Journal of Applied Fluid Mechanics, Vol. 8, No. 1, pp. 159-171, 2015.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.36884/jafm.8.01.20437

Effects of Hall Current and Rotation on Hydromagnetic Natural Convection Flow with Heat and Mass Transfer of a Heat Absorbing Fluid past an Impulsively Moving Vertical Plate with Ramped Temperature

G. S. Seth[1†], S. Sarkar[1], S. M. Hussain[2] and G. K. Mahato[3]

[1]*Department of Applied Mathematics, Indian School of Mines, Dhanbad-826004, India*
[2]*Department of Mathematics, O. P. Jindal Institute of Technology, Raigarh-496109, India*
[3]*Department of Mathematics, CUTM, Bhubaneswar-752050, India*

†*Corresponding Author Email: gsseth_ism@yahoo.com*

(Received June 13, 2013; accepted February 5, 2014)

ABSTRACT

An investigation of the effects of Hall current and rotation on unsteady hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible and time dependent heat absorbing fluid past an impulsively moving vertical plate in a porous medium taking thermal and mass diffusions into account is carried out. Exact solution of the governing equations is obtained in closed form by Laplace Transform technique. Exact solution is also obtained in case of unit Prandtl number and unit Schmidt number. Expressions for skin friction due to primary and secondary flows and Nusselt number are derived for both ramped temperature and isothermal plates. Expression for Sherwood number is also derived. The numerical values of primary and secondary fluid velocities and species concentration are displayed graphically whereas that of skin friction and Nusselt number are presented in tabular form for various values of pertinent flow parameters.

Keywords: Hydromagnetic natural convection flow, Heat and mass transfer, Ramped temperature, Hall current, Coriolis force, Heat absorption.

NOMENCLATURE

B_0 : uniform magnetic field,
C' : species concentration,
g : accleration due to gravity,
G_c : solutal Grashof number,
K_1 : permeability parameter,
m: Hall current parameter,
P_r : Prandtl number,
T' : fluid temperature,
u' : primary fluid velocity,
c_p : specific heat at constant pressure,
D : molecular(mass) diffusivity,
G_r : thermal Grashof number,
k_1 : thermal conductivity,
K^2: rotation parameter,
M^2 : magnetic parameter,
t_0 : characteristic time,
S_c : Schmidt number,
w' : secondary fluid velocity.

Greek Symbols

ω_e : cyclotron frequency,
ν : kinematic coefficient of viscosity,
τ_x : primary skin friction,
σ : electrical conductivity,
τ_e : electron collison time,
ρ : density,
τ_z : secondary skin friction,
ϕ : dimensionless heat absorption coefficient.

1. INTRODUCTION

Natural convection flow induced due to thermal and concentration buoyancy forces in a fluid saturated porous medium has been extensively studied in the past due to its frequent occurrence in nature and for its wide applications in industry. Natural phenomena such as photosynthesis, calm-day evaporation, vaporization of mist and fog, drying of porous solids and sea-wind formation (where upward convection is modified by Coriolis forces) occur due to differences in temperature and concentration or a combination of these two. In addition to it, coupled heat and mass transfer can also explain the nature of ocean currents driven by differential heating and act as freight trains for salt as pointed by Bejan (1993) and the role of factory waste gas diffusion in a differential heating circulated air. Also there are several industrial applications where heat and mass transfer take place simultaneously such as in heat exchanger devices, cooling of molten metals, insulation systems, petroleum reservoirs, filtration, chemical catalytic reactors and processes, nuclear waste repositories, desert coolers, wet bulb thermometers, frost formation in vertical channels (Fossa and Tanda 2010) etc. Pioneering work on heat and mass transfer has been carried out by Eckert and Drake (1972). Gebhart *et al*. (1998) have candidly documented free convection boundary layer flow due to simultaneous heat and mass transfer with various geometries. Free convection flow involving heat and mass transfer from different geometries in non-porous and porous media are investigated by a number of researchers in the past. Mention may be made of research studies of Raptis (1982), Bejan and Khair (1985), Jang and Chang (1988), Nakayama and Hossain (1995), Yih (1997), Chamkha *et al*. (2001), Abdallah and Zeghmati (2011) and Bisht *et al*. (2011).

