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A Modified Time Advancement Algorithm for Optimizing Channel Flow Analysis in Direct Numerical Simulation Method

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ABSTRACT

In this research a direct numerical simulation (DNS) of turbulent flow is performed in a geometrically standard case like plane channel flow. Pseudo spectral (PS) method is used due to geometry specifications and very high accuracy achieved despite relatively few grid points. A variable time-stepping algorithm is proposed which may reduce requirement of computational cost in simulation of such wall-bounded flow. Channel flow analysis is performed with both constant and varied time-step for 128 × 65×128 grid points. The time advancement is carried out by implicit third-order backward differentiation scheme for linear terms and explicit forward Euler for nonlinear convection term. PS method is used in Cartesian coordinates with Chebychev polynomial expansion in normal direction for one non-periodic boundary condition. Also Fourier series is employed in stream-wise and span-wise directions for two periodic boundary conditions. The friction Reynolds number is about Re_τ=175 based on a friction velocity and channel half width. Standard common rotational form was chosen for discritization of nonlinear convective term of Navier-Stocks equation. The comparison is made between turbulent quantities such as the turbulent statistics, Reynolds stress, wall shear velocity, standard deviation of (u) and total normalized energy of instantaneous velocities in both time-discretization methods. The results show that if final decision rests on economics, the proposed variable time-stepping algorithm will be proper choice which satisfies the accuracy and reduces the computational cost.

Keywords: Channel flow, Pseudo spectral method, Direct numerical simulation, Variable time-stepping algorithm.

NOMENCLATURE

a	upper wall boundary condition	U^+	wall-units for velocity
b	lower wall boundary condition	u(x)	vector of velocity flow
C (u)	the constant term of NSE	$u_{tot}(x;t)$	total fluid velocity field
$\tilde{C}$	Fourier transform of C	$u(x; t)$	fluctuating velocity
c	constant of wall law	$\hat{\hat{u}}_{kx,ny,kz}$	spectral coefficients of u
e_x	unit vector in x direction	u_τ	wall-shear velocity

k_x	wave number in x direction	$\tilde{u}$	Fourier transform of *u*
k_z	wave number in z direction	u^*	friction velocity
J	number of discretized step	ι_i	instantaneous velocity
L (u)	linear term of NSE	y^+	wall-units for length
$\tilde{L}$	Fourier transform of L	α	coefficient of the discretization formula in SBDF3
L_x	period in stream wise direction	β	coefficient of the discretization formula in SBDF3
L_y	period in normal direction	ζ	coefficient of the discretization formula in SBDF3
L_z	period in span wise direction	ΔT	time step of recorded information
N (u)	nonlinear term of NSE	Δt	varied or constant time step
$\tilde{N}$	Fourier transform of N	Δt_0	initial time step in varied form
N_y	vumber of discretization in normal directions	$\Delta t_{\min}$	minimum values of time step in varied form
$p_{tot}(x;t)$	total pressure field	$\Delta t_{\max}$	maximum values of time step in varied form
p(x; t)	periodic fluctuating pressure	Δx^+	grid spacing in the stream-wise direction in wall unit
q	modified pressure	$\Delta y^+_{\min}$	minimum grid spacing in normal directions in wall unit
Re_L	Initial Reynolds number	$\Delta y^+_{\max}$	maximum grid spacing in normal directions in wall unit
Re_τ	friction Reynolds number	Δz^+	grid spacing in the span-wise directions in wall unit
T_m	mth Chebyshev polynomial	$\Pi_x(t)$	spatially-constant base pressure gradient
T	time of recorded information	Ω	computational domain
U	velocity of base flow	$\partial\Omega_0$	boundary of sub-domain Ω_0
U(y)	mean velocity profile	ν	viscosity

1. INTRODUCTION

In fluid dynamics turbulent flows are not the exception. Virtually most kinds of flows either generated by nature or by human industrial activity are turbulent. Turbulent flows consist of a wide range of scales from a few micrometers in high-speed aircrafts to parsec scales for astrophysical flows such as conventional matter in the Universe. The governing equations of such disordered flow in space and time are deterministic, but due to the nonlinear interactions, the evolution in time is very complicated.

Numerical methods can be used to study the properties of turbulent flows and improve the basic understanding of turbulent field. This would be a complete description of turbulent flow, where the flow variables (e.g. velocity and pressure) are known as a function of space and time with the resolution of all scales, which can only be obtained by solving the Navier-Stokes equations (NSE) numerically. This approach is called direct numerical simulation (DNS). In the last decades, DNS has started to gain popularity due to significant progress in the speed and capacity of the computers, and also the development of efficient and accurate algorithms. Direct numerical simulation is a time-dependent three dimensional solution in which the governing equations are computed as accurately as possible without using any turbulence models. Precise knowledge of every fluid particle's position, velocity, pressure and their derivatives is provided in a wide range of information in the flow field. These are extremely difficult to be measured in experiments. Yet DNS is used to simulate flow in a channel with rough-walls (Bhaganagar and Leighton ,2013).

