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Effect of Non-Uniform Heat Generation on Unsteady MHD Non-Darcian Flow over a Vertical Stretching Surface with Variable Properties

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ABSTRACT

The effect of non-uniform heat generation on an unsteady MHD laminar boundary layer flow of viscous, incompressible fluid over a vertical stretching plate embedded in a sparsely packed porous medium is investigated numerically. The flow in the porous medium is governed by Brinkman-Forchheimer extended Darcy model. The variation of porosity, permeability and thermal conductivity is assumed. By applying similarity analysis, the governing partial differential equations are transformed into a set of time dependent non-linear coupled ordinary differential equations and they are solved by Runge-Kutta Fehlberg Method along with shooting technique. The effects of governing parameters on the dimensionless velocity and temperature distributions for uniform permeability (UP) and variable permeability (VP) of the porous medium are discussed graphically. Also, the local skin friction coefficient and the rate of heat transfer are computed for various pertinent parameters governing the problem. Moreover, the numerical results obtained in this study is compared with the existing literature and found they are in good agreement.

Keywords: MHD, Brinkman-Forchheimer model, Stretching plate, Non-uniform heat source/sink, Variable porosity and permeability, Variable thermal conductivity.

NOMENCLATURE

a, b, c, d, d^* constants
A unsteady parameter
A^*, B^* dimensionless heat generation parameters
B_0 uniform magnetic field
C_b empirical constant of second order
C_f skin-friction
c_p specific heat at constant pressure
f dimensionless stream function
g acceleration due to gravity
Gr local Grashof number
$k(y)$ permeability of the porous medium
k_0 permeability of the porous medium at the edge of the boundary layer
$K(T)$ thermal conductivity
K_∞ thermal conductivity far away from the sheet
M magnetic parameter
Nu_x Nusselt number
Pr Prandtl number
Re local Reynolds number
q_w local heat flux at the sheet
Q^* volumetric non-uniform heat generation
T & t fluid temperature and time
T_w given temperature at the sheet
T_∞ constant temperature of the fluid far away from the sheet
x, y axial and normal co-ordinate
u, v velocities in x, y directions respectively
U_w velocity of the stretching surface

Greek symbols

α_∞, α^* thermal diffusivity and ratio of viscosities
$\varepsilon(y)$ porosity of the porous medium
η similarity variable
ν kinematic viscosity
β^*, β inertial parameter, thermal expansion co-eff.
μ dynamic viscosity
ψ stream function
ρ density
σ permeability parameter
τ_w shear stress
θ dimensionless temperature variable
λ buoyancy parameter

1. INTRODUCTION

The study of MHD flow and heat transfer over a stretching surface has gained considerable interest because of its extensive engineering applications, such as in the extrusion of a polymer sheet from a die, glass fiber and paper production. These flows occur in many manufacturing processes in modern industry, such as hot rolling, hot extrusion, wire drawing and continuous casting. Considering of its importance, those flow have been studied by several research groups (Sakiadis 1961; Sparrow and Cess 1961; Sing and Cowling 1963; Riley 1964). The continuing interest in heat transfer and fluid flow through porous media is mainly due to several engineering and geophysical fields such as cooling of nuclear reactors, enhanced oil recovery, thermal insulation drying of porous solids, solid matrix heat exchanges, geothermal and petroleum resources, ceramic processing, filtration processes, chromatography, etc.

The problem of mixed convection flow past a stretching sheet embedded in porous medium arise in some metallurgical processes which involve the cooling of continuous strips or filaments by drawing them through quiescent fluid and the rate of cooling can be better controlled and final product of desired characteristics can be achieved if the strips are drawn through porous media (Pal and Mondal 2011). A comprehensive review of convection through porous medium was reported by Nield and Bejan (1992) and by Ingham and Pop (1998). Lai and Kulacki (1991) have studied the boundary layer mixed convection flow of heat and mass transfer over a vertical plate embedded in a saturated porous medium with constant wall temperature and constant heat flux. Recently, Makinde and Aziz (2010) investigated the mixed convection flow from a vertical plate embedded in a porous medium with magnetic effect and convective boundary condition.

One common feature of all the above-mentioned fluid saturated porous media studies deal with the Darcy flow model. But in many practical applications, for example packed sphere beds, the porous medium is bounded by an impermeable wall has higher flow rates, and reveals non-homogeneous porosity variation near the wall, making Darcy's law inapplicable. Such a case, the boundary and inertial effects should be included. The boundary and inertia effects on forced convective heat transfer from a flat plate were first examined by Vafai and Tien (1981). These effects were shown to decrease the velocity in the thermal boundary layer and reduce the heat transfer rate. Recently, Pal and Mondal (2011) studied the effect of radiation on the heat and mass transfer past a stretching sheet in a non-darcian porous medium. They found that the radiation parameter enhances the skin-friction coefficient and sherwood number whereas the local nusselt number decreases.

