



A Reynolds Stress Closure for Compressible Turbulent Flow

H. Khlifi^{1†} and T. Lili²

¹*Département de Physique, Faculté des Sciences de Tunis, Campus Universitaire, 1060, Tunis, Tunisie*

²*Département de Physique, Faculté des Sciences de Tunis, Campus Universitaire, 1060, Tunis, Tunisie.*

†*Corresponding Author Email: khlifhachmi@yahoo.fr*

(Received April 25, 2010; accepted March 13, 2011)

ABSTRACT

Several studies of compressible flows show that the pressure-strain is the main indicator of the structural compressibility effects. Undoubtedly, this term controls the change in the Reynolds stress anisotropy. Regarding the model of Adumitroaie et al., the slow part of the pressure strain correlation like the Rotta model uses the standard coefficient C_1 . The model predictions do not show large differences when compressibility increases. Correction of this coefficient using the turbulent Mach number is proposed. The two forms model of Adumitroaie et al. (with and without correction of C_1) are considered to study compressible mixing layers. The obtained results show that the predictions of the proposed compressibility correction model agree with the experiment results of Goebel and Dutton.

Keywords: Turbulence flow, Compressible, Pressure-strain, Models of turbulence, Mixing layer.

NOMENCLATURE

| | | | |
|----------|------------------------------------|-----------------|--------------------------------------|
| a | speed of sound | u_i | velocity in the direction, x_i |
| b_{ij} | Reynolds stress anisotropy | γ | specific heat ratio |
| C_p | specific heat at constant pressure | ε_s | solenoidal part of the dissipation |
| C_v | specific heat at constant volume | μ | viscosity coefficient |
| d' | fluctuation of the dilatation | ρ | density |
| K | turbulent kinetic energy | ε | dissipation rate of turbulence |
| p | pressure | δ_{ij} | Kronecker delta |
| R | specific gas constant | κ | thermal conductivity |
| R_{ij} | Reynolds stress | τ_{ij} | viscous stress tensor |
| Mc | convective Mach number | ε_c | dilatational part of the dissipation |
| M_t | turbulent Mach number | $(\cdot)'$ | Favre fluctuation |
| T | temperature | $(\cdot)''$ | Reynolds fluctuation |
| t | time | | |

1. INTRODUCTION

Compressible turbulence modelling plays an important role in the study of many applications such as aerospace and engineering problems. The techniques and methodologies that have been proved useful in the incompressible flows may represent a reasonable way to develop compressible turbulence models. The direct extension of incompressible models was used in simulating different compressible flows. That one was observed when the standard ($K - \varepsilon$) and the Reynolds stress closures were extended to compressible flows with an explicit account of compressibility effects, by considering dilatational terms models. These terms which have been recognized as important indicators of compressibility are the pressure-dilatation correlation

and compressible dissipation rate appearing in the turbulent kinetic energy transport equation. It has been shown that this practice of modelling, called compressibility correction models, may be able to reproduce the compressibility phenomenon at small values of Mach number. But, when the compressibility effects are more significant, the extended models do neither predict correctly the decrease in spreading rate of mixing layers, as it is observed in the experiments of Goebel *et al.* (1991) and Elliot *et al.* (1990), nor the reduction in the growth rate of turbulent kinetic energy Sarkar (1995). The deficiencies of such closure is due principally to the use of the incompressible models of the pressure strain correlation which controls the level of Reynolds stress anisotropy. However, new models taking into account structural compressibility effects are needed for the pressure strain correlation. The present

work focuses on this major issue. In this context, [Adumitroaie et al. \(1999\)](#) use the incompressible modelling approach to modify the linear pressure strain. In the present study, recent compressible models for the pressure-dilatation correlation have been used to modify the [Rotta model \(1951\)](#) for the slow part of the pressure-strain correlation. This modification make the standard coefficient C_1 dependant on the turbulent Mach number. This model is linked to the [Adumitroaie model \(1999\)](#). The resultant model is applied to compressible mixing layers, showing an acceptable success to predict the reduced growth rate and the decrease of the Reynolds stress peaks when the convective Mach number increases.