The development of the theory in fluid mechanics is closely coupled to experiments. It was not until the 19th century that mathematicians developed the fundamental equations called the NSE, which governs viscous flow. After developing the digital computer, researchers start to simulate turbulent flows, numerically.

The simulations of turbulence in three dimensional flows are complicated. It dramatically increases the number of required grid points compared to the two-dimensional flow calculation. Still with such features, the method demands high computer memory compared to the conventional techniques. In recent years, the developments of supercomputers facilitate such numerical simulations of fluid flows in different cases. However, with present resources, a direct simulation is feasible for simulation of transient, three-dimensional (3D) flows in or around simple geometries at low and moderate Reynolds numbers. Nowadays both finite-difference schemes and spectral schemes are used for direct numerical simulations of the fluid flows .Yet it seems spectral methods are the best tools to achieve higher accuracy on a simple domain and demand less computer memory than other alternatives.

The first attempt of the three dimensional DNS was made by Orszag and Patterson (1972) to perform computation of isotropic turbulence at a Reynolds number (based on Taylor microscale) of 35. More recently, the DNS of the fully developed turbulent channel flow started for the wall turbulence. The DNS of plane channel flow was performed by Kim *et al*. (1987). Their Reynolds number based on the friction velocity ut and the channel half width d was Reτ=180. Since then, the channel flow has proven to be an extremely useful framework for the study of wall-bounded turbulence. It has often been performed because of its simple geometry and fundamental nature to understand the transport mechanism. Afterwards various Reynolds number channel simulations have been performed by many others. Kuroda *et al*. (1989) and Kasagi *et al*. (1992) carried out the DNS for a slightly lower Reynolds number of Re_τ=150. Kim *et al*. (1990) also performed a DNS with a higher Reynolds number of Re_τ=395. Kawamura *et al*. (1998, 1999) performed the DNS to include the scalar transport with various Prandtl numbers for Re_τ=180 and 395. They also carried out the DNS for a higher Reynolds number of Re_τ=640 and reported preliminary results in Kawamura *et al*. (1999). In recent years, numerical simulations of fully developed turbulent channel flow at Reynolds numbers up to Re_τ=2320 by Iwamoto *et al*. (2005) and Re_τ=2000 by Hoyas and Jimenez (2006) are also reported.

There is a rich variety of strategies for time discretizing the NSE in DNS. Most commonly used time-discretization strategies are splitting techniques and coupled methods (monolithic methods). In fluid dynamics, the progenitor of splitting methods is the Chorin–Temam method in the late 1960s. In the late 1970s, second-order Adams–Bashforth for explicit terms and second-order Crank–Nicolson for implicit terms were common choices. Low-storage Runge–Kutta (third-order and fourth-order) became popular in the 1980s for explicit terms. In the 1990s, the third-order backward difference scheme came into use for implicit terms that seemed to be more accurate based on a higher order discretization of the time derivative (Canuto *et al*., 2007). This paper presents results from DNS of fully developed plane channel flow with backward difference scheme, which is a prototypical flow used to study physical and numerical modeling of wall-bounded flows frequently. The mathematics of numerical method are based on the spectral channel flow algorithm, proposed by Canuto *et al*.(2006) , with Spectral discretization in spatial directions (Fourier×Chebyshev×Fourier) and finite-differencing in time. The differential equations are Helmholtz equations. The tau-equation solution with tau correction is also applied in discretized form of each Fourier mode. Primitive variables (3D velocity and pressure) are used to integrate the incompressible NSE. The time advancement is carried out by three-stage scheme based on a backward difference algorithm treatment of the linear terms, combined with explicit extrapolation of the nonlinear convection terms. Variable time stepping algorithm is proposed to minimize the computational cost of the integration by maximizing the time step while keeping the Courant–Friedrichs–Lewy condition (CFL condition) near a threshold. It should be noticed that variable time stepping algorithm is applied especially in Parallel CFD, like: Bazavan (2007) and Chien et.al. (1997). Also this algorithm has been used in Gibson et. al. research on turbulent Plane Couette flow to reduce run time in time advancement calculation (Gibson et. al.,2007,2008,2009).

The primitive variable form of the three-dimensional incompressible NSE has several equivalent versions due to the precise manner of expressing the nonlinear terms. The more common alternatives are the convection form, the divergence form, the skew-symmetric form and the rotation form (Zang, 1991). Zang performed tests to compare four alternative formulations of the convective terms in an incompressible NSE in a PS method. It has been shown that the rotational form had poor performance due to aliasing errors. To avoid this loss of accuracy, the number of grid points should be increased in order to produce a meaningful solution, or more accurate form like skew-symmetric scheme should be employed. However, using the skew-symmetric form in place of the rotation adds a significant cost to the calculation. There is a modified technique that reduces or removes the aliasing error from discrete Fourier coefficients of Pseudo spectral methods referred as the 2/3-rule. In this research, the rotational form is being used with de-aliasing in x and z directions by 2/3-rule method to make the process more economical. The effect of aliasing error and proposed de-aliased method in this code has been discussed in Rajabi and Kavianpour (2012) in details.

