

# Stokes Flow around Rotating Axially Symmetric Pervious Body

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## ABSTRACT

In this paper, the problem of slowly rotating pervious axially symmetric body with source at its centre placed in an incompressible viscous fluid has been tackled. The method of separation of variables has been used and the general solution in terms of Legendre functions and Whittaker's polynomial is given. As a first approximation, for  $n = 1$ , the results are in confirmation with spherical body. It is found that the effect of source at the centre is to reduce the resulting moment. Further, it has been conjectured that the results of couple for other bodies ( i.e., for  $n \geq 2$  ) can also be obtained on the same ground but presently it is beyond the scope of the paper and would likely to appear in the future paper.

**Keywords:** Slow rotation, Axially symmetric bodies, Source, Viscous fluid, Stokes approximation.

## NOMENCLATURE

|            |  |                   |                             |
|------------|--|-------------------|-----------------------------|
| $e_x$      | unit vector along x -axis              | $r, \theta, \phi$ | spherical polar coordinates |
| $e_r$      | unit vector along radial direction     | $s=Q/va$          | source parameter            |
| $e_\theta$ | unit vector along transverse direction | $u$               | velocity vector             |
| $e_\phi$   | unit vector along azimuthal direction  | $W_{k,t}$         | Whittaker's function        |
| $M$        | moment                                 | $\rho$            | fluid density               |
| $M(a,b,z)$ | Kummer function                        | $\sigma_{r\phi}$  | viscous stress              |
| $n$        | positive integer                       | $\nu$             | kinematic viscosity         |
| $p$        | pressure                               | $\Omega$          | angular velocity            |
| $Q$        | strength of fluid source               | $\nabla^2$        | Laplacian operator          |

## 1. INTRODUCTION

The problem of determining the couple experienced by an axially symmetric body, rotating steadily in a viscous and incompressible fluid has engaged the attention of many authors in the amongst others by Collins (1955), Childress (1964), Davis (1980), Datta (1973), Datta and Srivastava (2000), Dennis *et al.* (1981), Hackborn (1990), Happel and Brenner (1965), Jeffery (1915), Lamb (1952), Kanwal (1961), Macaraeg (1986), Ranger (1971), Rieutord and Valdetaro (1997), Smith (1981), Tanzosh and Stone (1994), Vasudevaiah and Majhi (1982), Datta and Srivastava (2000), Kim and Choi (2002), Felderhof (2007), Murthy *et al.* (2007), Marcello (2008), Wan (2009), Arbaret *et al.* (2011), Ashmawy (2011), Chang and Keh (2012).

Slow rotation of spheroids(including the disc) in an infinite fluid was first solved by Jeffrey (1915) using curvilinear coordinates. His approach was later

extended to the spherical lens, torus, and other axisymmetric shapes. Collins (1955) have drawn an analogy between the slow steady rotation of a solid of revolution about its axis in a viscous fluid and the motion, with constant velocity. The flow generated by an axi-symmetric body due to the rotation about its axis of symmetry in a viscous fluid was studied by R.P. Kanwal (1961) by treating the flow to be Stokesian(i.e. by neglecting non linear convective terms) and found a general formula for couple as a limit on the toroidal velocity. Rubinow and Keller (1961) have considered the force on a spinning sphere which is moving through an incompressible viscous fluid by employing the method of matched asymptotic expansions to describe the asymmetric flow. Brenner (1961) also obtained some general results for the drag and couple on an obstacle which is moving through the fluid. Childress (1964) has investigated the motion of a sphere moving through a rotating fluid and calculated a correction to

