Journal of Applied Fluid Mechanics, Vol. 14, No. 3, pp. 681-690, 2021.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.47176/jafm.14.03.31815

Stability of Solids in Stepped Flume Nappe Flows: Subsidies for Human Stability in Flows

H. de B. Ribeiro[1], A. L. A. Simões[1†], L. D. da Luz[1], L. S. G. Mangieri[2] and H. E. Schulz[3,4]

[1] *Federal University of Bahia, Salvador, Bahia, 40210-630, Brazil*
[2] *Salvador University, Salvador, Bahia, 41770-235, Brazil*
[3] *Hydro-Engineering Solutions (Hydro-LLC), Auburn, Alabama, 36830, USA*
[4] *São Paulo University, São Carlos, São Paulo, 13566-590, Brazil*

†Corresponding Author Email: andre.simoes@ufba.br

(Received May 12, 2020; accepted October 8, 2020)

ABSTRACT

Knowing the details of the interaction between people and runoff flows caused by heavy rainfall or by floods due for example by the rupture of reservoirs or dams is essential to prevent accidents with humans. There are information in the literature on the equilibrium capacity of individuals partially immersed in flows occurring in flat-bottomed channels, but there are many gaps regarding the use of urban draining staircases during the occurrence of rainfalls that generate runoff over their steps, and their impact on people. This study considered the effect of the flow on the stability of five obstacles positioned on one of the steps of a reduced model of a draining staircase. The results were used to calculate dimensionless parameters which involve the mass and height of the obstacle, the water density, critical depth of the flow and step height. These parameters were justified by a fundamental toppling and drag formulation, and good correlations between the obtained dimensionless parameters were obtained following adequate power laws. Comparisons between the data obtained in the present reduced model of staircase and literature data of flat bottom channels showed similar behaviors. Finally, a scaling procedure to compare results of different scales and situations was also presented. Excellent correlations using different literature data and those of the present study were obtained.

Keywords: Stepped chute; Draining staircases; Safety in floods; Stability of solids in flows.

NOMENCLATURE

A	frontal area
b	width of the block
C_d	drag coefficient
e	block thickness
F	force
Fr_t	Froude number
f, G, J	generic functions
g	acceleration due to gravity
h_1	water column
h_c	critical depth
H	block height
H^*	dimensionless block height
$H^*_{\max}$	maximum H^*
$H^*_{\min}$	minimum H^*
H^*_n	normalized $H^*=H^*/H^*_{max}$
Io	slope
l	step length
m	mass of the block
M^*	dimensionless mass
$M^*_{\max}$	maximum M^*
$M^*_{\min}$	minimum M^*
M^*_{n}	normalized M^*
P	weight of the block
q	unit discharge
R	correlation coefficient
Re_t	Reynolds number
s	step height
V	mean flow velocity
α	angle of attack
φ_i	algebraic coefficients (i=1,2,3)
λ_i	algebraic coefficients (i=1,2)
μ_i	algebraic coefficients (i=1,2,3)
θ	friction coefficient
ν	kinematic viscosity
ρ	density of water
ρ_s	obstacle density
σ	expoent
ω_i	algebraic coefficients (i=1,2,3)

1. INTRODUCTION

Urban flooding has the potential to drag people, causing loss of life or serious injury. There is a growing concern across the globe with the accelerated changes in the built environments (urban regions), which have waterproofed large areas of the urban soil surfaces and restricted the regions of water evacuation, thereby generating areas of flooding and rapid flow in environments frequented by humans. Examples of the literature which emphasize this concern are Wade *et al.* (2005), Wallingford (2006), Cox *et al.* (2010), Smith and Rahman (2016), Kvočka *et al.* (2016), Yao *et al.* (2017), Martínez-Gomariz *et al.* (2019), and Rezende *et al.* (2019). People partially immersed in flows may lose their balance for a number of overlapping reasons: i) reduced friction of the shoe or foot with the floor, ii) transfer of momentum from the flow to the human body, which may cause tipping and dragging and iii) fluctuation due to the distribution of pressure on the body (Archimedes buoyancy). There is also the possibility of collision with floating objects and the instability of the human balance due to the generation and release of vortexes and the forces caused by these detachments (evidenced by Simões *et al.* 2016).

Considering the direct interaction between runoff and the human being, Foster and Cox (1973) studied the instability caused in six male children aged 9 to 13 years, heights between 1.27 and 1.45 m and masses between 25 and 37 kg. The tests were conducted in a channel with a length of 6.0 m, width of 0.6 m and depth of the cross section of 0.75 m. The authors discussed several aspects that can lead to instability and commented that even water depths less than 30 cm can generate instability for velocities above 1.5 m/s.

Following the pioneering work of Foster and Cox (1973), the study of Abt *et al.* (1989) was conducted to clarify the ability of men, women and a monolith to resist the aforementioned destabilizing factors. Twenty volunteers participated in the study, aged between 19 and 54 years, having heights between 1.52 m and 1.83 m and masses between 40.9 kg and 91.4 kg. The authors used a rectangular channel 2.44 m wide, 61 m long and 1.22 m deep, for bottom slopes of Io = 0.005 m/m (0.5%) and Io = 0.015 m/m (1.5 %). As a result of their research, Abt *et al.* (1989) proposed a relationship between discharge per unit length $q = hV$, (the product of the flow depth and the average velocity of the flow), and the product between the person's mass, m, and his height, H. The monolith data resulted in lower values for hV when compared to those found for humans.

The literature of the area (see, for example, Cox *et al.* 2010) mentions the Japanese language study conducted by Takahashi *et al.* (1992), who carried out experiments with 3 male people, with heights between 1.63 and 1.84 m and masses between 63 and 73 kg. The experimental device consisted of placing the volunteer on a platform equipped with force meters, and which was fixed in a 50 m long and 20 m wide basin. The authors proposed a computational model for the instability of human beings in flows that uses human characteristics normalized with the height. Their results are used in the literature to compare with data from several other authors.