Owing to adding time effect, the transient heat transfer is usually difficult to solve with either an analytical approach or numerical method. Examples of transient convective flows are numerous such as cooling of electronic devices in which the heat generation is not constant but time varying. First study on transient boundary layer on flat plate was made by Johnson and Cheng (1978). Raptis (1983) studied the case of two-dimensional free convection over a vertical plate embedded in a porous medium using the perturbation method. Al-Nimr and Masoud (1998) analyzed the problem of transient free convection flow over an impermeable vertical flat plate embedded in porous medium using Laplace transformation method. Anand Rao *et al.* (2012) investigated the effect of chemical reaction on unsteady MHD free convection flow over a semi-infinite vertical porous plate. Recently, Reddy *et al*. (2013) found the exact solutions of the unsteady MHD flow, heat and mass transfer over a moving vertical porous plate using Laplace transform technique.

The study of heat generation or absorption effects in moving fluid is important in view of several physical problems. Due to the fast growth of electronic technology, effective cooling of electronic equipment has become warranted and cooling of electronic equipment ranges from individual transistors to main frame computers and from energy suppliers to telephone switch boards. Several authors have investigated the heat transfer problems by considering temperature dependent heat source/sink (Chamkha and Ahmed 2011; Anand Rao 2012 ; Prakash *et al*. 2012, etc). Although, exact modelling of non-uniform heat generation is quite difficult, some simple mathematical model can express its average behavior for most physical situations. For example, in many situations there may be appreciable temperature difference between the surface and the ambient fluid. In such a case, non-uniform heat generation plays a crucial role and exerts strong influence on the heat transfer characteristics. Abo-Eldahab and El-Aziz (2004) studied the problem to involve a space-dependent exponentially decaying with internal heat generation or absorption. Abel *et al*. (2007) and Bataller (2007) investigated the effects of non-uniform heat source on visco-elastic fluid flow and heat transfer over a stretching sheet.

In most of the papers, the thermo physical properties of the ambient fluid were assumed to be constant. Based on previous investigations (Chiam 1996,1998 ; Mahmoud 2007 ; Rahman *et al*.2009; Sharma and Singh 2009 ; Mahanti and Gaur 2009 ; Prasad *et al*. 2013), it is well known that these physical properties may change with temperature, especially thermal conductivity. For lubricating fluids, heat generated by internal friction and the corresponding rise in the temperature affects the physical properties of the fluid and so the properties of the fluid are no longer assumed to be constant.

The increase in temperature leads to increase in the transport phenomena by reducing the physical properties across the thermal boundary layer and the heat transfer at the wall is also affected. Therefore to predict the flow and heat transfer rates, it is necessary to take into account the variable fluid properties.

The present study is to investigate the non-uniform heat generation effect on unsteady MHD boundary layer flow of an incompressible viscous fluid and heat transfer over a vertical surface embedded in the porous medium with variable porosity and variable thermal conductivity. By using similarity approach, the transport equations are transformed into non-linear ordinary differential equations and they are solved by Runge-Kutta-Fehlberg with shooting method. The present results are compared with previously obtained solutions and they are in good agreement. The behavior of the velocity, temperature, skin-friction and heat transfer has been discussed for a range of physical parameters.

2. MATHEMATICAL FORMULATION

Consider an unsteady two-dimensional laminar boundary layer flow over a continuous moving stretching plate in a viscous incompressible electrically conducting fluid saturated porous medium of variable porosity, permeability and thermal conductivity. A uniform magnetic field $B_0(t)=\hat{B}_0(1-ct)^{-1/2}$ is applied in the direction perpendicular to the stretching surface. Since the transverse applied magnetic field and magnetic Reynolds number are assumed to be small, the induced magnetic field can be neglected. The *x*-axis is taken along the stretching plate in the direction of the motion and the *y*-axis is perpendicular to the plate in the outward direction towards the fluid of ambient temperature T_∞ (See Fig. 1).

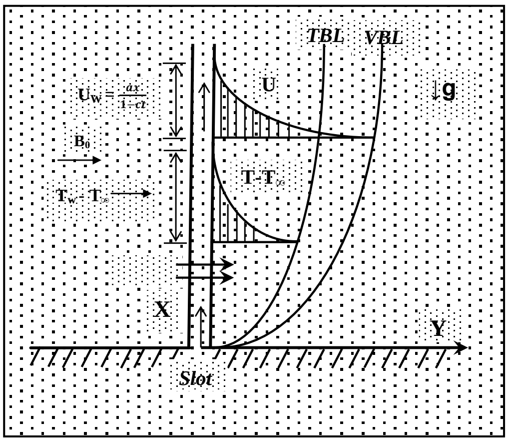

Fig. 1 Physical configuration & coordinate system

We assume that for time *t*<0 the fluid and heat flows are steady. The unsteady fluid and heat flows start at *t*=0, the plate is being stretched with the velocity $U_w(x,t)=ax/(1-ct)$ along the *x*-axis, where *a* (stretching rate) and *c* are positive constants having dimension t^{-1} (with *ct*<1, *c*≥0). The surface temperature of the plate varies with the distance *x* from the slot and time *t* in the form $T_w=T_\infty+(bx/1-ct)$ where *b* is a constant and has dimension temperature/length.

