right)W - R_A \alpha^2 T -$$

$$\frac{R_s}{L_e}\alpha^2 \Psi + R_n \alpha^2 \Phi = 0, \tag{22}$$

$$W + \left(\left(D^2 - \alpha^2\right) - s - 2\frac{N_A N_B}{L_n}D\right)T - \frac{N_B}{L_n}D\Phi$$

$$+N_{TC}\left(D^2 - \alpha^2\right)\Psi = 0, \tag{23}$$

$$W + \left(\frac{1}{L_e}\left(D^2 - \alpha^2\right) - s\right)\Psi + N_{CT}\left(D^2 - \alpha^2\right)T = 0, \tag{24}$$

$$\left(\frac{1}{L_n}\left(D^2 - \alpha^2\right) - s\right)\Phi + \frac{N_A}{L_n}\left(D^2 - \alpha^2\right)T = 0, \tag{25}$$

Equations (22) and (24), after using Eq. (25) become

$$\left(\frac{1}{L_n}\left(D^2 - \alpha^2\right) - s\right)\left(\left(D^2 - \alpha^2\right)^2 - \frac{s}{Pr}\left(D^2 - \alpha^2\right)\right)W$$

$$-\left(\left(\frac{1}{L_n}\left(D^2 - \alpha^2\right) - s\right)R_A \alpha^2 + \frac{N_A R_n}{L_n}\alpha^2\left(D^2 - \alpha^2\right)\right)T$$

$$-\left(\frac{1}{L_n}\left(D^2 - \alpha^2\right) - s\right)\frac{R_s}{L_e}\alpha^2 \Psi = 0, \tag{26}$$

$$\left(\frac{1}{L_n}\left(D^2 - \alpha^2\right) - s\right)\left(D^2 - \alpha^2 - s - 2\frac{N_A N_B}{L_n}D\right)T$$

$$+\frac{N_A N_B}{L_n^2}\left(D^2 - \alpha^2\right)DT + N_{TC}\left(\frac{1}{L_n}\left(D^2 - \alpha^2\right) - s\right)$$

$$\left(D^2 - \alpha^2\right)\Psi = 0, \tag{27}$$

with $D \equiv \dfrac{d}{dz}, \alpha = (k_x^2 + k_y^2)^{1/2}. \tag{28}$

Equations (24), (26) and (27) are solved using one term Galerkin weighted residual method. For free-free boundaries:

$$W = D^2 W = T = \Psi = 0 \text{ at } z = 0 \text{ and } z = 1. \tag{29}$$

We write

$$(W, T, \Psi) = (A, B, C)\sin \pi z, \tag{30}$$

satisfying boundary conditions given by Eq. (29) and using orthogonality to the functions; gives eigenvalue equation as

$$\alpha^2 \frac{R_s}{L_e}(J + s)\left(\frac{J}{L_n} + s\right) - (J + s)\left(\frac{J}{L_n} + s\right)$$

$$\left(J^2 + \frac{Js}{Pr}\right) - \alpha^2\left(\frac{J N_A R_n}{L_n} + R_A\left(\frac{J}{L_n} + s\right)\right) - \alpha^2 J$$

$$\left(\frac{J N_A R_n}{L_n} + R_A\left(\frac{J}{L_n} + s\right)\right)N_{TC}\left(\frac{J}{L_e} + s\right) - J N_{CT}$$

$$\left(\alpha^2 \frac{R_s}{L_e}\left(\frac{J}{L_n} + s\right) - J\left(\frac{J}{L_n} + s\right)\left(J^2 + \frac{Js}{Pr}\right)\right)N_{TC} = 0, \tag{31}$$

where $J = \pi^2 + \alpha^2$.