2. GOVERNING EQUATIONS

The general equations governing the motion of compressible fluid are the Navier stokes equations. They can be written as follows for mass, momentum and energy conservation:

$$\frac{\partial}{\partial t} \rho + (\rho u_i)_{,i} = 0 \quad (1)$$

$$\frac{\partial}{\partial t} (\rho u_i) + (\rho u_i u_j)_{,j} = (\sigma_{ij})_{,j} \quad (2)$$

$$\frac{\partial}{\partial t} (\rho e) + (\rho e u_j)_{,j} = (\sigma_{ij} u_j)_{,j} \quad (3)$$

$$e = c_v T + (u_i u_i / 2), \quad \sigma_{ij} = -p \delta_{ij} + \tau_{ij} \quad (4)$$

The Favre averaged-equations are :

$$\frac{\partial}{\partial t} \bar{\rho} + (\bar{\rho} \tilde{u}_i)_{,i} = 0 \quad (5)$$

$$\frac{\partial}{\partial t} (\bar{\rho} \tilde{u}_i) + (\bar{\rho} \tilde{u}_i \tilde{u}_j)_{,j} = -(\bar{\rho} R_{ij})_{,j} + (\bar{\rho} \tilde{\tau}_{ij})_{,j} \quad (6)$$

$$\frac{\partial}{\partial t} (\bar{\rho} \tilde{C}_v \tilde{T}) + (\bar{\rho} \tilde{C}_v \tilde{T} \tilde{u}_i)_{,i} = -\pi_d + \bar{\Phi} + (k \tilde{T}_{,i})_{,i} - \bar{Q}_{i,i} \quad (7)$$

where $R_{ij} = \overline{\rho u_i'' u_j''} / \bar{\rho}$

$$Q_i = C_v \overline{\rho u_i T''} \quad (8)$$

$$\bar{\Phi} = \overline{\tau_{ij} u_{i,j}} - \bar{p} \tilde{u}_{i,i} - \overline{p u_i''} \quad (9)$$

$$\tilde{\tau}_{ij} = \mu (\tilde{u}_{i,j} + \tilde{u}_{j,i}) - \frac{2}{3} \tilde{u}_{k,k} \delta_{ij} \quad (10)$$

$$\pi_d = \overline{p' u_i''} = \overline{p' d'} \quad , \quad d' = u_i' \quad (11)$$

To close the above equations, the Reynolds stress must be solutions of the following transport equations, namely

$$\frac{\partial}{\partial t} (\bar{\rho} R_{ij}) + (\bar{\rho} R_{ij} \tilde{u}_m)_{,m} = P_{ij} + D_{ij} + \phi_{ij} + \varepsilon_{ij} + T_{ij} \quad (12)$$

$$P_{ij} = -(\bar{\rho} R_{im} \tilde{u}_{j,m} + \bar{\rho} R_{jm} \tilde{u}_{i,m}) \quad (13)$$

$$\varepsilon_{ij} = -\overline{\tau_{im}'' u_{j,m}''} - \overline{\tau_{jm}'' u_{i,m}''} \quad (14)$$

$$D_{ij} = -(\overline{\rho u_i'' u_j''} + \overline{p' u_j''} \delta_{im} + \overline{p' u_j''} \delta_{jm} - \overline{\tau_{im}'' u_j''} - \overline{\tau_{jm}'' u_i''}) \quad (15)$$

$$\phi_{ij} = \overline{p' (u_{i,j}'' + u_{j,i}'')} \quad (16)$$

$$T_{ij} = -\overline{p_{,j}' u_i''} - \overline{p_{,i}' u_j''} + \overline{\tilde{\tau}_{im,m}'' u_j''} + \overline{\tilde{\tau}_{jm,m}'' u_i''} \quad (17)$$