The porous medium is isotropic and homogeneous. Local thermal equilibrium is assumed. With the inclusion of quadratic drag, inertial and boundary effects, the governing boundary layer equations for unsteady two dimensional flow can be written in the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\mu\varepsilon(y)}{\rho k(y)}u+\frac{\bar{\mu}}{\rho}\frac{\partial^2 u}{\partial y^2} \tag{2.2}$$

$$-\frac{C_b\varepsilon^2(y)}{\sqrt{k(y)}}u^2+g\beta(T-T_\infty)-\frac{\sigma_m B_0^2(t)}{\rho}u$$

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(K(T)\frac{\partial T}{\partial y}\right)+Q^* \tag{2.3}$$

The initial and boundary conditions are:

$$u=U_w(x,t),\ v=0,\ T=T_w(x,t)\quad at\ \ y=0 \tag{2.4}$$

$$u\to 0,\ T\to T_\infty\quad as\ \ y\to\infty \tag{2.5}$$

where *u* and *v* are the velocity components along *x* and *y* directions, respectively. *T* is the temperature of the fluid, ρ is the fluid density, $\bar{\mu}$ is the effective viscosity of the fluid, μ is the fluid viscosity, *k*(*y*) is the variable permeability of the porous medium, ε(*y*) is the porosity of the saturated porous medium, C_b is the empirical constant of the second-order resistance term due to inertia effect, *g* is the acceleration due to gravity, β is the coefficient of thermal expansion, c_p is the specific heat at constant pressure, σ_m is the magnetic permeability of the fluid and $B_0(t)$ is the applied magnetic field.

Savvas *et al*. (1994) observed that for liquid metals, the thermal conductivity varies linearly with temperature in the range 0 to 400^{0}C. Following Chiam (1996, 1998) and Savvas *et al*. (1994), we consider the specific model for variable thermal conductivity as

$$K(T)=K_\infty\left(1+\varepsilon\frac{T-T_\infty}{T_w-T_\infty}\right) \tag{2.6}$$

where K_∞ is the thermal conductivity of the fluid far away from the stretching surface, ε is a small parameter known as the variable thermal conductivity parameter.

The term Q^* in the right hand side of Eq.(2.3) is due to non-uniform heat generation which is defined (Abo-Eldahab and El-Aziz 2004; Pal 2011; Das 2012) as

$$Q^*=\frac{K_\infty U_w(x,t)}{x\nu}\left[A^*(T_w-T_\infty)e^{-\eta}+B^*(T-T_\infty)\right] \tag{2.7}$$

where A^* and B^* are the coefficient of exponentially decaying space and temperature-dependent heat source/ sink, respectively. It is to be noted that the case $A^* > 0$, $B^* > 0$ corresponds to internal heat generation and that $A^* < 0,\ B^* < 0$, corresponds to internal heat absorption.

We now introduce a stream function ψ(*x*, *y*, *t*), which is defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{2.8}$$

The mathematical analysis of the problem is simplified by introducing the dimensionless functions *f*, θ in terms of the similarity variable η [see Ishak *et al*. (2009a), Prakash *et al*. (2014)] as

$$\eta = y\sqrt{\frac{U_w}{\nu x}}, \quad \psi = \sqrt{\nu x U_w}\, f(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{2.9}$$

We consider the variable permeability *k*(η) and variable porosity ε(η) are to decrease exponentially with the normal distance to the wall, from a value close to one at the solid boundaries to k_0 and ε_0 value at the edge of the boundary layer (see Chandrasekhara and Namboodiri 1985).

$$k(\eta) = k_0(1 + d e^{-\eta})$$

$$\varepsilon(\eta) = \varepsilon_0(1 + d^* e^{-\eta}) \tag{2.10}$$

where k_0 and ε_0 are the permeability and porosity at the edge of the boundary layer respectively. For variable permeability, *d* and d^* are treated as constants having values 3.0 and 1.5 respectively and for uniform permeability $d = d^* = 0$.