the drag coefficient. [Thomas and Walter \(1964\)](#) considered the rotation of a sphere in a visco-elastic fluid  $B'$  (known as Walters fluid). The solution is obtained for the velocity using regular perturbation method with rotation Reynolds number as the perturbation parameter. [Manohar \(1965\)](#) considered the flow of slightly visco elastic fluid due to slow rotation of a sphere contained in a cylinder by neglecting nonlinear terms. [Wakiya \(1967\)](#) numerically evaluated the drag and angular velocity experienced by freely rotating spheres and compared with calculated from corresponding approximate formulae known before. [Barrett \(1967\)](#) has tackled the problem of impulsively started sphere rotating with angular velocity  $\Omega$  about a diameter. He modified the standard time-dependent boundary layer equation to give series solutions satisfying all the boundary conditions and gave solutions that are applicable at small times for non-zero Reynolds numbers. He found that the velocity components decay algebraically rather than exponentially at large distances. [Rao et al. \(1969\)](#) have tackled the problem of slow steady rotation of a sphere in a micropolar fluid. [Kanwal \(1970\)](#) has considered a disk performing simple harmonic rotary oscillations about its axis of symmetry in a non-conducting viscous fluid which is at rest at infinity. [Ranger \(1971\)](#) tackled the problem of axially symmetric flow past a rotating sphere due to a uniform stream of infinity. He has shown that leading terms for the flow consists of a linear superposition of a primary Stokes flow past a non-rotating sphere together with an anti symmetric secondary flow in the azimuthal plane induced by the spinning sphere. [Cooley \(1971\)](#) has investigated the problem of fluid motion generated by a sphere rotating close to a fixed sphere about a diameter perpendicular to the line of centers in the case when the motion is sufficiently slow to permit the linearization of the Navier-Stokes equations by neglecting the inertia terms. He used a method of matched asymptotic expansions to find asymptotic expressions for the forces and couples acting on the spheres as the minimum clearance between them tends to zero. In his paper, the forces and couples are shown to have the form  $a_0 \ln \varepsilon + a_1 + o(\varepsilon \ln \varepsilon)$ , where  $\varepsilon$  is the ratio of the minimum clearance between the spheres and the radius of the rotating sphere and where  $a_0$  and  $a_1$  are found explicitly. Using toroidal coordinates, an exact solution is derived for the velocity field induced in two immiscible semi-infinite fluids possessing a plane interface, by the slow rotation of an axially symmetric body partly immersed in each fluid by [Schneider et al. \(1973\)](#). [Takagi \(1974\)](#) has considered the flow around a spinning sphere moving in a viscous fluid. [Takagi \(1974\)](#) has studied the Stokes flow for the case in which two solid spheres in contact are steadily rotating with different angular velocities about their line of centers. [Takagi \(1977\)](#) further studied the problem of steady flow which is induced by the slow rotation of a solid sphere immersed in an infinite incompressible viscous fluid, on the basis of Navier-Stokes equations. He obtained the solution in the form of power series with respect to Reynolds number. [Ramkissoon \(1977\)](#) considered steady rotation of an axi-symmetric body in a micropolar fluid by neglecting non-linear terms. A fundamental singular solution due to a point force was derived and the method of associated matrices is used as an alternative method. He

derived a general formula for couple similar to that of [Kanwal \(1961\)](#). [Gierszewski and Cheffy \(1978\)](#) studied the problem of rotation of an isolated triaxial ellipsoid suspended in slow viscous flow. [Drew \(1978\)](#) has found the force on a small sphere translating relative to a slow viscous flow to order of the  $\frac{1}{2}$  power of  $Re$  for two different fluid flows far from the sphere, namely pure rotation and pure shear. For pure rotation, the correction of this order to the Stokes drag consists of an increase in the drag. [Kim \(1980\)](#) has calculated the torque and frictional force exerted by a viscous fluid on a sphere rotating on the axis of a circular cone of arbitrary vertex angle about an axis perpendicular to the cone axis in the Stokes approximation. [Fosdick and Kao \(1980\)](#) considered the slow steady rotation of a sphere in a simple fluid of order 4 in order to account the formation of a cap observed experimentally near the pole of the rotating sphere in a non-Newtonian fluid. [Smith \(1981\)](#) studied the influence of rotation in slow viscous flows. [Dennis et al. \(1981\)](#) considered the slow steady rotation of a sphere in a viscous fluid. The couple was obtained for a wide range of Reynolds number using finite difference like special technique. [Rao and Iyengar \(1981, 1983\)](#) studied the flow due to the slow rotation and rotary oscillations of a spheroid in a micropolar fluid neglecting the convective terms. [Davis and Brenner \(1986\)](#) have used the matched asymptotic expansion methods to solve the problem of steady rotation of a tethered sphere at small, non-zero Reynolds numbers. They obtained first order Taylor number correction to both the Stokes-law drag and Kirchhoff's law couple on the sphere for Rossby numbers of order unity. [O'Neill and Yano \(1988\)](#) derived the boundary condition at the surfactant and substrate fluids caused by the slow rotation of a solid sphere which is partially submerged in the substrate fluid. The problems of slow rotation and rotary oscillations of an approximate sphere in a micropolar fluid was studied by [Iyengar and Srinivasacharya \(1995, 2001\)](#). [Ranger \(1996\)](#) has found an exact solution of the Navier-Stokes equations for the axi-symmetric motion (with swirl) representing exponentially time-dependent decay of a solid sphere translating and rotating in a viscous fluid relative to a uniform stream whose speed also decays exponentially with time. He also described a similar solution for the two-dimensional analogue where the sphere is replaced by a circular cylinder of infinite length. [Tekasakul et al. \(1998\)](#) have studied the problem of the rotatory oscillation of an axi-symmetric body in an axi-symmetric viscous flow at low Reynolds numbers. They evaluated numerically the local stresses and torques on a selection of free, oscillating, axi-symmetric bodies in the continuum regime in an axi-symmetric viscous incompressible flow. [Datta and Srivastava \(2000\)](#) have tackled the problem of slow rotation of a sphere with fluid source at its centre in a viscous fluid. In their investigation, it was found that the effect of fluid source at the centre is to reduce the couple on slowly rotating sphere about its diameter. [Datta and Pandya \(2001\)](#) tackled the problem of axial symmetric rotation of a partially immersed body in a liquid with a surfactant layer. [Kim and Choi \(2002\)](#) conducted the numerical simulations for laminar flow past a sphere rotating in the streamwise direction, in order to investigate the effect of the rotation on the