Study of magnetohydrodynamic (MHD) natural convection flow with heat and mass transfer is of considerable importance due to its application in astrophysics, geophysics, aeronautics, electronics, meteorology, metallurgy, petroleum and chemical engineering etc. In addition to it, the thermal physics of hydromagnetic problems with mass transfer is of much significance in MHD energy generators, MHD flow-meters, MHD pumps, MHD accelerators, controlled thermonuclear reactors etc. Many cross galvano and thermo magnetic effects occur in the boundary zone between hydraulics and thermal physics and they are relevant in the study of semiconductor materials. Keeping in view the importance of such study, Hossain and Mandal (1985) studied mass transfer effects on unsteady hydromagnetic free convection flow past an accelerated vertical porous plate. Jha (1991) discussed hydromagnetic free convection and mass transfer flow past a uniformly accelerated vertical plate through a porous medium when magnetic field is fixed with the moving plate. Elbashbeshy (1997) studied heat and mass transfer along a vertical plate in presence of magnetic field. Chen (2004) analyzed combined heat and mass transfer in MHD free convection flow from a vertical surface with Ohmic heating and viscous dissipation. Ibhrahim *et al*. (2004) considered unsteady magnetohydrodynamic micropolar fluid flow and heat transfer past a vertical porous plate through a porous medium in the presence of thermal and mass diffusions with a constant heat source. Makinde and Sibanda (2008) investigated magnetohydrodynamic mixed convective flow with heat and mass transfer past a vertical plate embedded in a uniform porous medium with constant wall suction in the presence of uniform transverse magnetic field. Makinde (2009) studied MHD mixed convection flow and mass transfer past a vertical porous plate embedded in a porous medium with constant heat flux. Eldabe *et al*. (2011) studied unsteady MHD flow of a viscous and incompressible fluid with heat and mass transfer in a porous medium near a moving vertical plate with time dependent velocity. Sharma *et al*. (2012) discussed steady mixed convection flow of water at 4^oC along a non-isothermal vertical moving plate with transverse magnetic field.

It is noticed that there may be significant temperature difference between ambient fluid and surface of the solid in a number of fluid flow problems of physical interest. Therefore, it is appropriate to consider temperature dependent heat source and/or sink which may have strong influence on heat and mass transfer characteristics. Sparrow and Cess (1961) were the first to investigate temperature dependent heat absorption on steady stagnation point flow and heat transfer. Several physical problems exist for possible application in industry where heat generation and absorption take place viz. fire and combustion modeling, fluids undergoing exothermic and/or endothermic chemical reaction, development of metal waste from spent nuclear fuel, applications in the field of nuclear energy etc. Keeping in view the importance of such study Chamkha and Khaled (2000) investigated hydromagnetic combined heat and mass transfer by natural convection from a permeable vertical plate embedded in a fluid saturated porous medium in the presence of heat generation or absorption. Kamel (2001) considered unsteady hydromagnetic convection flow due to heat and mass transfer through a porous medium bounded by an infinite vertical porous plate with temperature dependent heat sources and sinks. Chamkha (2004) discussed unsteady two dimensional convective heat and mass transfer flow of a viscous, incompressible, electrically conducting and temperature-dependent heat absorbing fluid past a semi-infinite vertical permeable moving plate with thermal and concentration buoyancy effects. Reddy and Reddy (2011) studied steady two-dimensional MHD free convection and mass transfer flow past an inclined semi-infinite vertical surface embedded in a porous medium in the presence of heat generation. They found numerical solution of the problem using shooting technique. Reddy *et al*. (2012) investigated radiation effects on unsteady MHD flow of a viscous, incompressible and electrically conducting fluid past an exponentially accelerated infinite isothermal vertical plate with uniform mass diffusion in the presence of heat source. They obtained numerical solution of the problem using Crank-Nicolson finite difference scheme.