Substituting Eqs.(2.6) to (2.10) into Eqs.(2.1) to (2.5), we get the following transformed equations:

$$f''' + f f'' - f'^2 - A\left(f' + \tfrac{1}{2}\eta f''\right) + \lambda\theta - \frac{\alpha^*}{\sigma \mathrm{Re}}\left(\frac{1 + d^* e^{-\eta}}{1 + d e^{-\eta}}\right) f' - \beta^* \frac{(1 + d^* e^{-\eta})^2}{(1 + d e^{-\eta})^{1/2}} f'^2 - M f' = 0 \tag{2.11}$$

$$\left((1 + \varepsilon\theta)\theta'\right)' - \Pr(f'\theta - f\theta') - \Pr A(\theta + \tfrac{1}{2}\eta\theta') + (A^* e^{-\eta} + B^*\theta) = 0 \tag{2.12}$$

and the boundary conditions

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \tag{2.13}$$

$$f'(\infty) = 0, \quad \theta(\infty) = 0 \tag{2.14}$$

where prime denotes the differentiation with respect to η, $A = c/a$ is the unsteady parameter, $M = \frac{\sigma_m \hat{B}_0^2}{\rho a}$ is the local magnetic parameter, $\beta^* = \frac{C_b \varepsilon_0^2 x}{\sqrt{k_0}}$ is the local inertial parameter, $\lambda = Gr/\mathrm{Re}^2$ is the free convection parameter, $\Pr = \nu/\alpha_\infty$ is the Prandtl number, where $\nu = \bar{\mu}/\rho$ and $\alpha_\infty = K_\infty/\rho c_p$, $Gr = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}$ is the local Grashof number, $\mathrm{Re} = U_w x/\nu$ is the local Reynolds number, $\alpha^* = \mu/\bar{\mu}$ is the ratio of viscosities and $\sigma = k_0/x^2\varepsilon_0$ is the local permeability parameter.

The important characteristics of the flow are the skin-friction co-efficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\rho U_w^2/2}, \quad Nu_x = \frac{x q_w}{K_\infty(T_w - T_\infty)} \tag{2.15}$$

where the wall shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -K_\infty\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{2.16}$$

Substituting Eq.(2.16) in Eq.(2.15), we get

$$\frac{1}{2} C_f \sqrt{\mathrm{Re}_x} = f''(0), \quad Nu_x/\sqrt{\mathrm{Re}_x} = -\theta'(0) \tag{2.17}$$

where $\mathrm{Re}_x = U_w x/\nu$ is the local Reynolds number which is based on the surface velocity.

3. NUMERICAL PROCEDURE

The coupled system of equations Eq. (2.11) to (2.12) is highly non-linear. Most of the physical systems are inherently non-linear in nature and are of great interest to physicists, engineers and mathematicians. Problems involving non-linear ordinary differential equations are difficult to solve exactly. So, the governing equations together with the boundary conditions have to be solved numerically.

The system of equations subject to the boundary conditions (2.13)-(2.14) was solved numerically by Runge-Kutta-Fehlberg method along with shooting technique using MATLAB. Its accuracy and robustness has been repeatedly confirmed in various heat transfer papers. The asymptotic boundary conditions given by Eq.(2.14) were replaced by using a value of 6 for the similarity variable η_{max} as follows:

$$\eta_{\max} = 6, \quad f'(6) = 0, \quad \theta(6) = 0 \tag{3.1}$$

Table 1 Comparison of results of the wall temperature gradient with Ishak *et al.* (2009b) and Vajravelu *et al.* (2013) for $\left(M = \beta^* = A = 0,\ \frac{\alpha^*}{\sigma \mathrm{Re}} = 0,\ A^* = B^* = 0\right)$

ε	λ	*Pr*	Ishak *et al.*	Vajravelu *et al.*	Present results
0	0	0.72	0.8086	0.808836	0.80883589
		1.0	1.0000	1.000000	1.00000008
		3.0	1.9237	1.923687	1.92365749
		10.0	3.7207	3.720788	3.72064164
0	1.0	1.0	1.0873	1.087206	1.08726848
	2.0		1.1429	1.142298	1.14233042
	3.0		1.1853	1.185197	1.18528379
0.1	1.0	1.0	—	1.018446	1.01861813

Pantokratoras (2009) noticed that the erroneous result is found by many researchers in the field of convective heat and mass transfer because of taking small far field asymptotic value of η_{max} during their numerical computation. Here, the choice of $\eta_{max}= 6$ ensured that all numerical solutions approached the asymptotic values correctly.

In order to check the accuracy of the numerical solution procedure used, a comparison of wall temperature gradient $-\theta'(0)$ for various values of Pr, ε, λ with those of Ishak *et al.* (2009b) and Vajravelu *et al*. (2013) under certain limiting conditions is shown in Table 1. From the table, the present results are found to be in good agreement.

4. Results and Discussion

In order to understand the physical nature of the problem and the effects of various parameters like Magnetic number M, free convection parameter λ, variable thermal conductivity parameter ε, the unsteadiness parameter A, the Prandtl number Pr, the space and time dependent heat generation parameters A^*, B^*, we have computed the numerical solutions of the velocity and temperature profiles.