In the sense of brining more data and confidence for the already proposed relevant parameters, Karvonen *et al.* (2000) conducted a study in which seven participants aged between 17 and 60 years, having heights between 1.60 m and 1.95 m, and masses between 48 kg and 100 kg, were tested in a channel 130 m long and 11 m width. The authors analyzed their results using the products hV and Hm, therefore following the analysis conducted by Abt *el al.* (1989), having found lower hV values in relation to those of Abt *et al.* (1989) for humans, and greater in relation to the monolith. The results showed that more data and analyses are needed.

The relevance of the studies in this field was pointed by Jonkman (2005), who analyzed the deaths caused by flood events in the world between the years of 1975 and 2000, reporting that 1826 events were known, and that they killed more than 175,000 people. The author also mentioned that the cases of tsunamis, rupture of dams and storm tides can be even more catastrophic in terms of loss of life. Later Jonkman and Penning-Rowsell (2008) described a set of experiments done on full scale in the River Lea (England), which was possible due to a floodgate system and a flood relief channel. The experiments were carried out in a section with about 1% slope and a width of approximately 70 m. A healthy male person was used for the experiments, with a height of 1.70 m and a mass of 68.25 kg. Two experiments with the person standing and four experiments with the person walking were performed and the empirical and numerical results of these two conditions were discussed by Jonkman and Penning-Rowsell (2008). The authors presented calculations for dragging (sliding) and tipping (moment of force) and argued that limit values of the flow variables (for instability) depend on the mass of the person.

Intending to link the problem of human security in flows to physical conceptual basis, Cox *et al.* (2010) cited theoretical studies that explored different aspects of the problem of human instability in flows, presenting equations based on physical principles, and also evaluations conducted with computational models. In this sense, the authors cited the works of Keller and Mitsch (1993), Lind *et al.* (2004), Ramsbottom *et al.* (2004, 2006) and Ishigaki *et al.* (2005, 2008a and 2008b, 2009), which gave the basis for their arguments. Conceptual models were also presented by Milanesi *et al.* (2014), who proposed a risk classification, generating limit curves to be adopted as vulnerability criteria.

Introducing the geometrical complexity of the human body in laboratory scale, Xia *et al.* (2014) used a reduced model of a human being with 0.30 m in height and mass equal to 0.334 kg, partially immersed in a rectangular channel. The authors showed that the adopted reduced model resulted in lower hV values when compared to those by Abt *et al*. (1989) and Karvonen *et al*. (2000), and concluded that the different results are due to the people's ability to progressively adapt to adverse flow conditions, a reaction that evidently does not exist in the employed inanimade reduced model.

Evolving in the conceptual discussion of the problem, Arrighi (2016) and Arrighi *et al*. (2017) presented a perhaps more substantiated study of the hydrodynamics of pedestrians in flood regions. The authors used physical principles and dimensional analysis to present their results as dependent on the Froude number. The authors defined adequate mobility parameters for their analyses, which showed good correlations with the Froude number defined in terms of the mean velocity and flow depth.

Considering the previous results and discussions of the literature, Shu *et al*. (2016) presented arguments about the geometric, kinematic and dynamic similarities in the experimental study conducted with human models generated by three-dimensional printing. The authors analyzed their own and literature data, presenting an equation for tipping instability that, according to the authors, also best represents the literature data.

With the exception of studies by Arrighi (2016), Arrighi *et al*. (2017) and Shu *et al*. (2016), according to the conculted literature, the quantities used to characterize the problem are presented in dimensional form (that is, not normalized in the sense of allowing exploring scale effects). Simões *et al*. (2016) applied the Vaschy-Buckingham theorem in the sense to generalize the formulation of human instability in flows. The authors selected a functional involving the Reynolds number, Froude number, drag coefficients, aspect ratios, relative roughness, slope of background, and three non conventional dimensionless parameters: the first related to the individual's age, the second related to the individual's psychical interaction with the flood and the third related to the individual's mass. Literature data of Abt *et al*. (1989), Karvonen *et al*. (2000) and Xia *et al*. (2014) were used in the study, which involved a sensitivity analysis between the different parameters, enabling Simões *et al*. (2016) to obtain equations that correlate the mentioned dimensionless parameters. Good correlations were presented between those parameters indicated as the most relevant in the sensitivity analysis. The study was thus adequate to indicate the mentioned representative parameters, in which the Froude number better represented the flow information, a conclusion aligned with the indications of Arrighi (2016) and Arrighi *et al*. (2017), although the studies were conducted independently.

Regarding the flow of superficial rainwater (runoff) in built environments, it is worth mentioning the solutions for human locomotion by feet between nearby areas located on hillsides, as they occur for example in Salvador and Rio de Janeiro, two big Brazilian cities in Brazil. The usual solution is the use of staircases as a means to simplify and to help the locomotion on the hillsides. Because of their location, these staircases also serve as “not designed drainage channels” during the occurrence of floods. This condition induced the Brazilian architect João Filgueiras to study the problem and to conceive the so-called “draining staircases”, a concept presented in 1979, whose fundamental objective is to allow the simultaneous use of the stairs as water drainage and human locomotion. The water drains under the stairs, while the people use the upper surfaces. It allows the transit of people without entering into contact with potentially dangerous flows. The application of the concept was immediate, and the survey of Mangieri (2012) about the existing drainage stairs in the city of Salvador showed that four different models are used. However, no design methodologies were found for the adequate dimensioning of the drainage characteristics of the implemented stairs, or for the analyses of the stability of the users in flow conditions. As a matter of fact, the consulted literature for the present study showed that there are no studies directed to the human vulnerability or instability in the situation of flows over stairs. However, taking into account that the use of urban staircases is a historic option already perpetuated for dislocations in urban environments (see, for example, Taşke, 2002), and that flows over staircases in heavy rains are being more frequent due to the growing of the cities, involving even stairs of underground subway stations (see, for example, Compton *et al.* 2009; or Yu *et al.* 2019), this gap must be filled.