3. MODELS OF TURBULENCE

3.1 Models of the Pressure-Strain Correlation

*Model of [Launder Reece and Rodi\(LRR\) \(1975\)](#)

$$\begin{aligned} \phi_{ij}^* &= C_1 \bar{\rho} \varepsilon_s b_{ij} + \frac{4}{5} \bar{\rho} k (\tilde{S}_{ij} - \frac{1}{3} \tilde{S}_{ll} \delta_{ij}) \\ &+ 2 \bar{\rho} k (1 - C_3) [b_{im} \tilde{S}_{jm} + b_{jm} \tilde{S}_{im}] \\ &+ \bar{\rho} k (1 - C_4) [b_{im} \tilde{\Omega}_{jm} + b_{jm} \tilde{\Omega}_{im}] \end{aligned} \quad (18)$$

where:

$$\tilde{S}_{ij} = 0.5(\tilde{U}_{i,j} + \tilde{U}_{j,i}), \quad \tilde{\Omega}_{ij} = 0.5(\tilde{U}_{i,j} - \tilde{U}_{j,i})$$

and $b_{ij} = (R_{ij} / 2K - \delta_{ij} / 3)$

*Model of [Adumitroaie et al. \(1999\)](#)

[Adumitroaie et al. \(1990\)](#) assumed that incompressible modeling approach of the pressure strain correlation can be used to develop turbulent models taking into account compressibility effects. Considering a none zero divergence for the velocity fluctuation called the compressibility continuity constraint and using different models for the pressure dilatation which is proportional to the trace of the pressure strain, their model for the linear part of this term is written as:

$$\begin{aligned} \phi_{ij}^* &= C_1 \bar{\rho} \varepsilon_s b_{ij} + (\frac{4}{5} + \frac{2}{5} d_1) \bar{\rho} k (\tilde{S}_{ij} - \frac{1}{3} \tilde{S}_{ll} \delta_{ij}) \\ &+ 2 \bar{\rho} k (1 - C_3 + 2d_2) [b_{im} \tilde{S}_{jm} + b_{jm} \tilde{S}_{im} \\ &- \frac{2}{3} d_2 b_{nl} \tilde{S}_{nl} \delta_{ij}] + \bar{\rho} k (1 - C_4 - 2d_2) \\ &[b_{im} \tilde{\Omega}_{jm} + b_{jm} \tilde{\Omega}_{im} - \frac{4}{3} d_2 b_{ij} \tilde{S}_{nl}] \end{aligned} \quad (19)$$

The compressibility coefficients d_1 and d_2 are determined on the basis of some compressible closures of the pressure-dilatation correlation.

3.2 A Modeling for the Slow Part of the Pressure-Strain Correlation

In this section, the purpose is to investigate the compressibility effect in the slow part of the pressure-strain appearing in the turbulent Reynolds stress transport equation. Classically, for the compressible turbulent flows, the pressure-strain correlation can be divided in two terms as

$$\overline{p' u_i' u_j'} = (\overline{p' u_i' u_j'})_{inc} + (\overline{p' u_i' u_j'})_{comp} \quad (20)$$

where $(\overline{p' u_i' u_j'})_{inc}$ and $(\overline{p' u_i' u_j'})_{comp}$ are the incompressible and compressible parts of $(\overline{p' u_i' u_j'})$ respectively.

The trace of these terms are:

$$(\overline{p' u_i' u_i'})_{inc} = 0 \quad (21)$$

$$(\overline{p' u_i' u_i'})_{comp} = \overline{p' d'} \quad (22)$$

It is natural that the first term in the right hand side of Eq. (20) is represented by using the simplest Rotta's model as:

$$\overline{(p'u'_{i,j})_{inc}} = -C_1 \varepsilon b_{ij} \quad (23)$$

Where C_1 is a constant model assigned to a value of about 3.