The influence of the magnetic parameter *M* on the velocity and temperature profiles with fixed values of other parameters for both steady and unsteady cases are depicted in Fig.2 and Fig.3. It can be seen that with the fixed value of unsteady parameter, the effect of increasing of magnetic parameter is to decrease the velocity profile near the plate $(0\leq\eta\leq6)$. This is due to the fact that the transverse magnetic field gives rise to a resistive-type of force called the Lorentz force. This force has a tendency to slow down the motion of the fluid which results in reducing the velocity profiles. Further, the temperature profile increases with the increasing of magnetic parameter, due to interaction of applied magnetic field and fluid particles. These results qualitatively agree with the expectations. Moreover, with the fixed values of magnetic parameter, the effect of increasing values of unsteady parameter *A* is to decrease the velocity and temperature field and hence it reduce the momentum and thermal boundary layer thickness.

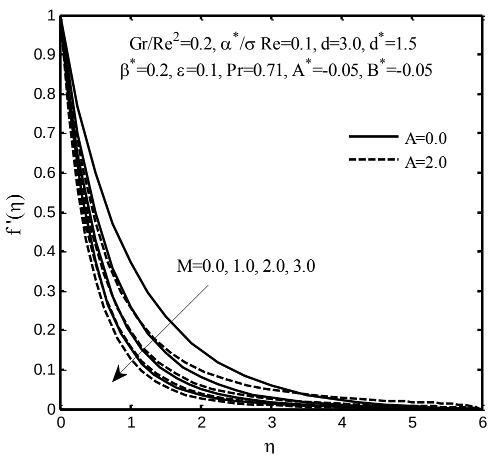

Fig. 2 Velocity profile for different values of magnetic and unsteady parameters

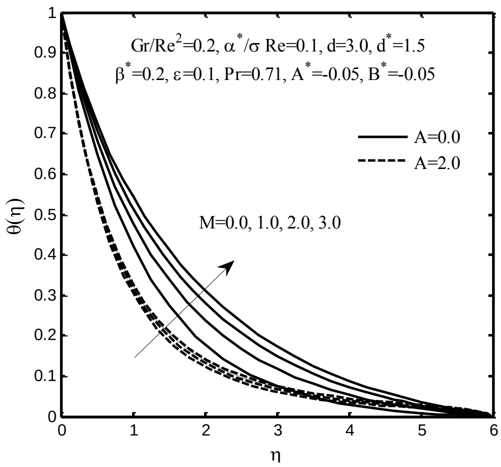

Fig. 3 Temperature profile for different values of magnetic and unsteady parameters

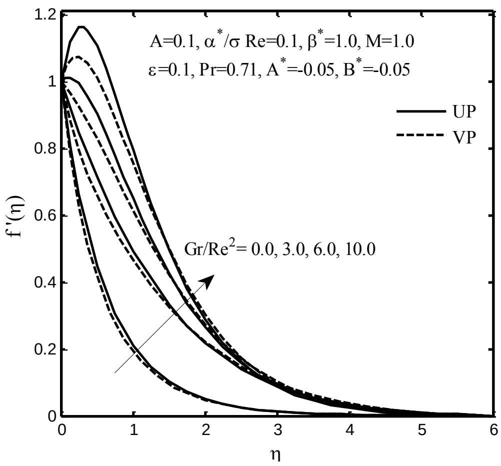

Fig. 4 Velocity profile for different values of buoyancy parameter

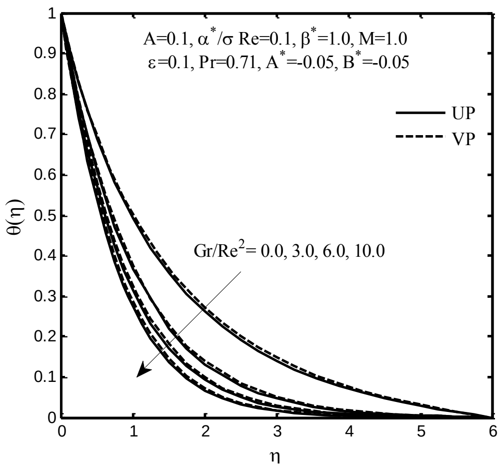

Fig. 5 Temperature profile for different values of buoyancy parameter

The effect of free convection parameter $\lambda = {Gr}/{Re^2}$ for the cases of UP and VP on velocity and temperature fields are shown in Fig.4 and Fig.5. It can be observed that an increase in the buoyancy parameter is to increase the velocity field and also increase the boundary layer thickness. Also in the case of higher buoyancy parameter, a peak is observed near the stretching boundary which exponential decreases away from the stretching boundary layer. This means that the ambient fluid velocity near the surface is higher than the stretching surface velocity. Whereas in temperature field, an increase in λ is to decrease in thermal boundary layer thickness. Moreover, when comparing the velocity and temperature profile for variable permeability (VP) with uniform permeability (UP) for a fixed value of λ, there is a decrease in momentum boundary layer thickness and increase in thermal boundary layer thickness.