This work presents a methodology for studying the instability of adequate shaped solids in flows over staircases considering the results of a semi-empirical dimensionless formulation for the assessment of the stability of the obstacles subjected to these flows. The following specific objectives were established:

1) to build, test and use an experimental equipment suitable for this study;
2) to formulate mathematically the problem of stability and to check it with experimental results, introducing the empirical information in the conceptual formulation;
3) to analyze the equations between the dimensionless parameters for two distinct situations: considering the obstacle on the steps (data from the present study), and comparing the present data with literature results for flat surfaces.

2. Basic Dimensionless Formulation

Simões *et al*. (2016) presented a classification of different types of instability for human beings in flows. Among them it is mentioned the direct impact of the water on the body, which can cause tipping and dragging of the human being.

The tipping threshold is quantified by the balance of moments of force, while the drag threshold is quantified by the balance of forces. For the present study, the scheme of Fig. 1 is considered.

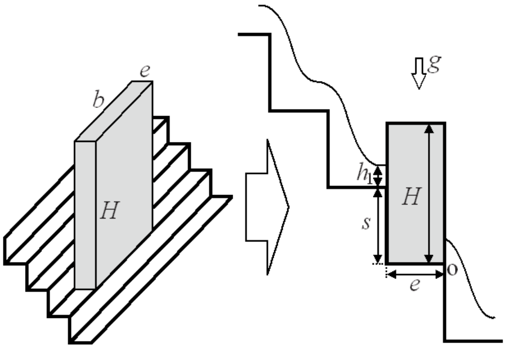


Fig. 1. Scheme of the variables used in the formulation.

The balance of the moments of forces was performed around the point "o" in Fig. 1 (outer edge of the step). In this case, the force F that produces the clockwise moment is due to the water hitting the block and the pressure of the column of water indicated by h_1, being calculated as:

$$F = C_d \rho V^2 A + 0.5 \rho g h_1 A \ , \qquad (1)$$

C_d is the drag coefficient, ρ is the density of the water, V is the horizontal velocity of the water, g is the acceleration of gravity and A is the frontal area of the block that is into contact with the water. This area is calculated as $A = bh_1$, with b being the width of the block and h_1 being the depth of the water next to the block.

The force that produces the counterclockwise moment is the weight P of the block, given by Eq. 2:

$$P = mg, \qquad (2)$$

in which m is the mass of the block. For the balance of moments, each component of force must be multiplied by the corresponding arm, providing:

$$C_d \rho V^2 A \left(s + \frac{h_1}{2}\right) + \frac{1}{2} \rho g h_1 A \left(s + \frac{h_1}{3}\right) = \frac{mge}{2}. \qquad (3)$$

The characteristic velocity of the water caused by falling down a step is $\sqrt{2gs}$. In this study V was taken as proportional to the characteristic velocity. Additionally, the height h_1 attained by the water impinging the block depends on this velocity, represented here by a generic function f(flow). Taking the depth h_1 normalized with the height of the step that generates the characteristic velocity, s, we thus have:

$$\frac{h_1}{s} = f(\text{flow}) \quad \text{or } h_1 = sf \ . \qquad (4)$$

With these considerations, Eq. (3) becomes:

$$4\alpha C_d \frac{b}{e}\, s^3 f \left(1 + \frac{f}{2}\right) + \frac{b}{e}\, s^3 f^2 \left(1 + \frac{f}{3}\right) = \frac{m}{\rho}, \qquad (5)$$

in which α is the proportionality constant used for the velocity. To obtain a dimensionless equation relating basic characteristics of the flow, Eq. (5) was divided by h_c^3, with h_c being the critical depth of the flow. Defining $M^* = m/(\rho h_c^3)$ and rearranging the terms in the equation results in:

$$M^* = \frac{b}{e} f \left[4\alpha C_d + (2\alpha C_d + 1) f + \frac{f^2}{3}\right] \left(\frac{s}{h_c}\right)^3, \qquad (6)$$

in which M^* is the tipping force normalized with the weight of a cubic volume of water with sides equal to the critical depth (critical volume for brevity of nomenclature). Equation (6) relates the tipping force to the weight (of the block) that tends to avoid this tipping (via moments of force). The factor $G = \frac{b}{e} f \left[4\alpha C_d + (2\alpha C_d + 1) f + \frac{f^2}{3}\right]$ is an unknown function of the flow (remembering that f was defined as a generic function of the flow). Since the independent variable in Eq. (6) is s/h_c, it was assumed that the factor G can also be expressed in terms of this independent variable, that is, $G = G\,(s/h_c)$, which produces:

$$M^* = G\left(\frac{s}{h_c}\right)\left(\frac{s}{h_c}\right)^3. \qquad (7)$$

Equation (7) suggests that the equilibrium condition of moments of force can be expressed by the dimensionless parameters M^* and s/h_c.