To give a complete model for the total slow part of the pressure-strain correlation, an expression of the compressible term $\overline{(p'u'_{i,j})_{comp}}$ is required. For this, equation of the fluctuating dilatation (d') is to be taken into consideration, such equation can be written as

$$\begin{aligned} \frac{d}{dt} d' &= -2u'_{j,i} \tilde{U}_{i,j} - u'_{j,i} u'_{i,j} + \\ &\frac{1}{\bar{\rho}} p'_{,ii} + \frac{4}{3} v d'_{,ii} + \text{other terms} \end{aligned} \quad (24)$$

The other terms are unknown in relation with density fluctuation, mean gradient of density and pressure. We therefore write this equation as follow

$$\frac{d}{dt} d' = -2u'_{j,i} \tilde{U}_{i,j} + \text{other terms} \quad (25)$$

Multiplying both sides of Eq. (25) by p' and taking the Reynolds-averaging, we have

$$p' \frac{d}{dt} d' = -2\overline{p'u'_{j,i} \tilde{U}_{i,j}} + \text{other terms} \quad (26)$$

This equation can be written as follow

$$\frac{\overline{p'd'}}{\tau} = -2\overline{p'u'_{j,i} \tilde{U}_{i,j}} + \text{other terms} \quad (27)$$

Where τ is the characteristic time scale of the

dilatation fluctuation (see Taulbee *et al.* 1991), $\tau = \frac{K}{\varepsilon}$

Because $\overline{p'd'}$ is the trace of $\overline{(p'u'_{i,j})_{comp}}$, we can write

Eq. (27) as

$$\frac{\overline{p'd'}}{\tau} = -2\overline{(p'u'_{j,i})_{comp}} \tilde{U}_{i,j} + \text{other terms} \quad (28)$$

To obtain the final form of $\overline{(p'u'_{i,j})_{comp}}$ an expression of

$\overline{p'd'}$ is required. It is well known that for this term there are some turbulent models, in this study, we choose those according to the general formulation as

$$\overline{p'd'} = A_{ij} \tilde{U}_{i,j} + \dots \quad (29)$$

Where A_{ij} is a second rank tensor.

Sarkar *et al.* (1991)

$$\overline{p'd'} = 0,4M_t^2 \bar{\rho} R_{ij} - \frac{2}{3} K \delta_{ij} \tilde{U}_{i,j} + 0,2\bar{\rho} M_t^2 \varepsilon_s \quad (30)$$

Sarkar (1992)

$$\begin{aligned} \overline{p'd'} &= \alpha_1 M_t \bar{\rho} (R_{ij} - \frac{2}{3} K \delta_{ij}) \tilde{U}_{i,j} + \alpha_2 \bar{\rho} M_t^2 \varepsilon_s \\ &+ \alpha_3 M_t^2 \bar{\rho} K \tilde{U}_{ii} \end{aligned} \quad (31)$$

The model constants, α_1 , α_2 and α_3 take the values :

$$\alpha_1 = 0,15, \quad \alpha_2 = 0,2 \quad \text{and} \quad \alpha_3 = 0,0$$

Elbaz and Launder (1993)

$$\overline{p'd'} = 1,5M_t^2 \bar{\rho} (R_{ij} - \frac{2}{3} K \delta_{ij}) \tilde{U}_{i,j} \quad (32)$$

Ristorcelli (1997)

$$\overline{p'd'} = \frac{C_1 M_t}{C_1 + C_1 M_t^2} \bar{\rho} (R_{ij} - \frac{2}{3} K \delta_{ij}) \tilde{U}_{i,j} \quad (33)$$

From these models, we can deduce easily the general form of the A_{ij} -tensor, namely

$$A_{ij} - \frac{1}{3} A_{mmm} \delta_{ij} = \bar{\rho} F(M_t) (R_{ij} - \frac{2}{3} K \delta_{ij}) \quad (34)$$

Where $F(M_t)$ is a function of the turbulent Mach number, vanishing in an incompressible flow.