Investigation of hydromagnetic natural convection flow in a rotating medium is of considerable importance due to its application in various areas of geophysics, astrophysics and fluid engineering viz. maintenance and secular variations of Earth's magnetic field due to motion of Earth's liquid core, internal rotation rate of the Sun, structure of the magnetic stars, solar and planetary dynamo problems, turbo machines, rotating MHD generators, rotating drum separators for liquid metal MHD applications etc. It may be noted that Coriolis and magnetic forces are comparable in magnitude and Coriolis force induces secondary flow in the flow-field. Taking into consideration the importance of such study, unsteady hydromagnetic natural convection flow past an infinite isothermal moving plate in a rotating medium is studied by a number of researchers. Mention may be made of research studies of Raptis and Singh (1985), Kyth and Puri (1988), Tokis (1988), Nanousis (1992) and Singh *et al*. (2009). Recently, Ghosh *et al*. (2013) considered the effects of rotation on unsteady hydromagnetic free and forced convection in a channel subject to forced oscillation under an oblique magnetic field.

In all these investigations, analytical/numerical solution is obtained by considering conditions for the velocity and temperature at the plate as continuous and well defined. However, there exist several problems of physical interest which may require non-uniform or arbitrary conditions at the boundary. Keeping in view this fact, several researchers investigated fluid flow problems of free convection from a vertical plate with step discontinuities in the surface temperature. Schetz (1963) attempted initially to develop an approximate model for free convection flow from a vertical plate with discontinuous thermal boundary conditions. Later, several investigations on such fluid flow problems are carried out by using an experimental technique (Schetz and Eichhorn, 1964), by numerical methods (Hayday *et al*. 1967) and by series expansion methods (Kelleher 1971; Kao 1975). Lee and Yovanovich (1991) developed a new analytical model for laminar natural convection flow past a vertical plate with step change in wall temperature. Chandran *et al*. (2005) analyzed unsteady natural convective flow of a viscous and incompressible fluid near a vertical plate with ramped temperature. Seth and Ansari (2010) investigated unsteady hydromagnetic natural convection flow past an impulsively moving vertical plate with ramped temperature in the presence of thermal diffusion and heat absorption. Saha *et al*. (2010) presented a scaling analysis for unsteady natural convection boundary layer flow adjacent to an inclined flat plate with ramp cooling boundary condition. Narhari (2012) discussed unsteady free convection flow between two vertical plates with ramped temperature within one of the plates in the presence of thermal radiation and mass diffusion. Recently, Seth *et al*. (2013b) considered the effects of rotation on unsteady hydromagnetic free convection flow of a viscous, incompressible and optically thick radiating fluid past an impulsively moving vertical plate with ramped temperature in a porous medium.

It is noticed that when the density of an electrically conducting fluid is low and/or applied magnetic field is strong, Hall current is produced in the flow-field which plays an important role in determining flow-features of the fluid-flow problems because it induces secondary flow in the flow-field. Keeping in view this fact, significant contributions on hydromagnetic free convection flow past a flat plate with Hall effects under different thermal conditions are made by a number of researchers in the past. Mention may be made of the research studies of Abo-Eldahab and Elbarbary (2001) and Takhar *et al*. (2003). It may be noted that Hall current induces secondary flow in the flow-field which is also characteristics of Coriolis force. Therefore, it seems to be important to compare and contrast the effects of these two agencies and also study their combined effects on such fluid flow problems. Recently, Seth *et al*. (2013a) investigated effects of Hall current and rotation on unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid past an impulsively moving vertical plate with ramped temperature in a porous medium taking into account the effects of thermal diffusion.