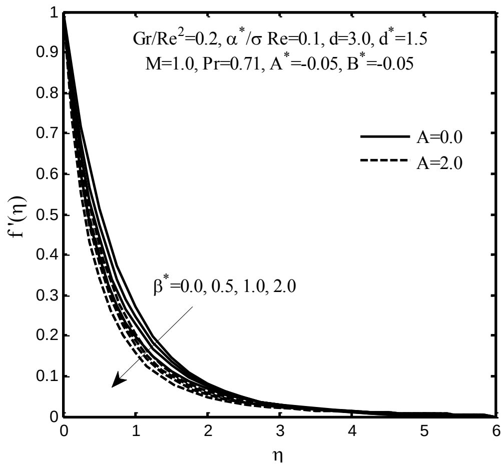

Fig. 6 Velocity profile for different values of inertial parameter and unsteady parameter

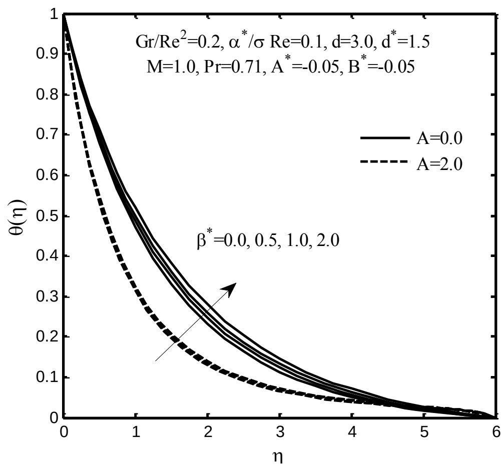

Fig. 7 Temperature profile for different values of inertial parameter and unsteady parameter

Figures 6 and 7 show the effect of inertial parameter on the velocity and temperature profiles for both steady and unsteady cases. It is observed that the inertia effect tends to decrease the velocity, since the fluid inertia provides an additional pressure loss in the flow field for both steady and unsteady cases. Moreover, the temperature increases as the inertial parameter increases due to the fact that the flow inertia force retards the momentum transport. This, in turn produces decrease in the velocity and increase in the fluid temperature. In addition, a slight increase in the thermal boundary layer thickness is observed as a result of increasing the inertial parameter for unsteady case comparing with steady one.

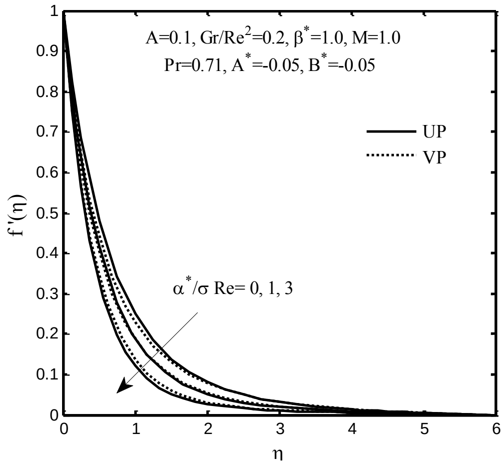

Fig. 8 Velocity profile for different values of $\alpha^*/\sigma Re$

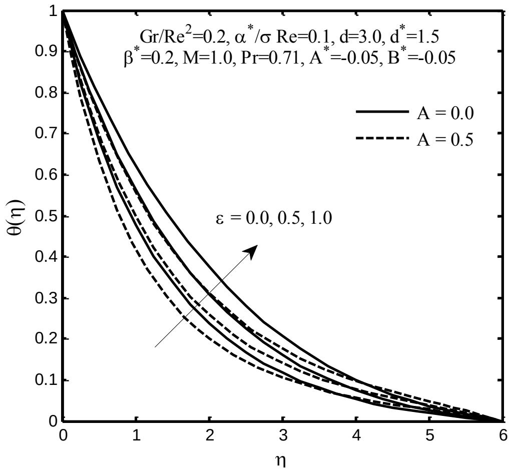

Fig. 9 Temperature profile for different values of ε

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The influence of $\alpha^*/\sigma \mathrm{Re}$ with UP and VP on the velocity profile is shown in Fig.8. It is observed that in the presence of porous medium causes higher retardation to the fluid which reduces the velocity and momentum boundary layer thickness. It is noted that, there is a significant increase in VP than UP for the higher $\alpha^*/\sigma \mathrm{Re}$. Fig.9 is plotted to demonstrate the temperature profile for the selected values of thermal conductivity parameter ε and unsteady parameter *A* with the fixed values of other parameters. It can be seen that an increasing ε is to increase the temperature profile due to the assumption of temperature dependent thermal conductivity, which makes reduction in the magnitude of the transverse velocity by a quantity $\frac{\partial}{\partial y} K(T)$. Also for each value of ε, the thermal boundary layer thickness reduces for unsteady state comparing with steady one.