5. RESULTS AND DISCUSSIONS

5.1 Stationary Convection

For non-oscillatory motions $s = 0$, this gives the expression for R_A from Eq. (31) as

$$R_A = \frac{J^3\left(1 - N_{CT}N_{TC}L_e\right) - \alpha^2 R_s\left(1 - N_{CT}\right)}{\alpha^2\left(1 - L_e N_{TC}\right)} - R_n N_A. \tag{32}$$

Note that non-dimensional parameters; Prandtl number and nanofluid Lewis number do not appear in Eq. (32). Also, we have two nanofluid parameters R_n and N_A which enter in the expression (32) in the product form. Thus the value of thermal Rayleigh number is decreased with the presence of nanoparticles in the system. Letting $\alpha^2 = \pi^2 x$, Eq. (32) becomes:

$$R_A = \pi^4 \frac{(1+x)^3 \left(1 - N_{CT} N_{TC} L_e\right) - x R_s \left(1 - N_{CT}\right)}{x \left(1 - L_e N_{TC}\right)}$$
$$- R_n N_A. \qquad (33)$$

The minimum value of R_A is attained at $x = 1/2$ ($\alpha = \pi/\sqrt{2}$) which is independent of solute and nanofluid parameters. Using Eqs. (12), let us write

$$R_n N_A = \frac{D_T \left(\rho_p - \rho\right)(T_1 - T_0)\boldsymbol{g}\, d^3}{D_B \mu \alpha_f T_0} \qquad (34)$$

where $D_B = \dfrac{k_B T}{3\pi\mu d_p}$ and $D_T = \tilde{\beta}\dfrac{\mu}{\rho}\phi$ with

$\tilde{\beta} = 0.26\dfrac{k}{2k + k_p}$ as given by Nield and Kuznetsov

(2010). Keeping in mind the fact that nanoparticle volume fraction is so small that the properties of fluid will not get affected by addition of nanoparticles and are assumed to coincide with base fluid properties, we write

$$R_n N_A = \frac{\left(\rho_p - \rho\right)}{2k + k_p} A; \qquad (35)$$

where A is a constant which is independent of the physical properties of nanoparticles. Thus there are two physical properties of the nanoparticles; density and conductivity which appear in the expression of R_A and hence influence the stability of the fluid directly. Note that for fixed conductivity of nanoparticles, density of nanoparticles destabilizes the fluid where as for fixed density of nanoparticles, conductivity stabilizes it.

5.2 Oscillatory Convection

Oscillatory motions are possible if buoyancy forces are in opposite directions which is not the case with the present formulation. Let us verify the absence of oscillatory motions. For the mode of instability occurring through oscillatory convection: $s = i\omega \neq 0$. Comparing real and imaginary parts of Eq. (31), we get complex expressions as triple diffusion problem is much more complicated. To study the problem analytically, let us make approximations: $L_e = 1$, N_{CT}, $N_{TC} \to 0$ and Pr, $L_n \to \infty$ which give real and imaginary parts as

$$R_A \alpha^2 = 2 J^3 - R_s \alpha^2, \qquad (36)$$

$$\omega^2 + J^2 = 0. \qquad (37)$$

For oscillatory convection, ω must be real which is not possible and hence convection through oscillations are not possible as expected.

5.3 Validation of Results and Efficacy of Modified Model

For binary convection in the absence of nanoparticles, expression (32) for thermal Rayleigh number becomes

$$R_A = \frac{J^3 \left(1 - N_{CT} N_{TC} L_e\right) - \alpha^2 R_s \left(1 - N_{CT}\right)}{\alpha^2 \left(1 - L_e N_{TC}\right)}. \qquad (38)$$

which coincides with the expression given by Nield and Kuznetsov (2011) and Gupta *et al.* (2015) for thermal convection. Note that oscillatory mode of heat transfer is not possible as is the case discussed in Nield and Kuznetsov (2014a,b). The thermal Rayleigh number as given by Nield and Kuznetsov (2010, 2011, 2014b) doesn't give due effect of diffusivity ratio (Lewis number is large as compared to diffusivity ratio). Hence conductivity of the nanoparticles does not show any impact on the stability of the system in their model under consideration.