As suggested in the above cited models, $F(M_t)$ takes the following expressions:

Sarkar (1991)

$$F(M_t) = 0,4M_t^2 \quad (35)$$

Sarkar (1992)

$$F(M_t) = 0,15M_t \quad (36)$$

Elbaz and Launder (1993)

$$F(M_t) = 1,5M_t^2 \quad (37)$$

Ristorcelli *et al.* (1997)

$$F(M_t) = \frac{C_1 M_t}{C_1 + C_2 M_t^2} \quad (38)$$

Substituting $\overline{p'd'}$ by its expression in Eq. (29) and using identification between terms which affect the mean gradient velocity in Eq. (28), we obtain

$$-2\overline{(p'u'_{i,j})_{comp}} - \frac{1}{3} \overline{p'd'} \delta_{ij} = \frac{1}{\tau} (A_{ij} - \frac{1}{3} A_{mmm} \delta_{ij}) \quad (39)$$

Using Eq. (34), we deduce for the compressible slow part of the pressure-strain correlation the final form model as follow

$$\overline{(p'u'_{i,j})_{comp}}^* = \bar{\rho} C' F(M_t) \frac{\varepsilon}{K} (R_{ij} - \frac{2}{3} K \delta_{ij}) \quad (40)$$

Where $\overline{(p'u'_{i,j})_{comp}}^* = \overline{(p'u'_{i,j})_{comp}} - \frac{1}{3} \overline{p'd'} \delta_{ij}$

Regarding the models of $F(M_t)$, we choose the following model

$$F(M_t) \propto M_t^2 \quad (41)$$

and finally, the deduced model for the term $\overline{(p'u'_{i,j})_{comp}}$ is:

$$\overline{(p'u'_{i,j})_{comp}}^* - \frac{1}{3} \overline{p'd'} \delta_{ij} = \bar{\rho} \beta M_t^2 \varepsilon b_{ij} \quad (42)$$

Thus, the proposed extension of the Rotta's model taking account of compressibility can be written as

$$\phi_{ij}^* = -C_1 (1 - \beta M_t^2) \bar{\rho} \varepsilon_s b_{ij} \quad (43)$$

4. SIMULATION OF COMPRESSIBLE MIXING LAYERS

For the stationary compressible mixing layers, the basic averaged equations are:

$$\overline{(\rho \tilde{u}_j)_{,j}} = 0 \quad (44)$$

$$(\overline{\rho \tilde{u}_i \tilde{u}_j})_{,j} = -(\overline{\rho R_{ij}})_{,j} \quad (45)$$

$$(\rho \tilde{u}_i \tilde{T})_{,i} = -\pi_d + \rho \varepsilon - (\overline{\rho u_i^* T^*})_{,i} \quad (46)$$

$$(\overline{\rho \tilde{u}_k R_{ij}})_{,k} = P_{ij} + \phi_{ij}^* + (\overline{\rho u_i^* u_j^* u_k^*})_{,k} - \frac{2}{3} \varepsilon \delta_{ij} + \frac{2}{3} \pi_d \delta_{ij} \quad (47)$$

$$\phi_{ij}^* = \phi_{ij} - \frac{2}{3} \pi_d \quad (48)$$

The gradient diffusion hypothesis is used to model:

-The turbulent heat flux

$$(\overline{\rho u_k^* T^*}) = -C_T \frac{K}{\varepsilon} R_{km} \tilde{T}_{,k} \quad (49)$$

-The diffusion term

$$D_{ij} = (C_s \frac{K}{\varepsilon} \overline{\rho u_i^* u_j^*})_{,m} \quad (50)$$

Classically, the second order closure suggests to determine the dissipation term \mathcal{E}_{ij} by using isotropic dissipation model:

$$\varepsilon_{ij} = \frac{2}{3} \varepsilon \delta_{ij} \quad (51)$$

Recently, a concept of the dissipation was proposed by Sarkar (1991):

$$\varepsilon = \varepsilon_s + \varepsilon_c \quad (52)$$

$$\varepsilon_s = \nu \omega_i \omega_i, \quad \varepsilon_c = \frac{4}{3} \nu d^2 \quad (53)$$