characteristics of flow over the sphere. [Tekasakul and Loyalka \(2003\)](#) have investigated the rotary oscillations of several axisymmetric bodies in axisymmetric viscous flows with slip. A numerical method based on the Green's function technique is used and analytic solutions for local stress and torque on spheres and spheroids as function of the frequency parameter and the slip coefficients are obtained. They have analyzed that in all cases, slip reduces stress and torque, and increasingly so with the increasing frequency parameter. [Davis \(2006\)](#) obtained the expression for force and torque on a rotating sphere close to and within a fluid-filled rotating sphere. [Felderhof \(2007\)](#) considered the Stokes problem of impulsively twisted sphere in an incompressible viscous fluid. [Murthy et al. \(2007\)](#) have discussed the flow generated by the slow steady rotation of a permeable sphere about its axis of symmetry in an incompressible micropolar fluid. [Marcello \(2008\)](#) has introduced new exact analytic solutions for the rotational motion of a axially symmetric rigid body having two equal principal moments of inertia and subjected to an external torque which is constant in magnitude. [Felderhof \(2011\)](#) studied the transient flow caused by a sudden impulse or twist applied to a sphere immersed in a viscous incompressible fluid. [Arbaret et al. \(2011\)](#) discussed the effect of shape and orientation on rigid particle rotation and matrix deformation in simple shear flow. [Ashmawy \(2011\)](#) discussed the rotational motion of an arbitrary axisymmetric body in a viscous fluid using a combined analytical-numerical technique. [Mendez \(2011\)](#) discussed the problem of a flow model for the settling velocities of non spherical particles in creeping motion.

The purpose of this paper is to study slow rotation of a axisymmetric body, assumed to be pervious, with source at its centre. Let 'Q' be the source strength and  $\Omega$  be the angular velocity of the slowly rotating axially symmetric body. We assume that the source strength 'Q' is of the same order as the angular velocity  $\Omega$  of the rotating body so that non-linear inertia terms could be neglected. In this situation the total flow consists of only the source solution superimposed on the Stokes solution and the Stokes drag and couple are not affected by the source. On the other hand when we assume that 'Q' is large enough so that  $Q\Omega$  is not negligible, the inertia terms being non-linear, cannot be altogether omitted, the equation, however, can still be linearized by assuming that the velocity perturbation due to the source flow on account of the Stokes flow is small so that the terms containing square of angular velocity can be neglected. This assumption is justifiable at least in the vicinity of the axi-symmetric body where the Stokes approximation is valid too. The present problem corresponds to the problem of Stokes flow past a sphere with source at its centre investigated by [Datta \(1973\)](#) and slow rotation of sphere with source at its centre by [Datta and Srivastava \(2000\)](#). The results of which have found application in investigating the diffusiophoresis target efficiency for evaporation or condensing drop by [Placek and Peters \(1980\)](#).