While there is a balance of moments, there is also balance of forces (equilibrium of moments implies in equilibrium of forces in this geometry and condition). In this case, considering the impulsive forces expressed by Eq. (1) and the resistive horizontal forces (the weight multiplied by the friction coefficient θ), the equilibrium condition imposes that:

$$C_d \rho V^2 A + \frac{1}{2} \rho g h_1 A = \theta m g. \qquad (8)$$

Or, rearranging:

$$\frac{H}{h_c} = \frac{\rho}{\rho_s} \frac{f}{\theta} \frac{s}{e} \left(2\alpha C_d + \frac{1}{2} f\right) \frac{s}{h_c}. \qquad (9)$$

The factor $J = \frac{\rho}{\rho_s} \frac{f}{\theta} \frac{s}{e} \left(2\alpha C_d + \frac{1}{2} f\right)$ is also an unknown function of the flow. Following the same argument of Eq. (7), since the independent variable in Eq. (9) is s/h_c, the factor J was expressed in terms of this independent variable, that is, $J = J\,(s/h_c)$, leading to:

$$\frac{H}{h_c} = J\left(\frac{s}{h_c}\right)\frac{s}{h_c}. \qquad (10)$$

Equations (7) and (10) are adequately aligned with each other, considering that the mass m of the block depends on its vertical dimension H. Thus, when relating m and H to the same independent variable s/h_c, the similar forms obtained for Eqs. (7) and (10) show the adequacy of the present analysis.

It is usual in dimensional analysis to express unknown functions through power laws. The functions G and J of Eqs. (7) and (10) were then substituted by power laws, in the form:

$$M^* = \mu_1 \left(\frac{s}{h_c}\right)^{\mu_2} \left(\frac{s}{h_c}\right)^3 = \mu_1 \left(\frac{s}{h_c}\right)^{\mu_3}, \qquad (11)$$

$$\frac{H}{h_c} = \varphi_1 \left(\frac{s}{h_c}\right)^{\varphi_2} \frac{s}{h_c} = \varphi_1 \left(\frac{s}{h_c}\right)^{\varphi_3}, \qquad (12)$$

in which $\mu_3 = \mu_2 + 3$ and $\varphi_3 = \varphi_2 + 1$.

As a consequence of Eqs. (11) and (12), the dimensionless parameter M^* (instability force normalized with the weight of the critical volume of water) can be expressed in different ways:

$$M^* = \mu_1 \left(\frac{s}{h_c}\right)^{\mu_3} \text{ (Presented in Eq. 11)}, \qquad (13a)$$

$$M^* = \lambda_1 \left(\frac{H}{h_c}\right)^{\lambda_2}, \lambda_1 = \frac{\mu_1}{\varphi_1^{\mu_3/\varphi_3}} \text{ and } \lambda_2 = \frac{\mu_3}{\varphi_3}, \qquad (13b)$$

$$M^* = \omega_1 \left(\frac{H}{h_c}\right)^{\omega_2} \left(\frac{s}{h_c}\right)^{\omega_3}. \qquad (13c)$$

Eq. (13c) was obtained combining the previous coefficients, adopting $M^{*\sigma}$ for Eq. (13a) and $M^{*(1-\sigma)}$ for Eq. (13b), and multiplying the resulting equations.

The validity of this dimensionless formulation was tested with experimental data generated in the present study and with data obtained from the literature in this field.

3. Experimental Setup

3.1 Material and Methods

The experimental study was conducted at the Hydraulics Laboratory of the Federal University of Bahia, in a physical model with the following characteristics: rectangular stepped chute, with s = 1.99 cm (step height), l = 2.4 cm (floor length), b = 15 cm (channel width). At the inlet a rectangular weir was used to measure the flow supplied by the pump, as shown in Fig. 2. A capture of part of the flow rate through a grid drain was added to control the condition of the flow over the steps. The flow rate deviated by the grid was calculated from measurements of volume and time. The flow over the stepped chute was calculated through the mass conservation equation (mass balance).

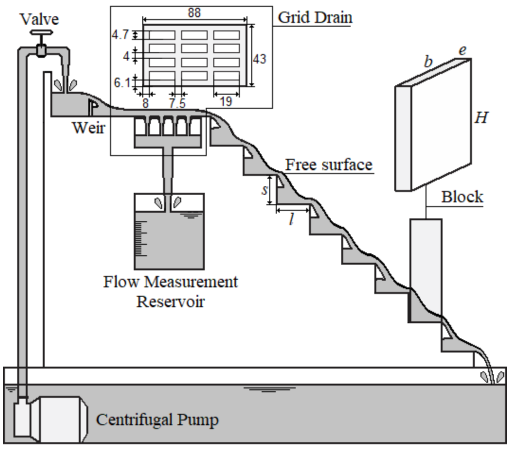


Fig. 2. Physical model and experimental apparatus.

The submersible centrifugal pump used to recirculate the water in the model furnished a flow rate per unit length between 0.68 and 1.72 L/(sm), controlled by a valve. The values of s/h_c, corresponding to these two limits of the flow rates were s/h_c = 5.5 and s/h_c = 2.9, respectively. s/l and s/h_c are used to define the boundaries between the flow regimes. The flows studied here are classified as nappe flow regime. Figure 3a shows the maximum and minimum values of s/h_c compared to the classification curves of Ohtsu *et al*. (2001) and Simões *et al*. (2011). Figure 3b illustrates the occurrence of nappe flow over the first step in this study.