Aim of the present investigation is to study the effects of Hall current and rotation on unsteady hydromagnetic natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting and heat absorbing fluid past an impulsively moving vertical plate embedded in a fluid saturated porous medium taking into account the effects of thermal and mass diffusions when temperature of the plate has a temporarily ramped profile. Natural convection flow resulting from such ramped temperature profile of a plate may have influence on several engineering problems specially where initial temperature profiles are of much significance in designing of electromagnetic devices and so many natural phenomena which occur due to natural convection flow with heat and mass transfer and heat generation/ absorption.

2. Formulation of the Problem and its Solution

Consider natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting and heat absorbing fluid past an infinite vertical plate embedded in a uniform porous medium taking Hall current into account. Coordinate system is chosen in such a way that x' - axis is considered along the plate in upward direction and y' - axis normal to plane of the plate in the fluid. A uniform transverse magnetic field B_0 is applied in a direction which is parallel to y' - axis. The fluid and plate rotate in unison with uniform angular velocity Ω about y' - axis. Initially i.e. at time $t' \le 0$, both the fluid and plate are at rest and are maintained at a uniform temperature T_∞'. Also species concentration within the fluid is maintained at uniform concentration C_∞'. At time $t' > 0$, plate starts moving in x'- direction with uniform velocity U_0 in its own plane. The temperature of plate is raised or lowered to $T_\infty' + \left(T_W' - T_\infty'\right) t'/t_0$ when $0 < t' \le t_0$, and it is maintained at uniform temperature T_w' when $t' > t_0$ (t_0 being characteristic time). Also, at time $t' > 0,$ species concentration at the

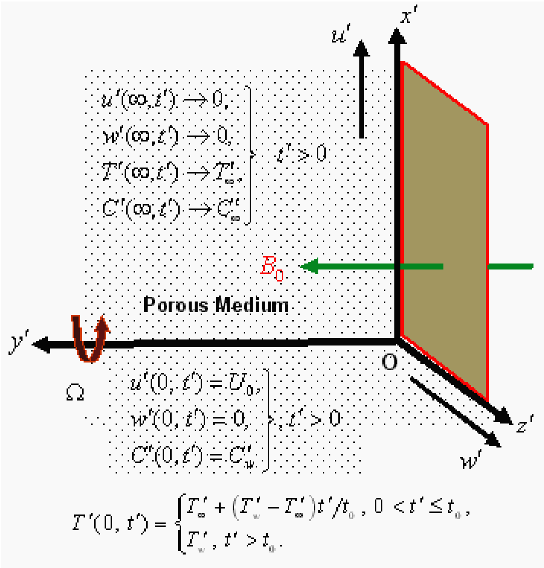

Fig 1. Geometry of the Problem

surface of the plate is raised to uniform species concentration C_w' and is maintained thereafter. The geometry of the problem is presented in Fig. 1.

Since plate is of infinite extent in x' and z' directions and is electrically non-conducting, all physical quantities except pressure, depend on y' and t' only. Also no applied or polarized voltages exist so the effect of polarization of fluid is negligible. This corresponds to the case where no energy is added or extracted from the fluid by electrical means (Cramer and Pai, 1973). It is assumed that the induced magnetic field generated by fluid motion is negligible in comparison to the applied one. This assumption is justified because magnetic Reynolds number is very small for liquid metals and partially ionized fluids which are commonly used in industrial applications (Cramer and Pai, 1973).