## 2. FORMULATION OF THE PROBLEM

Let us consider a pervious axisymmetric body with source of strength Q at its centre generating radial flow

field around it in an infinite expanse of incompressible fluid of density  $\rho$  and kinematic viscosity  $\nu$ . The body also rotates with small steady angular velocity  $\Omega$  so that terms of order  $O(\Omega^2)$  may be neglected but the terms of order  $O(Q\Omega)$  retained.

The motion is governed by Navier-Stokes equations

$$\mathbf{u} \cdot \text{grad} \mathbf{u} = -(1/\rho) \text{grad} p + \nu \nabla^2 \mathbf{u}, \tag{1}$$

and continuity equation

$$\text{div} \mathbf{u} = 0, \tag{2}$$

together with no-slip boundary condition

$$\mathbf{u} = \Omega \hat{\mathbf{e}}_x \times \mathbf{r} \hat{\mathbf{e}}_r, \text{ on the surface, } r=r(\theta), \tag{3}$$

where 'a' is axial length, and the condition of vanishing of velocity at far off points

$$\mathbf{u} = 0, \text{ at infinity as } r \rightarrow \infty. \tag{4}$$

It will be convenient to work in spherical polar coordinates  $(r, \theta, \phi)$  with x-axis as the polar axis. We non-dimensionalize the space variables by 'a' (axial length), velocity by  $a\Omega$  and pressure by  $\rho\nu\Omega$ . The velocity vector, in non-dimensional form, may be express as

$$\mathbf{u} = \frac{Q}{a^2 r^2} \hat{\mathbf{e}}_r + a\Omega(u_r \hat{\mathbf{e}}_r + u_\theta \hat{\mathbf{e}}_\theta) + a\Omega u_\phi \hat{\mathbf{e}}_\phi \tag{5}$$

and pressure(non-dimensional form) as

$$p = \rho\nu\Omega [p_0 + p_1(r,\theta)] \tag{6}$$

where  $u_r, u_\theta, u_\phi$  are velocity components in spherical polar coordinates with unit vectors  $\hat{\mathbf{e}}_r, \hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_\phi$ .

On substituting the values of  $\mathbf{u}$  and  $p$  from Eq. (5) and Eq. (6) in Eq. (1), we get

$$\nabla^2 u_r - \frac{2u_r}{r^2} - \frac{2}{r^2 \sin\theta} \frac{\partial}{\partial \theta} (u_\theta \sin\theta) = \frac{\partial}{\partial r} \left[ p_0 + \frac{Q^2}{\rho\mu\Omega a^4 r^4} + p_1 + \frac{su_r}{r^2} \right] \tag{7a}$$

$$\nabla^2 u_\theta + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2 \sin^2\theta} - \frac{s}{r^3} \left[ \frac{\partial(ru_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right] = \frac{1}{r} \frac{\partial}{\partial \theta} \left( p_1 + \frac{su_r}{r} \right) \tag{7b}$$

$$\nabla^2 u_\phi - \frac{u_\phi}{r^2 \sin^2\theta} = \frac{s}{r^3} \frac{\partial}{\partial r} (ru_\phi) \tag{7c}$$

where  $s = Q/\nu a$  is source parameter and  $\nabla^2$  is the Laplacian operator in spherical polar coordinates defined as

$$\nabla^2 \equiv \frac{1}{r^2} \left[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right] \quad (8)$$

On assuming the polar axis, in spherical polar coordinates, in the direction of angular velocity  $\Omega$ , we have  $u_r = u_\theta = 0$ . The Eq. (7a) and Eq. (7b) provides

$$p_0 = -\frac{Q^2}{2\nu\Omega a^4 r^4} + k(\theta) \quad \text{and} \quad p_1 = p_1(r) \quad (9)$$

where  $k(\theta)$  is constant function of integration. So the expression of pressure will be

$$p = \rho\nu\Omega \left[ -\frac{Q^2}{2\nu\Omega a^4 r^4} - p_1(r) \right] + k(\theta) \quad (10)$$

Now Eq. (7c) is to be solved under the boundary conditions

$$\begin{cases} u_\phi = a\Omega \sin \theta & \text{at } r = r(\theta) \\ u_\phi \rightarrow 0 & \text{as } r \rightarrow \infty \end{cases} \quad (11)$$