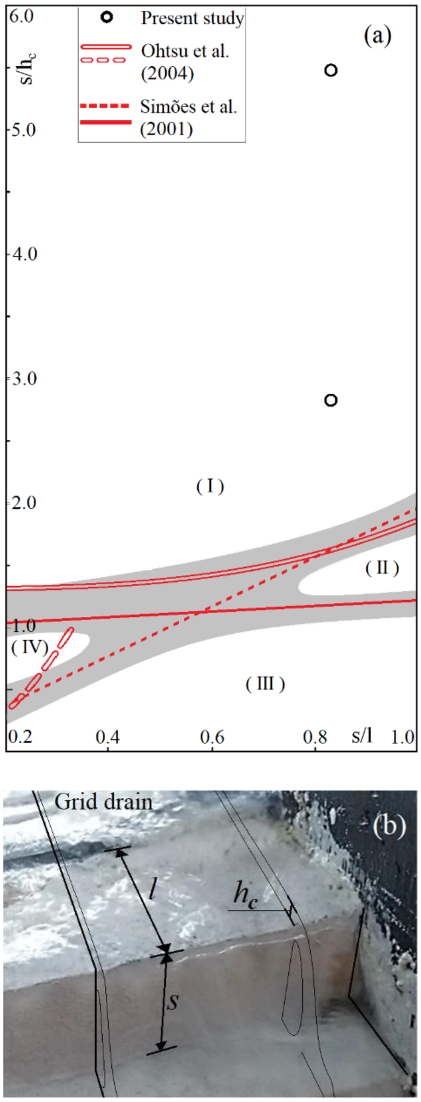


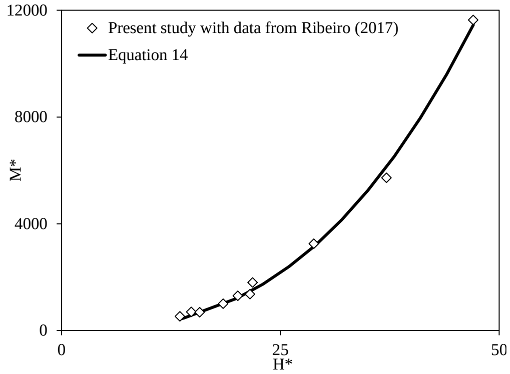
Fig. 3. (a) Flow regime zones I, II, III, and IV by Ohtsu *et al*. (2001) and Simões *et al*. (2011), and limit measured points. I: nappe flow; II: transition flow; III: type A skimming flow; IV: type B skimming flow; (b) first step flow lengths to obtain the real regime zones (Ribeiro, 2017).

Prismatic obstacles were constructed using plaster and wood, with the dimensions shown in Table 1. Dimensions and masses were chosen to attain an average density close to 970 kg/m³, similar to the average density of the human body with lungs filled with air (Daibert, 2008) and a body mass index BMI = mass/height², equal to 19 kg/m², a value between those considered adequate for good health, 18.5 and 25 kg/m², according to Anjos (1992).

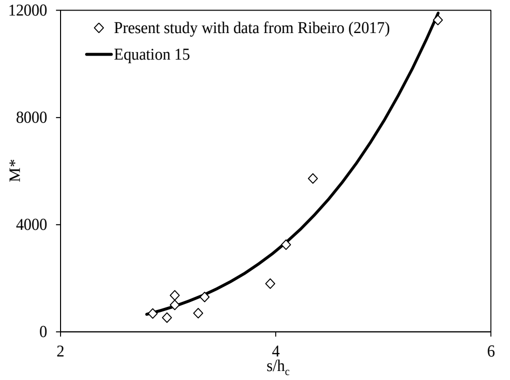
Table 1 Characteristics of obstacles

H [m]	e [m]	b [m]	m [kg]	ρ_s [kgm^{-3}]	BMI [kgm^{-2}]
0.09	0.024	0.073	0.1539	976.0	19
0.11	0.024	0.090	0.2299	967.6	19
0.12	0.024	0.098	0.2736	969.4	19
0.14	0.024	0.114	0.3724	972.2	19
0.17	0.024	0.139	0.5491	968.2	19

The experiments were carried out by placing one of the obstacles on the second step (from bottom to top) and measuring the flow that caused its tipping. The dimensionless parameters shown by Simões *et al.* (2016), and in section 2, $M^* = m/(\rho h_c^3)$, $H^* = H/h_c$, and s/h_c, together with the needed critical depth $h_c = (q^2/g)^{1/3}$, were then calculated. Here q is the flow rate per unit length, and g is the gravity acceleration.



(a)



(b)

Fig. 4. Relationship between the dimensionless parameters M^*, H^* and s/h_c, having uncertainty of 2.2% for M^* (error propagation analysis).

4. Results And Analysis

The obtained results show that there is a very good correlation between M^*, H^* and s/h_c, as can be seen in Fig. 4. Power laws were adjusted for both M^* and H^*, as suggested by Eqs. (13a) and (13b), leading to Eqs. (14) and (15) with good adherence to the experimental data, and correlation coefficients equal to 0.999 and 0.983, respectively.

$$M^* = 0.44H^{*2.64}, \tag{14}$$

$$M^* = 7.87(s/h_c)^{4.29}. \tag{15}$$

The behavior of M^* following Eq. (13c) was also evaluated. In this case Eq. (16) was obtained, with excellent adherence to the experimental data, as can be seen in Fig. 5, and correlation coefficient equal to 0.999.

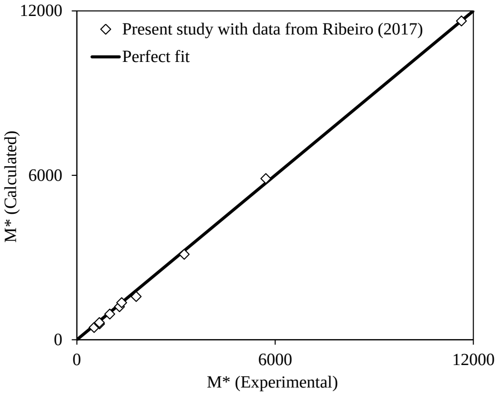


Fig. 5. Comparison between experimental data and Eq. (15).