The dissipation solenoidal ε_s can be modelled by using the traditional model, namely

$$\frac{\partial}{\partial t} (\overline{\rho \varepsilon_s}) + (\overline{\rho \tilde{U}_m \varepsilon_s})_{,m} = \frac{\varepsilon_s}{k} (C_{\varepsilon 1} R_{mm} \tilde{U}_{m,n} - \overline{\rho C_{\varepsilon 2}} \varepsilon_s) + (C_{\varepsilon 3} \frac{k}{\varepsilon} R_{mn} \varepsilon_{s,n})_{,m} \quad (54)$$

In this work, the model of Sarkar is used to express the pressure dilatation correlation and the turbulent dilatation of dissipation:

$$\varepsilon_c = 0,5 \varepsilon_s M_t^2 \quad (55)$$

For the model of Adumitroaie et al.(1999), the compressible parameters d_1 and d_2 are determined by using the model of Sarkar (1992)

$$d_2 = 0.15 M_t \quad \text{and} \quad d_1 = 0. \quad (56)$$

5. RESULTS AND DISCUSSION

The two free streams of the fully developed compressible mixing layer are characterized typically by the convective Mach number M_c , the parameters

$s = \frac{\rho_2}{\rho_1}$ and $r = \frac{U_2}{U_1}$ are respectively the density and velocity ratios, the experiments conditions of Goebel et al.(1991) are listed in Table.1

The values of the constants models used in the present simulation are:

$$C_{\varepsilon 1} = 1,4, C_{\varepsilon 2} = 1,8, C_{\mu} = 0,09, C_{\varepsilon} = 0,25, C_T = 0,26.$$

In this study, we consider predictions of two models for the pressure-strain correlation: the Launder Reece and Rodi (LRR) model(1975) and the Adumitroaie et al.

model (1999). Two versions of the Adumitroaie et al. model will be considered : the original version (without correction of the C_1 -coefficient and the version where the proposed M_t -correction of C_1 is included.

Table 1 Experience of Goebel and Dutton

| Mc | 0.2 | 0.46 | 0.69 | 0.86 | 1 |
|-----------------------------|------|------|------|------|------|
| $r = \frac{U_2}{U_1}$ | 0.78 | 0.57 | 0.18 | 0.16 | 0.16 |
| $s = \frac{\rho_2}{\rho_1}$ | 0.76 | 1.55 | 0.57 | 0.6 | 1.14 |

The fundamental parameter characterizing the effects of compressibility on the mixing layer is the growth rate $\frac{d\delta}{dx}$, δ denotes the momentum thickness of the mixing layer.

Figure 1 shows the comparison between the computed normalized growth rate by its incompressible counterpart $G = \frac{d\delta}{dx} / (\frac{d\delta}{dx})_{M_c=0}$ with different experiment results available in the literature and with those obtained by empirical formula of Dimotakis (1991):

$$G = 0,8 \exp(-M_t^2) + 0,2 \quad (57)$$

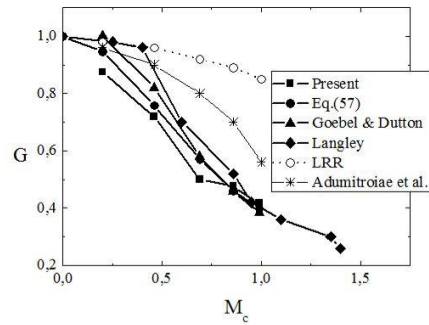


Fig.1. Normalized growth rate G versus M_c

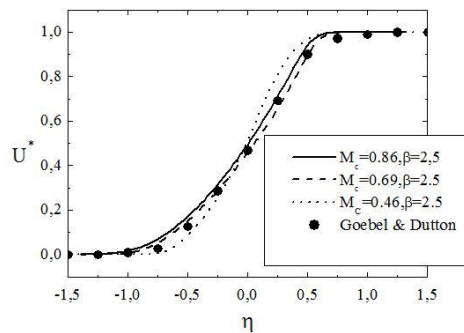


Fig. 2. Similarity of the mean velocity

The calculated growth rate G decreases with increasing convective Mach number: a phenomenon which has often been observed in experimental studies of compressible mixing layers. The LRR model over predicts the growth rate G and similar results are obtained for the Reynolds stress similarity (results are not included in Fig. 2 and Fig.3). With the proposed correction of the slow part of the pressure-strain

correlation, the model of Adumitroaie et al. is now able to properly predict the growth rate G .