Keeping in view the assumptions made above, governing equations for natural convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting and heat absorbing fluid in a uniform porous medium in a rotating frame of reference, under Boussinesq approximation, taking Hall current into account are given by

$$\frac{\partial u'}{\partial t'}+2\Omega w'=\nu\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma B_0^2}{\rho\left(1+m^2\right)}\left(u'+mw'\right)-\frac{\nu}{K_1'}u'$$
$$+g\beta'\left(T'-T'_\infty\right)+g\beta^*\left(C'-C'_\infty\right), \tag{1}$$

$$\frac{\partial w'}{\partial t'}-2\Omega u'=\nu\frac{\partial^2 w'}{\partial y'^2}+\frac{\sigma B_0^2}{\rho\left(1+m^2\right)}\left(mu'-w'\right)-\frac{\nu}{K_1'}w', \tag{2}$$

$$\frac{\partial T'}{\partial t'}=\frac{k_1}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2}-\frac{Q_0}{\rho c_p}\left(T'-T'_\infty\right), \tag{3}$$

$$\frac{\partial C'}{\partial t'}=D\frac{\partial^2 C'}{\partial y'^2}, \tag{4}$$

where $u', w', \nu, \sigma, \rho, K_1', g, \beta', \beta^*, T', C', D,$ $m=\omega_e\tau_e, \omega_e, \tau_e, k_1, c_p$ and Q_0 are, respectively, fluid velocity in x' - direction, fluid velocity in z' - direction, kinematic coefficient of viscosity, electrical conductivity, density, permeability of porous medium, acceleration due to gravity, coefficient of thermal expansion, coefficient of expansion for species concentration, fluid temperature, species concentration, molecular(mass) diffusivity, Hall current parameter, cyclotron frequency, electron collision time, thermal conductivity, specific heat at constant pressure and heat absorption coefficient.

Initial and boundary conditions for the problem are specified as

$$u'=w'=0, T'=T'_\infty, C'=C'_\infty \text{ for } y'\ge 0 \text{ and } t'\le 0, \tag{5a}$$

$$u'=U_0,\ w'=0 \text{ at } y'=0 \text{ for } t'>0, \tag{5b}$$

$$T'=T'_\infty+\left(T'_w-T'_\infty\right)t'/t_0 \text{ at } y'=0 \text{ for } 0<t'\le t_0, \tag{5c}$$

$$T'=T'_w \text{ at } y'=0 \text{ for } t'>t_0, \tag{5d}$$

$$C'=C'_w \text{ at } y'=0 \text{ for } t'>0, \tag{5e}$$

$$u', w'\to 0,\ T'\to T'_\infty,\ C'\to C'_\infty \text{ as } y'\to\infty \text{ for } t'>0. \tag{5f}$$

Equations (1) to (4), in non-dimensional form, assume the following forms

$$\frac{\partial u}{\partial t}+2K^2w=\frac{\partial^2 u}{\partial y^2}-\frac{M^2}{\left(1+m^2\right)}\left(u+mw\right)-\frac{u}{K_1}$$
$$+G_rT+G_cC, \tag{6}$$

$$\frac{\partial w}{\partial t}-2K^2u=\frac{\partial^2 w}{\partial y^2}+\frac{M^2}{\left(1+m^2\right)}\left(mu-w\right)-\frac{w}{K_1}, \tag{7}$$

$$\frac{\partial T}{\partial t}=\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2}-\phi T, \tag{8}$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}, \tag{9}$$

where

$$y=y'/U_0t_0,\ u=u'/U_0,\ w=w'/U_0,\ t=t'/t_0,$$
$$M^2=\sigma B_0^2\nu/\rho U_0^2,\ K^2=\nu\Omega/U_0^2,\ K_1=K_1'U_0^2/\nu^2,$$
$$T=\left(T'-T'_\infty\right)/\left(T'_w-T'_\infty\right),\ C=\left(C'-C'_\infty\right)/\left(C'_w-C'_\infty\right),$$
$$G_r=g\beta'\nu\left(T'_w-T'_\infty\right)/U_0^3,\ P_r=\nu\rho c_p/k_1,$$
$$G_c=g\beta^*\nu\left(C'_w-C'_\infty\right)/U_0^3,\ \phi=\nu Q_0/\rho c_p U_0^2$$

and $S_c=\nu/D$. $M^2, K^2, K_1, G_r, G_c, P_r, \phi$ and S_c are, respectively, magnetic parameter, rotation parameter, permeability parameter, thermal Grashof number, solutal Grashof number, Prandtl number, dimensionless heat absorption coefficient and Schmidt number.