$$M^* = 0.493H^{*2.4}(s/h_c)^{0.484}, \tag{16}$$

Returning to dimensional variables from Eq. (16) produces, for the flow rate per unit length:

$$q = 9371.5\left(\frac{m}{\rho}\right)^{12.93}\frac{\sqrt{g}}{H^{31.03}s^{6.259}}. \tag{17}$$

The coefficients have their origin explained in the derivation of Eq. (13c) and have empirical values based on the analyzed experimental results. For tipping or sliding velocity we have then:

$$V = 9371.5\left(\frac{m}{\rho}\right)^{12.93}\frac{\sqrt{g}}{h_1 H^{31.03}s^{6.259}}. \tag{18}$$

This procedure exposes the variable q used by Abt *et al.* (1989) and the relation with the person's mass, m, and his height, H. The form of dependence, however, follows the here proposed relationship. Note that Eqs. (17) and (18) also define a Reynolds number for tipping or sliding, in the form:

$$Re_t = 9371.5\left(\frac{m}{\rho}\right)^{12.93}\frac{\sqrt{g}}{\nu H^{31.03}s^{6.259}},\text{ or} \tag{19}$$

$$Re_t = 9371.5\left(\frac{m}{\rho H^3}\right)^{12.93}\left(\frac{H}{s}\right)^{6.259}\frac{\sqrt{gH^3}}{\nu}. \tag{20}$$

As noted, the Reynolds number of tipping/sliding Re_t is dependent on the characteristics of the obstacle (human being, as the final objective of the security studies), expressed by the mass m and height H, as well as the characteristics of the flowing fluid, expressed by the density ρ and the kinematic viscosity ν. This is the limiting Reynolds number that corresponds to the particular case of instability of this study. The mass and height variables presented in Eqs. (19) and (20) are related to the average limiting velocity of Eq. (18) that produced the instability.

This velocity (to attain instability) depends on the measured variables, and so also the limiting Reynolds number. In the case of stepped chutes, as considered in the present study, it also depends on the height of the step, *s*. In the present study the Reynolds number was not considered as an *a priori* variable, but emerged from the data analysis based on the balance equations for the forces and for the moments of forces. Equations (16) and (20) have the same information, and Eq. (20) presents them linked to the immediately measurable characteristics (not involving the calculation of the critical depth). The tipping Reynolds number can also be obtained directly from the measurements of water speed and depth, associated with the viscosity of the water (which depends on the temperature). When using Eq. (14) the Reynolds number is given by Eq. (21).

$$Re_t = 30.59\left(\frac{m}{\rho H^3}\right)^{4.167}\frac{\sqrt{gH^3}}{\nu}. \tag{21}$$

This equation shows that it is possible to consider Ret also in cases in which the step height is not involved (flat bottoms, for example). Further, by dividing and multiplying the right member of Eq. (21) by $qh_1^{1/2}$, the Reynolds number is simplified and Eq. (21) may be written in terms of a tipping/sliding Froude number Fr_t

$$Fr_t = 30.59\left(\frac{m}{\rho H^3}\right)^{4.167}\left(\frac{H}{h_1}\right)^{3/2}. \tag{22}$$

Equations (21) and (22) show that the two dimensionless parameters Re_t and Fr_t may be related to the sliding and tipping events in flood security studies. This coincides with the discussion of Simões *et al.* (2016), in which these two parameters (Re_t and Fr_t) were also considered in the question of human instability in floods.

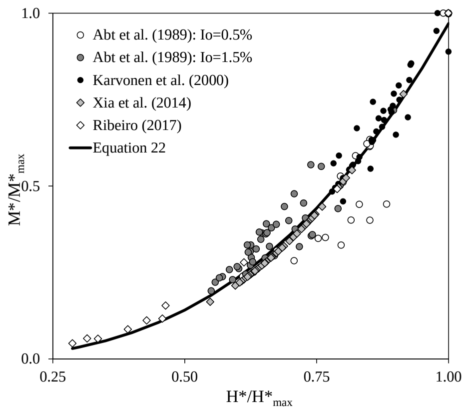
4.1 Comparison with Literature Studies

Data from Abt *et al*. (1989), Karvonen *et al* (2000) and Xia *et al*. (2014) were analyzed using the dimensionless M^* and H^* as defined in the present work. To minimize the scale effects between the different physical models and situations, M^* and H^* were normalized with adjusted maximum values obtained for each set of data. The normalized parameters thus have a maximum theoretical variation between 0 and 1, being expressed as $M^*_n = M^*/M^*_{max}$ and $H^*_n = H^*/H^*_{max}$. Because M^*_{max} and H^*_{max} are adjusted values, the extremes 0 and 1 may not be reached in the data analysis. This procedure proved to be adequate for the analyzed data set, as can be seen in Fig. 6a. Following the indication of Eq. (13b), the general behavior of the experimental data was adjusted to the power law expressed by Eq. (23). The correlation coefficient between experimental and calculated data is R = 0.97, and Fig. 6b shows the good approximation obtained by using the power law to calculate M^*_n. The minimum and maximum values of H^* and M^* are shown in Table 2.