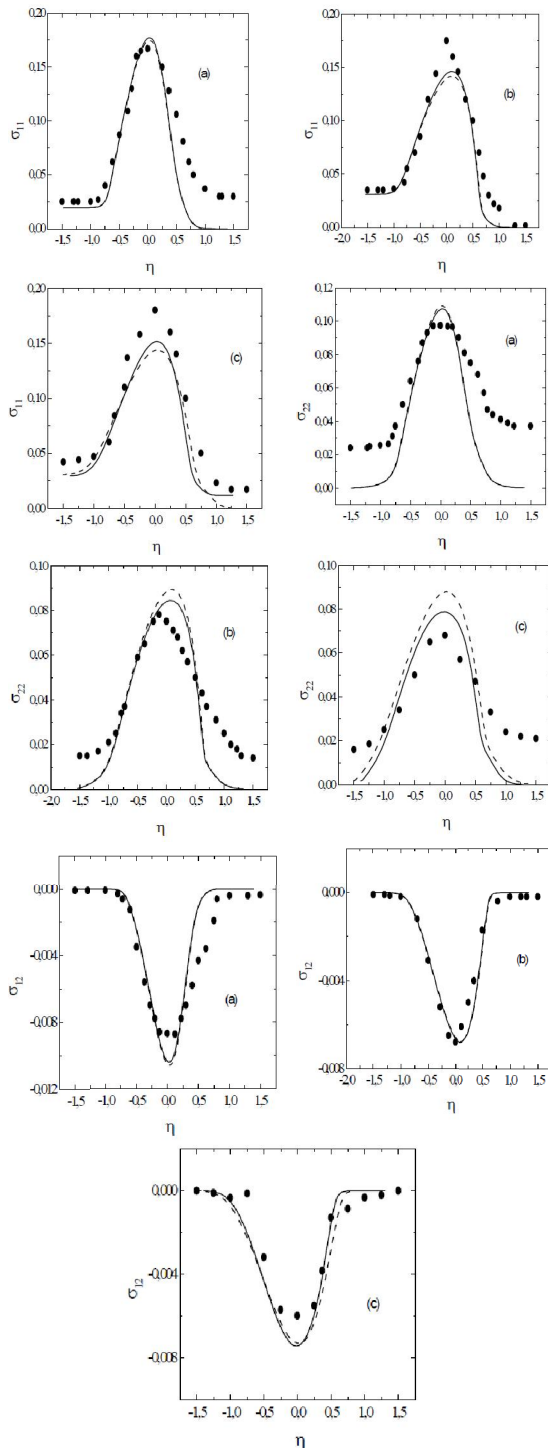


Fig. 3. Similarity profiles of Reynolds stress: (a) $M_c = 0,46$, (b) $M_c = 0,69$, (c) $M_c = 0,86$. (solid line : $\beta = 2,5$, dashed line : $\beta = 0,0$, symbol: experience of *Goebel et al.*)

The normalized stream mean velocity $U^* = \frac{U - U_2}{U_1 - U_2}$ is represented in relation to the similarity variable $\eta = (y - y_c) / \delta$ in *Fig. 2*, where y is the local cross

stream coordinate and y_c is the cross-stream coordinates corresponding to $U^* = 0,5$.