Temperature profile for the selected values of space and temperature-dependent heat source/sink parameters are predicted in Fig.10 and Fig.11. It is evident from these figures, the temperature in the thermal boundary layer increases with increase in A^*, B^* for a given value of B^*, A^* in both cases of UP and VP. The heat sink $(A^*, B^* < 0)$ parameters lead to decrease in the thermal boundary layer whereas the boundary layer thickness increases with increase in (A^*, B^* >0). Moreover, it is observed that the variation between UP and VP is more for space dependent heat sink parameter $A^* < 0$ than heat source parameter $A^* > 0$, whereas reverse trend is observed in B^*.

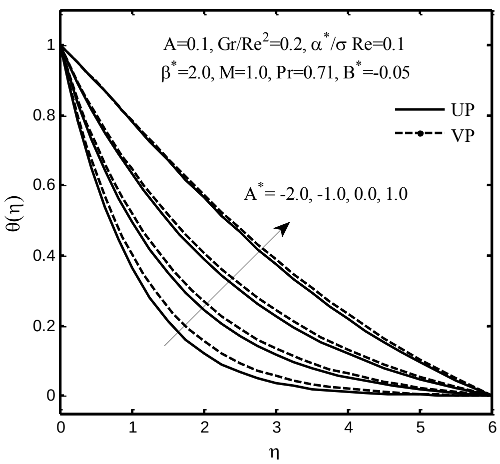

Fig. 10 Temperature profile for different values of space dependent heat source/sink parameters

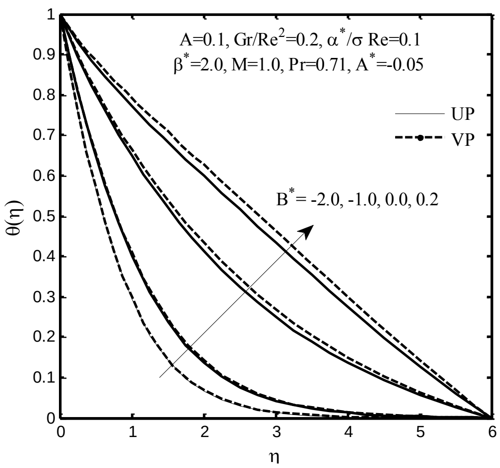

Fig. 11 Temperature profile for different values of temperature dependent heat source/sink parameters

The variation of temperature distribution within the boundary layer for various values of *Pr*, *A* in both cases of UP and VP are depicted in Fig.12. As *Pr* increases, the temperature is decreasing at a steeper rate in the flow region, which shows that the rate of cooling is much faster and the thermal boundary layer thickness becomes thinner for higher values of *Pr*. In the case of steady state and low Prandtl number, the temperature distribution for VP is slightly increasing behavior than in UP. But, there is no such a significant behavior between UP and VP with increase in *Pr* and *A*.

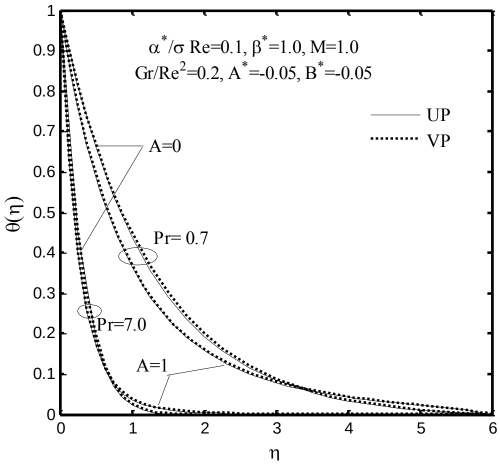

Fig. 12 Temperature profile for different values of Prandtl number and unsteady parameter

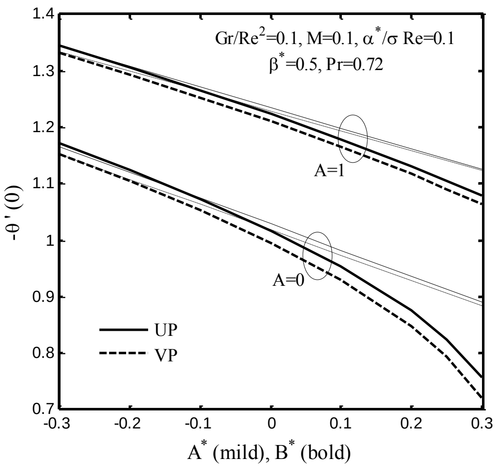

Fig. 13 Variation of Nusselt number with unsteady, non-uniform heat generation parameters

The variation of temperature gradient with unsteady parameter *A*, non-uniform heat generation parameters A^*, B^* is plotted in Fig.13. It is pointed out that the rate of heat transfer increases with an increase in unsteady parameter and decreases with an increase in non-uniform heat generation parameters A^*, B^*. Also, the heat transfer rate is low in VP comparing with UP with an increase of heat generation parameters. Moreover, the heat transfer rate for time-dependent heat generation parameter $B^* > 0$ decreases very fastly than in space dependent heat generation parameter $A^* > 0$. The positive value of heat transfer rate shows that the heat is transferring from plate to the fluid i.e. cooling of the plate occurs. Thus it can be concluded that the heat sink parameters $(A^*, B^* < 0)$ can be effectively used for the fast cooling of the plate, as expected. It is pointed out that the unsteady term makes to cool the plate faster.