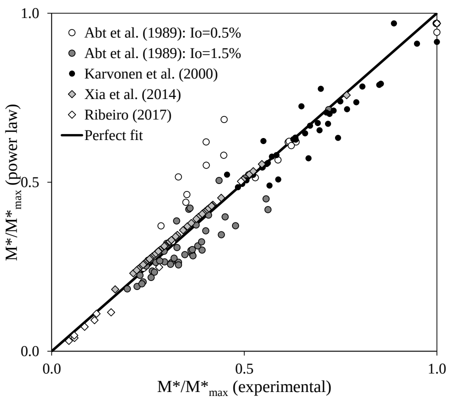
$$M^*_n = 0.97{H^*_n}^{2.78}. \tag{23}$$

Table 2 Minimum and maximum values of the parameters M^* and H^*

Author	H^*_{min}	M^*_{min}	H^*_{max}	H^*_{min}
Abt *et al.* (1989), Io=0.5%	2.33	0.20	3.82	0.83
Abt *et al.* (1989), Io=1.5%	2.57	0.22	4.67	1.12
Karvonen *et al.* (2000)	3.56	0.57	4.56	1.25
Xia *et al.* (2014)	4.30	0.99	7.84	5.97
Ribeiro (2017)	13.52	521.88	47.05	11637.50



(a)



(b)

Fig. 6. Relationship between normalized dimensionless, M^*/M^*_{max} and H^*/H^*_{max} and Eq. (23) (a); comparison between the Eq. (23) and the experimental data (b).

The dimensionless parameters M^* and H^* showed a great capacity to unify data from our own experiments and from different sources in the literature. The normalization with maximum values of M^* and H^* adjusted for each experiment emphasized the subjacent similarity between the profiles obtained for the data of different sources. This is seen as a very positive characteristic of the methodology here described.

5. CONCLUSION

Theoretical analyses were performed about the phenomenon of tipping/dragging of obstacles in flows, together with semi-empirical proposals of power laws, and correlation analyses of experimental data obtained in a reduced model of a draining staircase. The correlation analyses followed the indications of the theoretical study, and evidenced the dependence between the dimensionless parameters M^*, H^* and s/h_c for the tested obstacles. From the theoretical analyses, the equations were expressed as $M^* = F(H^*)$, $M^* = F(s/h_c)$ and for $M^* = F(H^*, s/h_c)$ using power laws.

The three functional forms were checked, generating excellent correlations and adherence between experimental data and calculated results.

Considering the functional form $M^* = F(H^*)$, in which s/h_c is not an explicit variable, it was observed that the experimental data of the present study, for a drainage staircase, could be analyzed together with data of the literature, for flat bottom channels. A procedure was proposed to minimize the effects of different scales and experimental conditions between the data of different sources. In this sense, a normalization of M^* and H^* was performed using maximum adjusted values of these parameters for each independent set of data. When plotted together, the normlized profiles showed similar evolution for M^*/M^*_{max} against H^*/H^*_{max} independently of their origin. It is understood that this is a promising way to conduct joint analyzes to assess the vulnerability criteria of human beings exposed to urban floods.

Considering the Reynolds and Froude numbers, the present analysis allows relating both parameters to the formulation of sliding or tipping in open flows.

In terms of specific results, it was shown that M^* consistently grows with H^* and s/h_c following power laws. For the study conducted in the draining staircase the exponents were 2.64 and 4.29, respectively, evidencing the strong dependence between the different parameters. The correlation coefficients presented the values of 0.999 and 0.983, respectively.

For the joint analysis of the present data and those of the literature using the normalized parameters M_n^* and H_n^*, the exponent of the power law was 2.78, once more showing the strong dependence between the parameters defined in this study. The correlation coefficient presented the value of 0.97.

ACKNOWLEDGEMENTS

To FAPESB, for the support related to the infrastructure project (PIE0021/2016). The last co-author thanks CNPq for the Scientific Productivity Scholarship 307105/2015-6, which guaranteed the necessary dedication to this study.

REFERENCES

Abt, S. R., R. J. Wittier, A. Taylor and D. J. Love (1989). Human stability in a high flood hazard. *Water Resources Bulletin* 25(4), 881-890.

Anjos, L. A. (1992). Índice de massa corporal como indicador do estado nutricional de adultos: revisão da literatura. *Revista de Saúde Pública* 26(6), 431-436. (In Portuguese)

Arrighi, C. (2016). *Vehicles, pedestrians and flood risk: a focus on the incipient motion due to the mean flow*. PhD. thesis, University of Braunschweig and University of Florence.

Arrighi, C., H. Oumeraci and F. Castelli (2017). Hydrodynamics of pedestrian's instability in floodwaters. *Hydrology and Earth System Sciences-Discussions* 21, 515–531.

Compton, K. L., R. Faber, T. Y. Ermolieva, J. Linerooth-Bayer and H. P. Nachtnebel (2009). *Uncertainty and disaster risk management*. RR-09-02, International Institute for Applied Systems Analyses, Laxenburg, Austria.

Cox, R. J., T. D. Shand and M. J. Blacka (2010). *Australian Rainfall & Runoff Project 10*. The University of New South Wales, School of Civil and Environmental Engineering.

Daibert, J. B. C. (2008). *Os benefícios da natação para bebês*. Monography (Graduation). Instituto de Biociências de Rio Claro, Universidade Estadual Paulista. (In Portuguese)

Foster, D. N. and R. J. Cox (1973). *Stability of children on roads used as floodways*, Technical Report 73/13, Water Research Laboratory, The University of New South Wales, School of Civil and Environmental Engineering.

Ishigaki, T., R. Kawanaka, Y. Onishi, H. Shimada, K. Toda and Y. Baba(2008b). Assessment of safety on evacuation route during underground flooding, *Proceedings of the 16th APD-IAHR, Nanjing, China, pp.141-146*.

Ishigaki, T., Y. Asai, Y. Nakahata, H. Shimada, Y. Baba and K. Toda (2009). Evacuation of aged persons from inundated underground space, *Proceedings of the 8th International Conference on Urban Drainage Modelling, Tokyo*.

Ishigaki, T., Y. Baba, K. Toda and K. Inoue(2005). Experimental study on evacuation from underground space in urban flood, *Proceedings of the 31st IAHR Congress, Seoul*, pp.261-262.

Ishigaki, T., Y. Onishi, Y. Asai, K. Toda and H. Shimada (2008a). Evacuation criteria during urban flooding in underground space, *Proceedings of the 11th ICUD,* Scotland, UK.