In this work, a new variable time stepping method is applied to reduce the computational cost in PS method by adding an accessory (supplementary) algorithm. This algorithm is used to minimize the computational cost of integration by maximizing the time step, while the CFL number keeps near a threshold. The CFL number and time step are bound in a given range to control the stability. This time step determines as a fraction of a fixed time-interval to keep CFL number maximum under above condition. Here, a direct numerical simulation for turbulent channel flow with rotational forms of nonlinear terms and 3rd order semi backward difference algorithm (SBDF3) time-discretization methods is computed via variable and constant time stepping algorithms. The results of turbulence intensity and other quantitative values are compared to choose more economic, accurate and stable algorithm.

2. PROBLEM FORMULATION

The channel flow is a remarkable example of wall bounded problems. In this study, a plane Poiseuille flow with two parallel stationary plates and parabolic stream-wise velocity profile base flow is considered. The flow geometry and the coordinate system are shown in Fig. 1.

Flow fields are allocated in terms of their physical grid sizes $128\times65\times128$ and integrated on the computational domain $\Omega = [0,4\pi]\times[-1,1]\times[0,2\pi]$. The flow domain has finite height, with the centerline at zero, but two infinite stream- wise and span-wise directions periodically continued in x and z directions. No-slip boundary conditions at y= ±1 and $U = 1-y^2$ as the base flow are assumed. The initial Reynolds number is 4000 so the viscosity set to be $\nu = 1/4000$. The velocity flow represents by vector-valued Fourier×Chebyshev×Fourier expansions whose mathematical form is as follows;

$$u(x)=\sum_{k_x=-N_x/2+1}^{N_x/2}\ \sum_{n_y=0}^{N_y-1}\ \sum_{k_z=-N_z/2+1}^{N_z/2} \hat{\tilde{u}}_{kx,ny,kz}\, T_m(y)\, e^{2\pi i(k_x x/L_x + k_z z/L_z)}$$

(1)

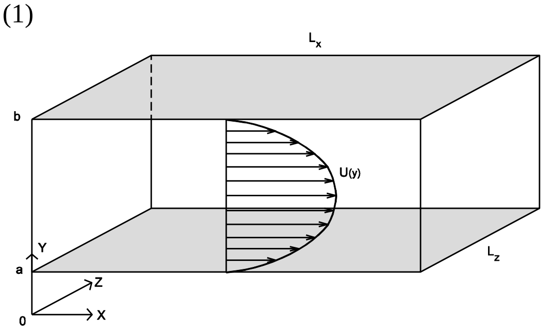

Fig. 1. Schematic of channel flow

where, x=(x,y,z). The double tilde/hat notation on the spectral coefficients $\hat{\tilde{u}}_{kx,ny,kz}$ indicates that the coefficients result from a combined Fourier transform in xz and a Chebyshev transform in y direction. Here, Tm is the mth Chebyshev polynomial rescaled to the interval [-1, 1]. The spectral coefficients of u can be computed efficiently from the function values taken at a discrete set of Chebyshev grid points in the form of;

$$y_n = \cos\left(\frac{n\pi}{N_y-1}\right) \qquad n\in(0,N_y-1) \qquad (2)$$

The discretization in the horizontal directions are done using Fourier series expansions thus assuming periodicity, which is reasonable if the flow is homogeneous in these directions. The NSE for an incompressible flow in the channel flow geometry can be written in the following form:

$$\frac{\partial u_{tot}}{\partial t} + u_{tot}\cdot\nabla u_{tot} = -\nabla p_{tot} + \nu\nabla^2 u_{tot} \qquad (3)$$

$$\nabla u_{tot} = 0 \qquad (4)$$

Where utot(x; t) is the total fluid velocity field with u, v and w components in three dimensions and ptot(x; t) is the total pressure field. The first and second equations represent conservation of momentum and incompressibility of the fluid, respectively. The velocity satisfies the no-slip boundary conditions (u=0) at both upper and lower channel walls (y=a,b). The boundary conditions in the x and z directions are periodic: $u_{tot}(x+L_x,y,z;\ t)= u_{tot}(x,y,z;\ t)$ and $u_{tot}(x,y,z+L_z;\ t)= u_{tot}(x,y,z;\ t)$.