This modified model gives the expression for R_A as

$$R_A = \frac{J^3 \left(1 - N_{CT} N_{TC} L_e\right) - \alpha^2 R_s \left(1 - N_{CT}\right)}{\alpha^2 \left(1 - L_e N_{TC}\right)} - R_n N_A. \qquad (39)$$

This expression strongly depends on R_n as well as N_A and hence both the physical properties (density and conductivity) contribute towards deciding the stability of the system. In the literature no one has revised the model in the light of the fact that conductivity must also influence the stability of the fluid layer. Thus the present modified model seems to be more realistic than earlier defined models.

6. NUMERICAL RESULTS AND DISCUSSIONS

Numerical computations are carried out using Eq. (32) for metallic (Al, Cu, Ag, Fe) and non-metallic (Al$_2$O$_3$, SiO$_2$, CuO, TiO$_2$) nanoparticles in water based nanofluids using the software Mathematica. The increase in the values of thermal Rayleigh number for an increase in values of a particular parameter establishes stabilizing effect of that parameter on the system while decrease in values of Rayleigh number with an increase in values of parameter establishes the destabilizing effect. Table 1 shows the physical properties of nanoparticles under consideration. The values of nanofluid parameters (R_n and N_A) appearing in Eq. (32) are calculated using Table 1 and Buongiorno (2006) for fixed depth of the layer and are written in Table 2. Note that values of nanoparticle Rayleigh number depend on depth of the fluid layer and their higher values lead to negative values of thermal Rayleigh number and hence system becomes unstable. The value of diffusivity ratio for typical nanofluid is fixed. Further, metallic and non-metallic

Table 1 Physical characteristics of water and metallic/non-metallic nanoparticles under consideration. (refer: Turkyilmazoglu, M. (2012), Jang, S.P. and S.U.S Choi (2007))

Physical Properties	Water	Al	Cu	Ag	Fe	Al$_2$O$_3$	SiO$_2$	CuO	TiO$_2$
$\rho\,(Kg\,/\,m^3)$	997.1	2700	9000	10500	7900	3970	2600	6510	4250
k (W/ mK)	0.613	237	401	429	80	40	10.4	18	8.9

Table 2 Nanofluid parameters for metallic/non-metallic nanoparticles in water based nanofluids under consideration.(refer: Table 1 and Buoingiorno (2006))

Nanofluid Parameters	Al	Cu	Ag	Fe	Al$_2$O$_3$	SiO$_2$	CuO	TiO$_2$
R_n	17.17	80.13	95.95	69.69	30	16.15	55.65	32.82
N_A	1.6	0.5	0.5	2.64	5	17.5	10.83	20

nanoparticles have been considered with the existing data available in the literature. We fix solute parameters as: $L_e = 2$, $R_s = 50$, $N_{CT} = 2$, $N_{TC} = 0.01$.

Figure 2 shows the influence of non-metallic nanoparticles on water based binary nanofluid system which establishes the stability pattern as: alumina-water> silica-water > > copper oxide-water > titanium oxide-water. The stability of the system for alumina and silica is much higher than copper oxide and titanium oxide The reason may be due to the higher densities of oxides of copper and titanium.
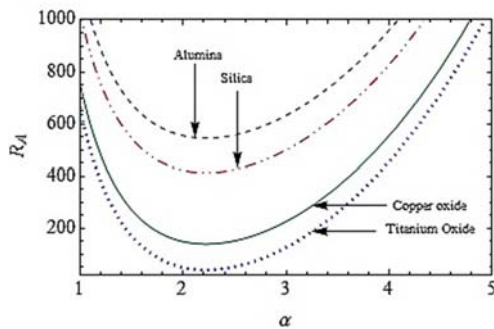

Fig. 2. Variation in R_A for metallic oxide nanoparticles.