### 3. SOLUTION

Let us take the solution of Eq. (7c) in the form

$$u_\phi = F(r, s) \cdot G(\theta) \quad (12)$$

After evaluating the constituent parts, Eq. (7c) reduces to

$$\frac{FG}{r^2} \left[ \frac{1}{F} \frac{d}{dr} \left( r^2 \frac{dF}{dr} \right) - \frac{s}{r} \frac{d}{dr} (rF) + \frac{1}{G} \frac{d}{d\mu} \left\{ (1-\mu^2) \frac{dG}{d\mu} \right\} - \frac{1}{1-\mu^2} \right] = 0 \quad (13)$$

where  $\mu = \cos \theta$ , on writing,

$$\frac{1}{F} \left[ \frac{d}{dr} \left( r^2 \frac{dF}{dr} \right) - \frac{s}{r} \frac{d}{dr} (rF) \right] = n(n+1) \quad (14)$$

Or

$$\frac{d}{dr} \left( r^2 \frac{dF}{dr} \right) - \frac{s}{r} \frac{d}{dr} (rF) - n(n+1)F = 0$$

with 'n' as positive integer. In particular, for  $n = 1$ , we get the equation, solution of which contribute in slowly rotating sphere with source at its centre. Viz.; the equation will be

$$\frac{d}{dr} \left( r^2 \frac{dF}{dr} \right) - \frac{s}{r} \frac{d}{dr} (rF) - 2F = 0 \quad (15)$$

Now, with the help of Eq. (14), Eq. (13) reduces to

$$\frac{d}{d\mu} \left\{ (1-\mu^2) \frac{dG}{d\mu} \right\} + \left\{ n(n+1) - \frac{1}{1-\mu^2} \right\} G = 0, \quad (16)$$

$\mu = \cos \theta$

in this case too, for  $n = 1$ , we can have

$$\frac{d}{d\mu} \left\{ (1-\mu^2) \frac{dG}{d\mu} \right\} + \left\{ 2 - \frac{1}{1-\mu^2} \right\} G = 0, \quad (17)$$

which has obvious solution,  $G(\theta) = \sin \theta$ .

In general, the solution of Eq. (16) can easily be expressed in terms of Legendre Functions (Abramowitz and Stegun, 1968)

$$\begin{aligned} G(\theta) &= G_n(\theta) \\ &= P_n^1(\mu), \quad \mu = \cos \theta \\ &= -\sqrt{1-\mu^2} P_n'(\mu) \\ &= -\frac{d}{d\theta} \{ P_n(\cos \theta) \}, \end{aligned}$$

which, for  $n = 1$ , provide  $G(\theta) = \sin \theta$ .

Now, we discuss the general solution of Eq. (14), we take,  $s/r = z$ , then Eq. (14) reduces to

$$\frac{d^2 F}{dz^2} + \frac{dF}{dz} - \left\{ \frac{1}{z} + \frac{n(n+1)}{z^2} \right\} F = 0 \quad (18)$$

which is second order linear differential equation in

$$\text{standard form with } Q(z) = -\left\{ \frac{1}{z} + \frac{n(n+1)}{z^2} \right\},$$

$P(z) = 1$ , Converting it into normal form, with  $F(z) = u(z) e^{-z^2}$ , this equation further reduces to

$$\frac{d^2 u}{dz^2} + \left\{ -\frac{1}{4} - \frac{1}{z} - \frac{n(n+1)}{z^2} \right\} u(z) = 0 \quad (19)$$

which is a Whittaker's equation (Abramowitz and Stegun, 1968) for

$$k = -1 \quad \text{and} \quad \frac{1}{4} - t^2 = -n(n+1) \quad (20)$$

in particular, for  $n = 1$ ,  $k = -1$ ,  $t = \pm 3/2$ ,  $n = 2$ ,  $k = -1$ ,  $t = \pm 5/2$ ,  $n = 3$ ,  $k = -1$ ,  $t = \pm 7/2$  and so on.