Total velocity and pressure fields can be broken into constant and fluctuating parts, so the velocity field is the sum of the base velocity or base flow U(y)ex, and the fluctuating velocity u(x; t). The total pressure field is obtained from a linear-in-x term $\Pi_x(t)x$ and a periodic fluctuating pressure p(x; t). The gradient of this decomposition relates the total pressure gradient to a spatially-constant base pressure gradient $\Pi_x e_x$ and a fluctuating pressure gradient $p_{tot}(x;\ t)$. Therefore;

$$u_{tot}(x;t) = U(y)\,e_x + u(x;t) \qquad (5)$$

$$p_{tot}(x;t) = x\frac{dP}{dx}(t) + p(x;t) = \Pi_x(t)x + p(x;t) \qquad (6)$$

$$\nabla p_{tot}(x;t) = \frac{dP}{dx}(t)\,e_x + \nabla p(x;t)$$

$$= \Pi_x(t)\,e_x + \nabla p(x;t) \qquad (7)$$

Substituting Eq. (5) and (7) into Eq. (3) gives:

$$\frac{\partial \mathrm{u}}{\partial t} + \nabla p = \nu \nabla^2 \mathrm{u} - \mathrm{u}_{tot} \cdot \nabla \mathrm{u}_{tot} + \left[\nu \frac{\partial^2 U}{\partial y^2} - \Pi_x \right] \mathrm{e}_x$$

(8)

There are several different forms for the nonlinear term $\mathrm{u}_{tot} \cdot \nabla \mathrm{u}_{tot}$ in Eq. (8) that are identical in continuous mathematics but have different properties when discretized. Four commonly used forms of them referred to as divergence, advective, skew-symmetric, and rotational forms. The nonlinear term in rotational forms defined as:

$$\left(\nabla \times \mathrm{u}_{tot}\right) \times \mathrm{u}_{tot} + \frac{1}{2} \nabla \cdot \left(\mathrm{u}_{tot} \mathrm{u}_{tot}\right) \quad (9)$$

Despite the equivalent of these forms in the continuous NSE, the rotation form may offer better physical properties in terms of conservation laws and has superior properties for iterative algorithm development. Rotational form is typically more stable than the convective form and less time consuming than the skew-symmetric form. However, it may lead to a less accurate approximate solution than skew-symmetric form due to aliasing errors that would remove by de-aliased techniques. Eq. (8) could be rewrite as:

$$\frac{\partial \mathrm{u}}{\partial t} + \nabla q = \nu \nabla^2 \mathrm{u} - \mathrm{N(u)} + \left[\nu \frac{\partial^2 U}{\partial y^2} - \Pi_x \right] \quad (10)$$

Here, the modified pressure (q) in rotational form is equal to:

$$q \triangleq p + 1/2\, \mathrm{u.u} \quad (11)$$

And N (u) is the rotational form of nonlinear term that would substitute in this code as follows;

$$\mathrm{N(u)} \triangleq \left(\nabla \times \mathrm{u}\right) \times \mathrm{u} + U \frac{\partial \mathrm{u}}{\partial x} + \nu \frac{\partial U}{\partial y} \mathrm{e}_x \ \text{Rotational} \quad (12)$$

Equation (10) is solved by the Chebychev-tau method for each wave number after it is Fourier transformed in the stream wise and spans wise directions (Canuto *et al*., 2007). Also, there is a need to add tau correction to the solution of the equations in their discretized form which is used to determine the pressure. In formulation of time integration scheme, linear term L (u) and the constant term C in NSE are defined by:

$$\mathrm{L\,u} \triangleq \nu \nabla^2 \mathrm{u} \quad (13)$$

$$\mathrm{C} \triangleq \left[\nu \frac{\partial^2 U}{\partial y^2} - \Pi_x \right] \mathrm{e}_x \quad (14)$$

With these definitions the Eq. (10) can be written as:

$$\frac{\partial \mathrm{u}}{\partial t} + \nabla q = \mathrm{Lu} - \mathrm{N(u)} + \mathrm{C} \quad (15)$$

After Fourier transform, Eq. (15) is equal to;

$$\frac{\partial \tilde{\mathrm{u}}}{\partial t} + \tilde{\nabla} \tilde{q} = \tilde{\mathrm{L}} \tilde{\mathrm{u}} - \mathrm{N(u)} + \tilde{\mathrm{C}} \quad (16)$$

Let $\tilde{\mathrm{u}}^n$ be the approximation of $\tilde{\mathrm{u}}$ at time $t = n\Delta t$ and let $\tilde{\mathrm{N}}^n \triangleq \mathrm{N}(\mathrm{u}^n)$ .Here, splitting method was used that leads to a three stage pressure-correction algorithm based on a backward difference algorithm (BDF) treatment of the linear terms. It is combined with an explicit extrapolation of the nonlinear convection terms. Also, 3rd order discretization of the time derivative based on BDF is used for linear terms. The formulation of three-stage scheme based on implicit treatment for the linear terms combined with an explicit extrapolation of the nonlinear convection terms, which refers as SBDF3 here, is:

$$\frac{1}{\Delta t} \left(\hat{\mathrm{u}}^{n+1} - \sum_{q=0}^{J-1} \alpha_q \mathrm{u}^{n-q} \right) = \sum_{q=0}^{J-1} \beta_q \mathrm{N}^{n-q} \quad \text{in } \Omega \quad (17)$$