The impact of metals on the stability of binary system is illustrated in Fig. 3. The stability pattern followed by metals under consideration is: aluminium-water > copper-water > silver-water >> iron-water. Instability of binary nanofluid with iron nanoparticles is much higher as compared to other metals due to its lesser conductivity. Interestingly, aluminium metal has highest stability as compared to other nanoparticles under consideration because of its balanced values of density and conductivity. Also, note that metallic nanoparticles make the system more stable than metal oxides except the metals like iron with lower thermal conductivity.
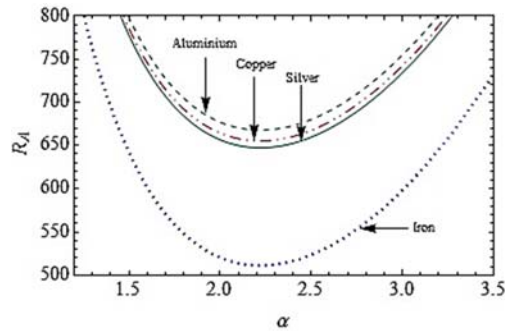

Fig. 3. Variation in R_A for metallic nanoparticles.

Figure 4 depicts the effect of nanoparticles diameter on the stability for alumina and copper nanoparticles. The increase in size of nanoparticles, entering in the system through D_B and hence N_A, destabilizes the system. This destabilizing influence is more for alumina than copper due to higher conductivity of copper nanoparticles. Further, the destabilizing impact of nanoparticle volume fraction (appears through R_n) on the fluid layer is shown in Fig. 5 and this destabilizing influence is more pronounced for alumina-water nanofluid due to its lower density as compared to copper nanoparticles.
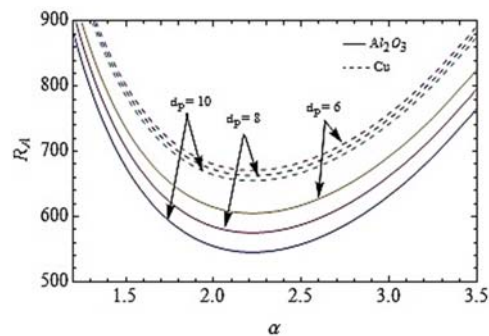

Fig. 4. Variation in R_A for size of nanoparticles.

Note that the solute concentration difference is entering in the expression for Rayleigh number through solute parameters (R_s, N_{CT}, N_{TC}). Fig. 6 shows the destabilizing effect of difference in solute concentration in the fluid layer. The difference in temperature at the boundaries affects the convection in the layer through two solute parameters (N_{CT}, N_{TC}) and one nanofluid parameter (N_A). Temperature difference stabilizes the system for metals and destabilizes for metal oxides as shown in Fig. 7. The reason for the contrast behavior is the higher conductivity of metals which leads to lower value of N_A.
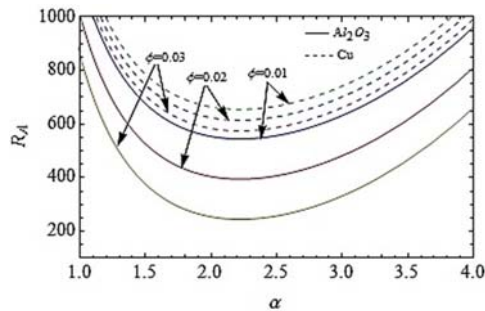
Fig. 5. Variation in R_A for nanoparticle volume fraction.
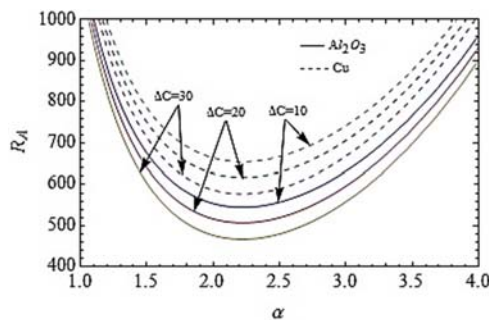
Fig. 6. Variation in R_A for solute difference.
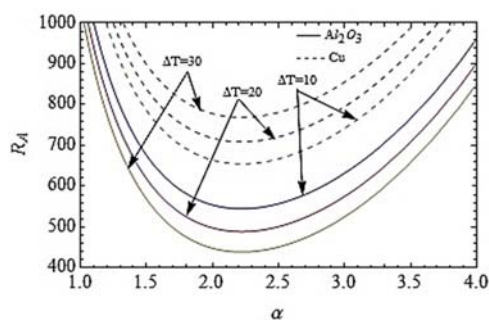

Fig. 7. Variation in R_A for temperature difference.