$$\text{And in general, } t^2 = \frac{1}{4} + n(n+1).$$

Now, the solution of Whittaker's equation will be in terms of Whittaker's function (Abramowitz and Stegun, 1968) defined as

$$W_{k,t}(z) = e^{-\frac{z}{2}} z^{\frac{1}{2}+t} U\left(\frac{1}{2} + t - k, 1 + 2t, z\right) \quad (21)$$

where

$$U(a, b, z) = \frac{\pi}{\sin \pi b} \left[ \frac{M(a, b, z)}{(2+a-b)! (b+1)!} - \left\{ z^{(1-b)} \right\} \frac{M(1+a-b, 2-b, z)}{(1+a)! (3-b)!} \right] \quad (22)$$

with  $a, b$  are real numbers and  $M(a, b, z)$  is Kummer function (Abramowitz and Stegun, 1968) defined as

$$M(a, b, z) = 1 + \frac{a}{b} z + \frac{(a)_2}{(b)_2} \frac{z^2}{2!} + \frac{(a)_3}{(b)_3} \frac{z^3}{3!} + \dots + \frac{(a)_n}{(b)_n} \frac{z^n}{n!} + \dots \quad (23)$$

with  $(a)_n = a(a+1)\dots(a+n-1)$ ,  $(a)_0 = 1$ . The restrictions on this Whittaker's functions are  $-\pi < \arg z < \pi$ ,  $k = \frac{b}{2} - 2$ ,  $t = \frac{b}{2} - \frac{1}{2}$ .

So the solution of Eq. (14) may be expressed in terms of Whittaker's function as

$$F(r, s) = e^{-\frac{s}{2r}} W_{k,t} \left( \frac{s}{r} \right), \quad k = -1, \quad (24)$$

$$t = \pm \sqrt{\frac{1}{4} + n(n+1)}$$

Therefore, the azimuthal component, of velocity can be expressed as

$$u_\phi = e^{-\frac{s}{2r}} W_{k,t} \left( \frac{s}{r} \right) \cdot G_n(\theta) \quad (25)$$

Now, we use this result to verify the solution obtained by authors (Datta and Srivastava, 2000) for the rotation of sphere with source at its center.

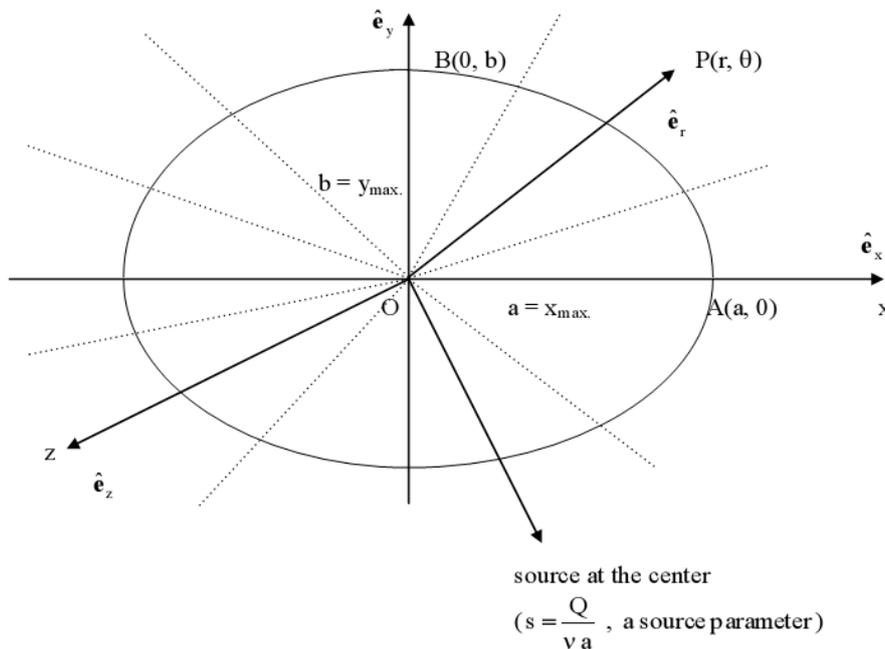


Fig. 1. Rotating axially symmetric pervious body having angular velocity  $\Omega$  with source at its center

#### 4. ROTATION OF THE SPHERE

##### 4.1 The case with $n=1$

With some manipulations, it can be easily analyzed that the azimuthal component of velocity in the case of sphere rotating with small angular velocity  $\Omega$ , under the specific boundary conditions on the surface of sphere,  $r=a$ ,

$$\mathbf{u} = a \Omega \hat{\mathbf{e}}_x \times \hat{\mathbf{e}}_r, \quad (26)$$

and

$$\mathbf{u} = 0, \text{ at infinity as } r \rightarrow \infty \quad (27)$$

comes out to be

$$u_\phi = r \sin\theta \left[ \frac{s^2}{r^2} - 2\frac{s}{r} + 2 - 2e^{-\frac{s}{r}} \right] \left[ s^2 - 2s + 2 - 2e^{-s} \right]^{-1} \quad (28)$$