Equation (17) is the first stage that consists of solving the explicit problem in the forms of BDF time-discretization formulas. In equation 17 J ≥ 1 is the number of steps, $\zeta_0, \alpha_0, \alpha_1, \ldots, \alpha_{J-1}$ are the coefficients of the BDF formula for the discretization, and

$\beta_0, \beta_1, \ldots, \beta_{J-1}$ are the coefficients of the extrapolation formula for nonlinear term. The time derivative $\frac{dy}{dt}$ at time $n+1$ is:

$$\left.\frac{dy}{dt}\right|_{t=t^{n+1}} \simeq \frac{1}{\Delta t}\left(\zeta_0 y^{n+1} - \sum_{q=0}^{J-1} \alpha_q y^{n-q}\right) \quad (18)$$

Thus, we have;

$$N^{n+1} = \sum_{q=0}^{J-1} \beta_q \mathrm{N}^{n-q} \quad (19)$$

The second and third stages are projection and diffusion step:

$$\text{projection step} \begin{cases} \frac{1}{\Delta t}\left(\hat{\hat{\mathrm{u}}}^{n+1} - \hat{\mathrm{u}}^{n+1}\right) + \nabla p^{n+1} = 0 \text{ in } \Omega \\ \nabla . \hat{\hat{\mathrm{u}}}^{n+1} = 0 \text{ in } \Omega \\ \hat{\hat{\mathrm{u}}}^{n+1} . n = 0 \quad \text{on } \partial\Omega_0 \end{cases} \quad (20)$$

$$\text{diffusion step} \begin{cases} \frac{1}{\Delta t}\left(\zeta_0 \mathrm{u}^{n+1} - \hat{\hat{\mathrm{u}}}^{n+1}\right) - \nu \, \Delta u^{n+1} = 0 \text{ in } \Omega \\ \mathrm{u}^{n+1} = 0 \quad \text{on } \partial\Omega_0 \end{cases} \quad (21)$$

Here $\hat{u}, \hat{\hat{u}}$ are used as intermediate velocity in sub steps and an exception to our usual practice of using ˆ's for chebishev coefficients. The initial time step is set to a definite Δt_0 and may vary during the integration $\Delta t = \Delta t_0 / n$ to adjust CFL number to the given bound. Maximum and minimum values of time step would define preliminary and the fraction n make the varied time step between the prescribed Δt_{max}=0.04 and Δt_{min}.=0.001 obtaining by try and error.

3. RESULTS AND DISCUSSION

Direct numerical simulation of turbulent Poiseuille channel flow is carried out on the periodic, rectangular, wall bounded domain with 128 × 65 × 128 grid points that obtained friction Reynolds number of $\mathrm{Re}_\tau = 175$ based on friction velocity and channel half width and initial viscosity of ν=1/4000. Fully spectral method of Fourier series in the homogeneous directions and Chebyshev polynomial expansion in the normal direction is used for the spatial derivatives with different time advancement scheme. Rotational form of nonlinear term $\mathrm{u}_{tot} \cdot \nabla \mathrm{u}_{tot}$ is employed with de-aliased XZ to vanish aliasing error at x and z directions. The third order semi-implicit backwards-differentiation algorithm mentioned as SBDF3 is used for time discritization. The results of turbulence statistics has been collected in both constant and variable time stepping algorithm. The comparison of turbulence intensities is made over the interval from T_0=100 to T_1=300 due to Abe *et al* (2001). The time step of recorded information was ΔT=10 sec. In varied time stepping algorithm, the initial time step Δt_0 is set to .025 and the variable time step is bounded from Δt_{min}=0.001 to Δt_{max}=0.04. Also the CFL condition is adjusted to [0.4, 0.8]. In constant time stepping algorithm, the maximum value of time step is chosen about Δt=0.015 that satisfy convergence and stability of the solution with determinate [0.2, 0.8] bounded CFL number. The results are spatially averaged along the x and z directions.

The verification and accuracy of the presented analysis in rotational form with varied time step algorithm is validated in Fig. 2, where turbulence statistics profiles are in excellent agreement with Kim et. al. results in fully developed turbulent channel flow.