The impact of physical parameters $\lambda, M, \frac{\alpha^*}{\sigma \mathrm{Re}}, \beta^*, \varepsilon$ on the skin friction co-efficient $f''(0)$ and the wall temperature gradient $-\theta'(0)$ is given in Table 2. The skin friction co-efficient is governed by the slope of the fluid velocity at the wall. As the fluid velocity increases, the slope of the velocity profile also increases, which tends to increase the skin friction co-efficient. Thus, skin friction increases with increase in ε, λ, whereas decreases with increase in $\beta^*, \alpha^*/\sigma\mathrm{Re}, M$. Also, the rate of heat transfer enhances with A, λ, but it decreases with increase in $\varepsilon, \beta^*, \alpha^*/\sigma\mathrm{Re}, M$. It is noted that from the values of UP, VP for each parameter, VP has more tendency to control the fluid velocity than UP. Thus, the study of variable permeability (VP) is more important in fluid flow through porous media.

Table 2 Skin friction and wall temperature gradient for different values of pertinent parameters
$(\mathrm{Pr} = 0.72, A^*, B^* = -0.05)$

A	$\lambda = \frac{Gr}{\mathrm{Re}^2}$	M	$\frac{\alpha^*}{\sigma \mathrm{Re}}$	β^*	ε	UP $f''(0)$	UP $-\theta'(0)$	VP $f''(0)$	VP $-\theta'(0)$
0	0.1	0.1	0.1	0.5	0.1	-1.182891	0.796237	-1.372596	0.781352
					0.5	-1.178497	0.635081	-1.368657	0.622924
1	0.1	0.1	0.1	0.5	0.1	-1.470428	1.063873	-1.637654	1.055938
					0.5	-1.467828	0.862374	-1.635273	0.855915
0	0.1	0.1	0.1	1.0	0.1	-1.311183	0.780905	-1.664145	0.753913
				2.0		-1.538455	0.754640	-2.149738	0.711683
1	0.1	0.1	0.1	1.0	0.1	-1.578446	1.056766	-1.895666	1.042267
				2.0		-1.775346	1.044597	-2.338814	1.021148
0	0.1	0.1	0.5	0.5	0.1	-1.338582	0.763102	-1.469628	0.757295
			1.0			-1.511511	0.728565	-1.580282	0.731771
1	0.1	0.1	0.5	0.5	0.1	-1.599271	1.051072	-1.717523	1.047347
			1.0			-1.747268	1.037452	-1.811530	1.037902
0	0.1	0.5	0.1	0.5	0.1	-1.338582	0.763102	-1.514629	0.749923
		1.0				-1.511511	0.728565	-1.674176	0.717152
1	0.1	0.5	0.1	0.5	0.1	-1.599271	1.051072	-1.757029	1.044068
		1.0				-1.747268	1.037452	-1.895335	1.031365
0	0.5	0.1	0.1	0.5	0.1	-0.998171	0.838582	-1.199267	0.823686
	1.0					-0.788033	0.875244	-1.002071	0.859801
1	0.5	0.1	0.1	0.5	0.1	-1.320863	1.080417	-1.495977	1.071941
	1.0					-1.140954	1.098823	-1.325673	1.089693

5. CONCLUSION

In this paper, we have investigated numerically the effect of non-uniform heat generation on unsteady MHD boundary layer flow of an incompressible viscous fluid and heat transfer over a vertical surface embedded in the porous medium with variable porosity and variable thermal conductivity. From the present investigation the following conclusions may be drawn:

1. An increase in the unsteady parameter is to decrease the thickness of the momentum and thermal boundary layers for all the governing parameters.
2. The velocity profile decreases with an increase in the values of magnetic parameter *M*, inertial parameter and the porous parameter, whereas reverse trend is seen with increasing the buoyancy parameter λ. Also, the temperature profile increases with an increase in the values of magnetic parameter, inertial parameter, variable thermal conductivity parameter, non-uniform heat source parameters whereas it decreases with an increase in buoyancy parameter, Prandtl number, non-uniform heat sink parameters.
3. The value of the local skin-friction coefficient increases with increase in buoyancy parameter whereas reverse effect is seen by increasing

magnetic parameter, the inertial parameter and the porous parameter. Moreover, the rate of heat transfer decreases with an increase in magnetic, thermal conductivity, inertial, porous and non-uniform heat source parameters.

4. The rate of heat transfer increases with an increase in the unsteady parameter *A*, the buoyancy parameter, non-uniform heat sink parameters $A^*, B^* < 0$. Thus fast cooling of the plate can be achieved by implementing these effects.
5. Variable permeability has more tendency to control the fluid velocity than by applying uniform permeability in the applications.

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