The calculated velocity profiles with the corrected version of the Adumitroaie et al. model are in reasonable agreement with experimental data. *Figure 3* shows the computed results of the Reynolds similarity intensities: the streamwise intensity $\sigma_{11} = \sqrt{R_{11} / (U_1 - U_2)^2}$, the transverse intensity $\sigma_{22} = \sqrt{R_{22} / (U_1 - U_2)^2}$, and the shear stress $\sigma_{12} = R_{12} / (U_1 - U_2)^2$, obtained from the two versions model of Adumitroaie et al. are compared with experiments results of Goebel and Dutton. It is clear that the two versions model lead to similar results for small value of convective Mach number ($M_c = 0,46$). But, when the compressibility effects are more significant ($M_c = 0,69$, $M_c = 0,86$), the effects of the proposed correction model are clearly manifested on the normal turbulent Reynolds stress, particularly on the transverse similarity intensity. But the shear stress is not affected by the proposed correction model.

6. CONCLUSION

In this study, the widely used second order closures has been used for the prediction of compressible mixing layer. The standard -stress closure with the addition of the dilatational terms: the pressure dilatation correlation and the turbulent dissipation of the dilatation yields very poor predictions of the changes in the Reynolds stress anisotropy magnitude. The deficiencies of this closure is due to the use of the incompressible models of the pressure-strain correlation. This term controls the structural compressibility effects on the turbulence.

A modification of the standard Rotta's model of the slow part of the pressure strain correlation has been proposed. The usual coefficient C_1 becomes dependent on the turbulent Mach number M_t . In general, the proposed compressibility model with the Adumitroaie model of the mean part of the pressure strain successfully predict the reduced growth rate and the decrease of the normal Reynolds stress peaks with increasing convective Mach number. Also the similarity velocity is well predicted by the proposed model. This leads to the conclusion that the compressibility correction of the slow part of the pressure strain correlation model is found out to be an important issue in the second order closure for the compressible turbulent flows.

ACKNOWLEDGEMENTS

This work is realized in the Fluid Mechanics Laboratory, F.S.T, Tunis, Tunisia.

REFERENCES

Adumitroaie, V., J.R. Ristorchelli and D.B. Taulbee(1999). Progress in Favre Reynolds stress closure. *Physics of Fluids A*(9), 2696-2719.

- Dimotakis, P.E.(1991). Turbulent free shear layer mixing and combustion. In Murhy and curran (eds) progress in astronautics and aeronautics. *AIAA, Washington DC*. 137.
- Elliot, G.S. and M. Samimy (1990). Effects of compressible effects in free shear layers. *Physics of Fluids A*(2), 1231.
- El-Baz, A.M. and B.E Launder (1993). Second moment modeling of compressible mixing layers. In W. Rodi and F.Martelli, editors, *Engineering Turbulence Modeling and Experiments 2*, 63-72.
- Favre, A.(1965). Statistical equations of turbulent gases. *Jour. Mekan.* 4, 361-390.
- Goebel , S.G., J.C. Dutton, H. Krier and J.P. Rence(1990). Mean and turbulent velocity measurements of supersonic mixing layers. *Experiments in Fluids* 8, 263-272.
- Launder , B.E., G.J. Reece and Rodi (1975). Progress in the development of a Reynolds stress turbulence closure. *Journal of Fluid Mech.* 68, 537-566.
- Ristorcelli, J.R. (1977). A pseudo-sound constitutive relationship for the dilatational covariance in compressible turbulence. *Journal of Fluid Mech.* 347, 37-70.
- Sarkar, S., Erlebacher, G., M.Y. Hussaini, and H.O. Kress, (1991). The analysis and modeling of dilatational terms in compressible turbulence. *Journal of Fluid Mech.* 277, 473-493.
- Sarkar, S. (1992). The pressure dilatation correlation in compressible flows. *Physics of Fluids A*(4), 2674-2682.
- Sarkar, S. (1995). The stabilizing effects of compressibility in turbulent shear flows. pressure dilatation correlation in compressible flows. *Journal of Fluid Mech.* 282, 163-186.
- Taulbee , D. and J. Van Osdol(1991). Modeling turbulent compressible flows . The mass fluctuating velocity and squared density. *AIAA paper* 91-0524.
- Rotta,J.(1951). Statische theorie nichthomogener . *Turbulenz Zeitschrift für Physik* 129, 547-572.