It should be noticed that the uniform grid spacing in the stream-wise and span-wise directions are $\Delta x^+ \approx 15$ and $\Delta z^+ \approx 7.5$ respectively. In the vertical direction, a non-uniform mesh distribution is used, with the minimum grid spacing of $\Delta y^+_{\min} \approx 0.2$ near the wall and the maximum of $\Delta y_{\max}^+ \approx 7.5$ spacing at the centerline of the channel, which are acceptable due to Moser et. al.(1999) and Moin & Mahesh (1998). The grid spacing are $\Delta x^+ \approx 12$, $\Delta y_{\max}^+ \approx 4.4$, $\Delta y_{\min}^+ \approx 0.05$ and $\Delta z^+ \approx 7$ in the wall units in Kim *et al*. (1987) within192 × 129 × 160 mesh points that obtain friction Reynolds number of $\mathrm{Re}_\tau = 175$ that appears to be sufficient for the Reynolds number under consideration.

Figure 3 illustrate stream-wise, normal and span-wise turbulence statistics (which is called as turbulence strength value) and Reynolds stress in both time advancement algorithms. Rotational form is used to extract the nonlinear term. The results shows that the trend of all profiles is completely similar and the qualitative agreement in both forms is obvious, detailed comparison in all regions of the channel reveals insignificant discrepancies between those schemes. The maximum of relative deviation in stream-wise turbulence strength (uu), which computed at each grid point of normal direction, is less than 5.5%. This deviation of averaged values is reached to 5.48% for vv, while the resulted differences are about4.55% in span-wise turbulence strength value (ww).

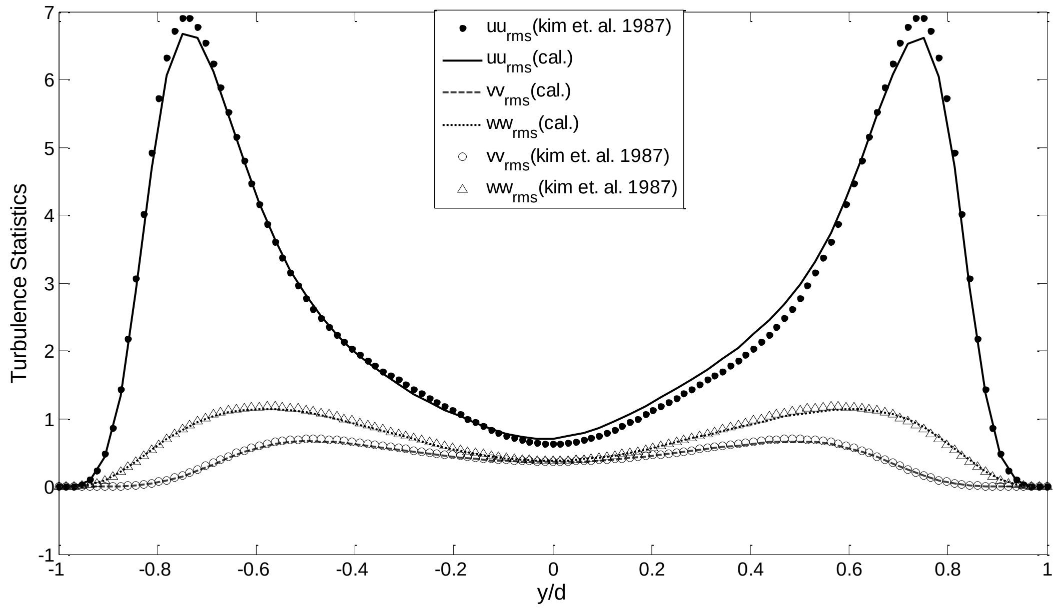

Fig. 2. Root-mean-square velocity fluctuations, symbols represent the data from Kim et.al. (1987)

The comparison of Reynolds stress uv revealed slight discrepancy. The maximum of relative deviation is about 4.88% in whole region except at the centre of channel where the uv values is close to zero. In the middle of channel, the differences between the uv values are insignificant (about 1.0e-6).