Characteristic time t_0 is defined according to the non-dimensional process mentioned above as $t_0 = \nu / U_0^{\ 2}$.
Initial and boundary conditions presented by Eqs. (5a) to (5f), in non-dimensional form, become

$$u = w = 0, T = 0, C = 0 \quad \text{for } y \geq 0 \text{ and } t \leq 0, \tag{10a}$$

$$u = 1, w = 0 \text{ at } y = 0 \quad \text{for } t > 0, \tag{10b}$$

$$T = t \text{ at } y = 0 \quad \text{for } 0 < t \leq 1, \tag{10c}$$

$$T = 1 \text{ at } y = 0 \quad \text{for } t > 1, \tag{10d}$$

$$C = 1 \text{ at } y = 0 \quad \text{for } t > 0, \tag{10e}$$

$$u \to 0, w \to 0; T \to 0; C \to 0 \quad \text{as } y \to \infty \text{ for } t > 0 \tag{10f}$$

Equations (6) and (7) are presented, in compact form, as

$$\frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial y^2} - \lambda F + G_r T + G_c C, \tag{11}$$

where

$$F = u + iw \text{ and } \lambda = M^2 (1 - im) / (1 + m^2) + 1/K_1 - 2iK^2 .$$

Initial and boundary conditions presented by Eqs. (10a) to (10f), in compact form, are given by

$$F = 0, T = 0, C = 0 \quad \text{for } y \geq 0 \text{ and } t \leq 0, \tag{12a}$$

$$F = 1, \text{ at } y = 0 \quad \text{for } t > 0, \tag{12b}$$

$$T = t \text{ at } y = 0 \quad \text{for } 0 < t \leq 1, \tag{12c}$$

$$T = 1 \text{ at } y = 0 \quad \text{for } t > 1, \tag{12d}$$

$$C = 1 \text{ at } y = 0 \quad \text{for } t > 0, \tag{12e}$$

$$F \to 0, \ T \to 0, \ C \to 0, \text{ as } y \to \infty \text{ for } t > 0. \tag{12f}$$

Equations (8), (9) and (11) after taking Laplace transform and using initial conditions presented by Eq. (12a) reduce to

$$\frac{d^2 \bar{T}}{dy^2} - P_r (s + \phi) \bar{T} = 0, \tag{13}$$

$$\frac{d^2 \bar{C}}{dy^2} - s S_c \bar{C} = 0, \tag{14}$$

$$\frac{d^2 \bar{F}}{dy^2} - (s + \lambda) \bar{F} + G_r \bar{T} + G_c \bar{C} = 0, \tag{15}$$

where

$$\bar{T}(y,s) = \int_0^\infty T(y,t) e^{-st} dt, \ \bar{C}(y,s) = \int_0^\infty C(y,t) e^{-st} dt,$$

$$\bar{F}(y,s) = \int_0^\infty F(y,t) e^{-st} dt \ \text{ and } s > 0$$

(s being the Laplace transform parameter).
Boundary conditions presented by Eqs. (12b) to (12f), after taking Laplace transform, become

$$\left.\begin{array}{l} \bar{F} = 1/s, \ \bar{T} = (1 - e^{-s})/s^2, \ \bar{C} = 1/s \ \text{ at } y = 0, \\ \bar{F} \to 0, \ \bar{T} \to 0, \ \bar{C} \to 0 \ \text{ as } y \to \infty. \end{array}\right\} \tag{16}$$