Then, the couple on the sphere required to maintain the motion is obtained by integration of viscous stress

$$\sigma_{r\phi} = \mu \Omega \left[ \frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r} \right]_{r=1} \quad (29)$$

$$= 2\mu \Omega \sin\theta \left[ -s^2 + s(1 - e^{-s}) \right] \left[ s^2 - 2s + 2 - 2e^{-s} \right]^{-1}$$

thus, the moment of the required couple is given by

$$M = -\int_{\theta=0}^{\pi} \sigma_{r\phi} a^3 2\pi \sin^3 \theta d\theta$$

$$\frac{16}{3} \pi a^3 \mu \Omega \left[ s(1-e^{-s}) - s^2 \right] \left[ 2 - 2s + s^2 - 2e^{-s} \right]^{-1} \quad (30)$$

which match with result of moment given by authors (Datta and Srivastava, 2000).

### 5. NUMERICAL DISCUSSION

When we take up the case of small 's' (source parameter), we get from Eq. (4) and Eq. (5)

$$M = 8 \pi a^3 \mu \Omega \left[ 1 - \frac{s}{12} \right] \quad (31)$$

which provides the classical value  $M_0 = 8\pi a^3 \mu \Omega$  for  $s=0$ .

Also, for large values of 's', we have the approximation as

$$M = \frac{16}{3} \pi a^3 \mu \Omega \left[ 1 + \frac{1}{s} \right] \quad (32)$$

We find that  $M \rightarrow \frac{2}{3} M_0$ , as  $s \rightarrow \infty$ .

The variation of moment coefficient  $C_M = \frac{M}{M_0}$  with

source parameter  $s = \frac{Q}{va}$ , where  $M_0$  is the moment for  $s = 0$ , has been shown in Fig. 2, which clearly shows that the effect of source at the centre of the sphere is to reduce the moment ultimately to two third of its value in the absence of source.

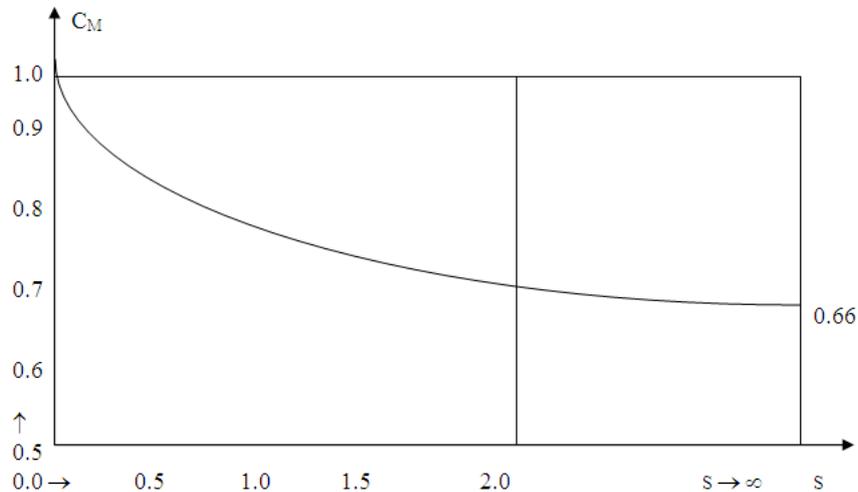


Fig. 2. Variation of moment coefficient  $C_M$  with source parameter 's'

### 6. CONCLUSION

The sections 4 and 5 of the paper are devoted to the verification of the general solution for azimuthal component of velocity in case of spherical body and subsequently other results such as viscous stress and moment have been discussed for the same body. Further, at this stage, we may only conjecture, that for  $n > 1$  ( $n \geq 2$ ), there are some other axi-symmetric bodies, which could be of spheroidal shape, slightly deformed sphere etc. We reserve this part of the work for future investigation. Based on section 5, we conjectured that the effect of fluid source at the centre of axially symmetric slowly rotating body is to reduce the moment.

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