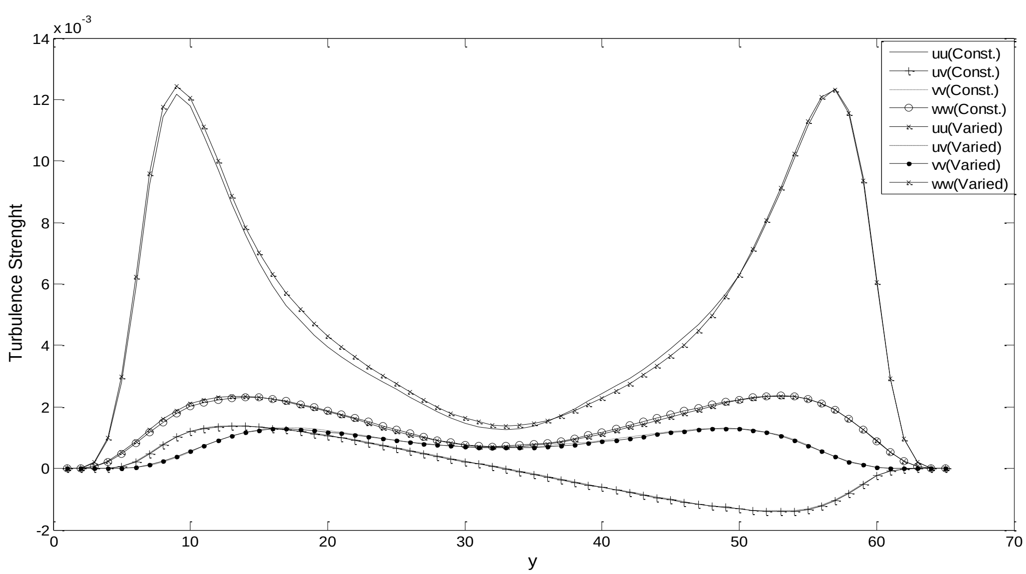

Fig. 3. Turbulence strengths with varied and constant time advancement algorithm

In Fig. 4, variation of CFL number during the process is shown. The figure shows sharper fluctuating at the beginning of the solution, which may make the solution unstable. But the source of instability is eliminated by decreasing the initial time step (dt) from 0.025 to 0.015 in constant time stepping algorithm.

Also, the time step could vary from 0.4 to .001 to satisfy the CFL condition in the given interval of CFL number [0.4, 0.8] in variable time stepping algorithm. These changes increase the overall CPU time from 1.97e+04 up to 2.937e+04 and decrease the dominant CFL condition from about 0.5 to 0.3 between varied time step and constant time step algorithm.

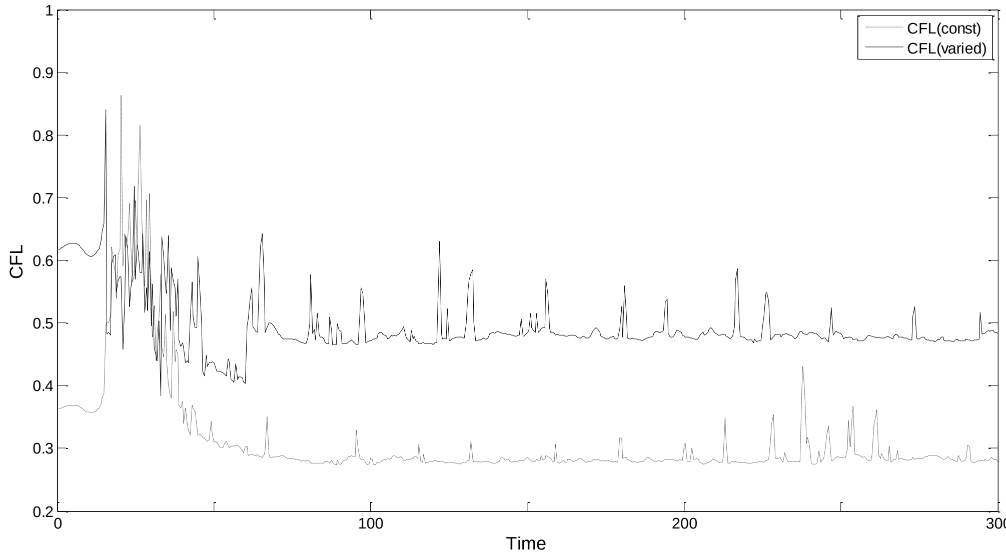

Fig. 4. Variation of CFL number in different time advancement scheme

In Fig. 5 the result of mean velocity from both time advancement methods are compared to scaling laws for the viscous sublayer and the inertial layer. Based on theoretical and experimental scaling laws, the velocity profile can be split into three distinguished regions: the viscous sublayer, the logarithmic layer and the defect layer. If the values scaled by wall units, then units of a turbulent channel flow obey:

$$U^+ = y^+ \qquad y^+ \prec 10 \quad \text{in the viscous sublayer}$$
$$U^+ = 2.5 \ln y^+ + c \; y^+ \succ 30 \quad \text{in the inertial layer} \tag{22}$$

c is the additive constant which varies between 5 to 5.5 based on Re_τ . The wall-units for length and velocity are set to:

$$U^+ \triangleq U/u^*, \; y^+ \equiv yu^*/\nu \tag{23}$$

Also the friction velocity u* is defined by

$$u^* \triangleq \sqrt{\nu \left.\frac{dU}{dy}\right|_{y=0}} \tag{24}$$

where $U(y)$ is the mean velocity profile. According to the results, the differences between the results of mean velocity profile of a turbulent channel flow computed with varied and constant time advancement algorithms against wall law are negligible in all regions.