Solution of Eqs. (13) to (15) subject to the boundary conditions (16) are given by

$$\bar{T}(y,s) = \frac{(1 - e^{-s})}{s^2} e^{-y\sqrt{P_r (s+\phi)}}, \tag{17}$$

$$\bar{C}(y,s) = \frac{1}{s} e^{-y\sqrt{s S_c}}, \tag{18}$$

$$\begin{aligned} \bar{F}(y,s) = {} & \frac{1}{s} e^{-y\sqrt{s+\lambda}} - \frac{G_1 (1 - e^{-s})}{s^2 (s - \beta_1)} \left\{ e^{-y\sqrt{s+\lambda}} - e^{-y\sqrt{P_r (s+\phi)}} \right\} \\ & - \frac{G_2}{s(s + \beta_2)} \left\{ e^{-y\sqrt{s+\lambda}} - e^{-y\sqrt{s S_c}} \right\}, \end{aligned} \tag{19}$$

Where

$G_1 = G_r / (1 - P_r)$, $G_2 = G_c / (1 - S_c)$,

$\beta_1 = (P_r \phi - \lambda) / (1 - P_r)$, $\beta_2 = \lambda / (1 - S_c)$.

Exact solution for the fluid temperature $T(y,t)$, species concentration $C(y,t)$ and fluid velocity $F(y,t)$ is obtained by taking inverse Laplace transform of solution (17) to (19) which is expressed in the following form

$$T(y,t) = T^*(y,t) - H(t-1) T^*(y, t-1), \tag{20}$$

$$C(y,t) = erfc\left(\frac{y}{2} \sqrt{\frac{S_c}{t}} \right), \tag{21}$$

$$\begin{aligned} F(y,t) = {} & \frac{1}{2} \left[e^{y\sqrt{\lambda}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t} \right) \right. \\ & \left. + e^{-y\sqrt{\lambda}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{\lambda t} \right) \right] \\ & - G_1 \left[F^*(y,t) - H(t-1) F^*(y, t-1) \right] \\ & - G_2 C^*(y,t), \end{aligned} \tag{22}$$

where

$$\begin{aligned} T^*(y,t) = {} & \frac{1}{2} \left[\left(t + \frac{y}{2} \sqrt{\frac{P_r}{\phi}} \right) e^{y\sqrt{\phi P_r}} erfc\left(\frac{y}{2} \sqrt{\frac{P_r}{t}} + \sqrt{\phi t} \right) \right. \\ & \left. + \left(t - \frac{y}{2} \sqrt{\frac{P_r}{\phi}} \right) e^{-y\sqrt{\phi P_r}} erfc\left(\frac{y}{2} \sqrt{\frac{P_r}{t}} - \sqrt{\phi t} \right) \right], \end{aligned}$$

$$\begin{aligned} F^*(y,t) = {} & \frac{1}{2} \left[\frac{e^{\beta_1 t}}{\beta_1^2} \left\{ e^{y\sqrt{(\lambda + \beta_1)}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(\lambda + \beta_1) t} \right) \right. \right. \\ & + e^{-y\sqrt{(\lambda + \beta_1)}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(\lambda + \beta_1) t} \right) \\ & - e^{y\sqrt{P_r (\phi + \beta_1)}} erfc\left(\frac{y}{2} \sqrt{\frac{P_r}{t}} + \sqrt{(\phi + \beta_1) t} \right) \\ & \left. - e^{-y\sqrt{P_r (\phi + \beta_1)}} erfc\left(\frac{y}{2} \sqrt{\frac{P_r}{t}} - \sqrt{(\phi + \beta_1) t} \right) \right\} \\ & - \frac{1}{\beta_1} \left\{ \left(t + \frac{1}{\beta_1} + \frac{y}{2\sqrt{\lambda}} \right) e^{y\sqrt{\lambda}} erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t} \right) \right. \\ & + \left(t + \frac{1}{\beta_1} - \frac{y}{2\sqrt{\lambda}} \right) e^{-y\sqrt{\lambda}} erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{\lambda t} \right) \end{aligned}$$