Jonkman, S. N. (2005). Global perspectives of loss of human life caused by floods. *Natural Hazards* 34, 151-175.

Jonkman, S. N. and E. Penning-Rowsell (2008). Human instability in flood flows. *Journal of the American Water Resources Association* 44(4), 11.

Karvonen, R. A., H. K. Hepojoki, H. K. Huhta and A. Louhio (2000). *The use of physical models in dam-break analysis*. Helsinki: Helsinki University of Technology, 2000. RESCDAM Final Report.

Keller, R. J. and B. Mitsch (1993*). Safety aspects of the design of roadways as floodways*, Research Report 69, Urban Water Research Association of Australia.

Kvočka, D., R. A. Falconer and M. Bray (2016). Flood hazard assessment for extreme flood events. *Nat. Hazards, Springer* 84, 1569–1599.

Lind, N., D. Hartford and H. Assaf (2004). Hydrodynamic models of human stability in a flood. *Journal of the American Water Resources Association* 40(1), 89-96.

Mangieri, L. S. G. (2012). *Avaliação dos sistemas de escadarias e rampas drenantes implantadas em assentamentos espontâneos na cidade do Salvador – Bahia*. MSc Dissertation - Escola Politécnica – Universidade Federal da Bahia, Salvador-BA (inPortuguese). (In Portuguese)

Martínez-Gomariz, E., L. Locatelli, M. Guerrero, B. Russo and M. Martínez (2019). Socio-Economic Potential Impacts Due to Urban Pluvial Floods in Badalona (Spain) in a Context of Climate Change. *Water* 11(12), 2658.

Milanesi, L., M. Pilotti and R. Ranzi (2014). A conceptual model of people's vulnerability to floods. *Water Resources Research* 51(1), 182-197.

Ohtsu, I., Y. Yasuda and M. Takahashi (2001). *Onset of skimming flow on stepped spillways* – Discussion. Journal of Hydraulic Engineering. 127, 522-524. Discussion on: Chamani, M. R., Rajaratnam, N. *Onset of skimming flow on stepped spillways*. ASCE, Journal of Hydraulic Engineering. 125(9), 969-971, Sept, 1999.

Ohtsu, I., Y. Yasuda and M. Takahashi (2004). Flows characteristics of skimming flows in stepped channels. *ASCE, Journal of Hydraulic Engineering* 130(9), 860-869.

Ramsbottom, D., P. Floyd and E. Penning-Towsell (2004). *Flood risks to people*, Phase 2 Draft Inception Report.

Ramsbottom, D., P. Floyd and E. Penning-Towsell (2006). *Flood risks to people*, Phase 2 Project Record, FD2321/PR, Department for Environment Food and Rural Affairs, United Kingdom.

Rezende, O. M., F. M. Miranda, A. N. Haddad and M. G. Miguez (2019). A framework to evaluate urban flood resilience of design alternatives for flood defence considering future adverse scenarios. *Water* 11, 1485-1513.

Ribeiro, H. de B. (2017). *Estabilidade de pedestres sobre escadarias drenantes*. Monography (Graduação). Escola Politécnica – Universidade Federal da Bahia, Salvador-BA. (In Portuguese)

Shu, C. W., S. S. Han, W. N. Kong and B. L. Dong (2016). Mechanism of toppling instability of the human body in floodwaters, *International Conference on Water Resource and Environment, IOP Conf. Series: Earth and Environmental Science*, 39, 23–26 July 2016, Shanghai, China.

Simões, A. L. A., H. E. Schulz and L. D. da Luz (2016). Dimensionless formulation for human stability in open flows. *RBRH* 21(4), 666-673.

Simões, A. L. A., H. E. Schulz, R. J. Lobosco and R. M. Porto (2011). *Stepped spillways: theoretical, experimental and numerical studies*. In: Schulz, H.E., Simões, A.L.A., Lobosco, R.J. Hydrodynamics - Natural Water Bodies, ISBN 978-953-307-893-9, InTech Open Access Publisher.

Smith, G. P. and P. F. Rahman (2016). *Approaches for estimating flood fatalities relevant to floodplain management*, WRL Technical Report 2015/09, The University of New South Wales, School of Civil and Environmental Engineering.

Takahashi, S., K. Endoh and Z. I. Muro (1992). Experimental study on people's safety against overtopping waves on breakwaters. *Report of the Port and Harbour Research Institute* 34(4), 4-31. (in Japanese)

Taşke, N. (2002). *Stairways as spatial elements in an urban environment*, MSc Dissertation, Izmir|Institute of Technology, Turkey, 214.

Wade, S., D. Ramsbottom, P. Floyd, E. Penning-Rowsell and S. Surendran (2005). Risks to people: developing new approaches for flood hazard and vulnerability mapping, *Proceedings of the 40th Defra Flood and Coastal Management Conference, HRPP340*.

Wallingford, H. R. (2006). *Flood Risks to People, Phase 2*, Defra/Environment Agency, FD2321/TR1.

Xia, J., R. A. Falconer, Y. Wang and X. Xiao (2014). New criterion for the stability of a human body in floodwaters. *Journal of Hydraulic Research* 52(1), 93-104.

Yao, L., L. Chen and W. Wei (2017). Exploring the Linkage between Urban Flood Risk and Spatial Patterns in Small Urbanized Catchments of Beijing. China, *International Journal of Environmental Research and Public Health* 14, 239-255.

Yu, H., C. Ling, P. Li, K. Niu, F. Du, J. Shao and Y. Liu (2019). Evaluation of waterlogging risk in an urban subway station. *Advances in Civil Engineering,* Hindawi 19, 1-12.