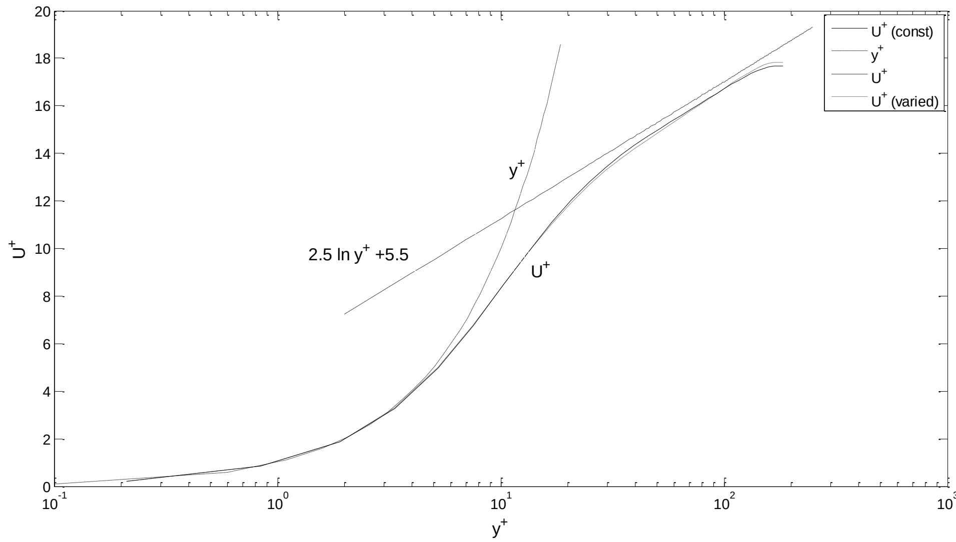

Fig. 5. The law of the wall, (Comparison of mean velocity profile in both time advancement schemes)

Table 1 reviews the values of wall-shear velocity, standard deviation, overall CPU time and total normalized energy of instantaneous velocities. The computed values are providing with both time advancement schemes. The standard deviation of u in each method is also computed by;

$$\sqrt{\frac{1}{N}\sum_{i=T_0}^{T_1}\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_a^{b}\int_0^{L_z}\left(\mathrm{u}_i-\mathrm{u}_{i\,mean}\right)\cdot\left(\mathrm{u}_i-\mathrm{u}_{i\,mean}\right)dx\ dy\ dz}$$

(25)

And the total normalized energy of instantaneous velocities is computed by:

$$\sum_{i=T_0}^{T_1}\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_a^{b}\int_0^{L_z}\mathrm{u}_i\cdot\mathrm{u}_i\,dx\ dy\ dz \quad (26).$$

The differences between computed values are unnoticeable. In both versions, the calculated values of wall shear velocity, energy and standard deviation of streamwise velocity are almost similar. The maximum relative deviation is about 1.78% for normalized energy of instantaneous velocities, 0.3% for standard deviation and 0.04% for wall shear velocities. It should be noticed that in comparison with ordinary constant time advancement algorithms, varied time advancement algorithms provided similar accuracy in analysis of plane channel flow problem. The comparison between these methods indicates that the proposed varied time step algorithms do greatly reduces the total time. It is believed that the use of constant time step algorithm for the solution of nonlinear term is time consuming. Therefore, this modification decreases the overall time from 2.937e+04 to 1.97e+04, which is about 49.1% reduction in time and thus, in computational cost.

Table 1 Specification of turbulent flow in varied and constant time advancement algorithms

Time Stepping Algorithm	**Scheme**	**Total Time (sec)**	**Wall-Shear Velocity(u_τ)**	**Standard Dev.**	**Energy(u)**
Varied	Rotational, SBDF3	1.97e+04	0.037694 (Relative dev.0.043%)	0.0790983 (Relative dev.0.320%)	0.0199848 (Relative dev. 1.74%)
Constant	Rotational, SBDF3	2.937e+04	0.037726 (Relative dev.0.042%)	0.078591 (Relative dev.0.323%)	0.0192868 (Relative dev. 1.8%)

4. CONCLUSION

The present work demonstrated the comparison of two time advancement schemes for three-dimensional incompressible NSE to find a better framework for simulation of the Poiseuille channel flow. Rotational form with aliasing errors removal was employed for discritization of nonlinear term in a fully pseudo spectral method. Also, the common time-discretization strategy of 3rd order backward difference algorithm was used.

The proposed variable time stepping algorithm seems to be a desirable modification and could reduce the computational time around 50% in plan Poiseuille flow. It was clearly shown that between the sampling techniques, the variable time stepping provides simultaneity more improvement in total CPU time, without loss of accuracy. Also, the results showed that this technique has less effect on stability problem by adjusting the variation of CFL condition in bounded limitation.

It is concluded that based on economic consideration, the de-aliased rotational form with variable time stepping algorithm is proper choice. The ordinary constant time stepping scheme requires greater computational time without much accuracy improvement. Therefore, from practical point of view, a combination of third order SBDF3 and rotational form in a variable time stepping algorithm is suggested in direct numerical simulation of Poiseuille channel flow.

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