To show the importance and significance of present model compared to the original model which fails to account for the effects of thermal conductivity on thermal Rayleigh number, influence of physical properties (conductivity and density) of

nanoparticles on the instability of the system are depicted in Figs. 8 and 9. Note that conductivity and density of nanoparticles affect the system through non-dimensional numbers N_A and R_n, respectively. The value of N_A decreases with an increase in conductivity and higher value of density of nanoparticles leads to higher R_n. The stabilizing impact of conductivity is shown in Fig. 8 while destabilizing influence of density is depicted in Fig. 9 which also validates the analytical results found in section 5.3.
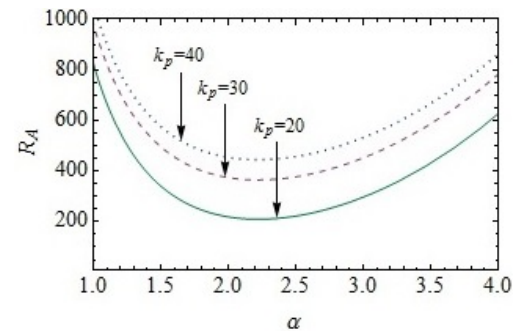

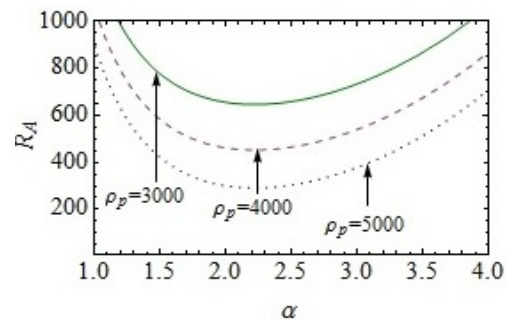**Fig. 8. Variation in R_A for nanoparticle conductivity.**

Fig. 9. Variation in R_A for nanoparticle density.

7. CONCLUSIONS

Modified model incorporating differential conductivity effects of nanoparticles is used to investigate the instability of a binary nanofluid layer analytically and numerically. The partial differential equations based on conservation laws are translated into an eigenvalue problem. Nanoparticle volume fraction is assumed to be constant in the initial state which gives the Rayleigh number R_A that strongly depends on R_n as well as N_A and hence both the physical properties (density and conductivity) contribute towards deciding the stability of the system. Expressions (36) and (37) are derived using valid approximations for considering the possibility of oscillatory convection. It is clear that oscillations are not possible and hence instability of the layer is through stationary mode only. The critical wave number is found to be independent of the presence of nanoparticles and solute in the fluid. The density of nanoparticles destabilizes the fluid where as conductivity stabilizes it. Numerical computations

are carried out using metallic and non-metallic nanoparticles. The stability pattern followed by non-metals is: alumina-water> silica-water > > copper oxide-water > titanium oxide-water and metals is: aluminium-water > copper-water > silver-water >> iron-water. Aluminium nanoparticles in spite of their lower conductivity as compared to silver and copper nanoparticles make the system more stable in water based nanofluids because of its lower density. Metallic nanoparticles stabilize nanofluids more than non-metallic nanoparticles except for the metals like iron with low thermal conductivity. The destabilizing influence of size of nanoparticles and volume fraction of nanoparticles is more pronounced for alumina nanoparticles than copper due to its lower density and lower conductivity. Also, difference in solute concentration destabilizes the layer for alumina and copper nanoparticles at the same rate. Temperature difference destabilizes the system for metal oxides and stabilizes for metals due to higher thermal conductivity of metals.

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