$$-\left(t+\frac{1}{\beta_1}+\frac{y}{2}\sqrt{\frac{P_r}{\phi}}\right)e^{y\sqrt{\phi P_r}}erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}+\sqrt{\phi t}\right)$$
$$\left.\left.-\left(t+\frac{1}{\beta_1}-\frac{y}{2}\sqrt{\frac{P_r}{\phi}}\right)e^{-y\sqrt{\phi P_r}}erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}-\sqrt{\phi t}\right)\right\}\right],$$

$$C^*(y,t)=\frac{1}{2\beta_2}\left[e^{y\sqrt{\lambda}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)\right.$$
$$+e^{-y\sqrt{\lambda}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{\lambda t}\right)-2erfc\left(\frac{y}{2}\sqrt{\frac{Sc}{t}}\right)$$
$$-e^{-\beta_2 t}\left\{e^{y\sqrt{\lambda-\beta_2}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda-\beta_2)t}\right)\right.$$
$$+e^{-y\sqrt{\lambda-\beta_2}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda-\beta_2)t}\right)$$
$$-e^{iy\sqrt{S_c\beta_2}}erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}+i\sqrt{\beta_2 t}\right)$$
$$\left.\left.-e^{-iy\sqrt{S_c\beta_2}}erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}-i\sqrt{\beta_2 t}\right)\right\}\right].$$

$H(t-1)$ and $erfc(x)$ are, respectively, the unit step function and complementary error function.

2.1 Solution in case of unit Prandtl number and unit Schmidt number

It is noticed that solution (22) for the fluid velocity is not valid for the fluids with unit Prandtl number and unit Schmidt number. Prandtl number is a measure of the relative strength of viscosity and thermal diffusivity of fluid whereas Schmidt number is a measure of the relative strength of viscosity and molecular (mass) diffusivity of fluid. Therefore, fluid flow problem with $P_r=1$ corresponds to those fluids for which both viscous and thermal boundary layer thicknesses are of same order of magnitude and $S_c=1$ corresponds to those fluids for which both viscous and concentration boundary layer thicknesses are of same order of magnitude. There are some fluids of practical interest which belong to this category (Chen, 2004). Substituting $P_r=1$ and $S_c=1$ in equations (8) and (9) and following the same procedure as before, exact solution for fluid temperature $T(y,t)$, species concentration $C(y,t)$ and fluid velocity $F(y,t)$ is obtained and is presented in the following form

$$T(y,t)=T_1(y,t)-H(t-1)T_1(y,t-1), \tag{23}$$

$$C(y,t)=erfc\left(\frac{y}{2\sqrt{t}}\right), \tag{24}$$

$$F(y,t)=\frac{1}{2}\left[e^{y\sqrt{\lambda}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)\right.$$
$$\left.+e^{-y\sqrt{\lambda}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{\lambda t}\right)\right]$$
$$-G_3\left[F_1(y,t)-H(t-1)F_1(y,t-1)\right]$$
$$-G_4C_1(y,t), \tag{25}$$

where

$$G_3=G_r/(\lambda-\phi),\quad G_4=G_c/\lambda,$$

$$T_1(y,t)=\frac{1}{2}\left[\left(t+\frac{y}{2\sqrt{\phi}}\right)e^{y\sqrt{\phi}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{\phi t}\right)\right.$$
$$\left.+\left(t-\frac{y}{2\sqrt{\phi}}\right)e^{-y\sqrt{\phi}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{\phi t}\right)\right],$$

$$C_1=\frac{1}{2}\left[\left(t+\frac{y}{2\sqrt{\lambda}}\right)e^{y\sqrt{\lambda}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)\right.$$
$$+e^{-y\sqrt{\lambda}}\left(t-\frac{y}{2\sqrt{\lambda}}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{\lambda t}\right)$$
$$\left.-2erfc\left(\frac{y}{2\sqrt{t}}\right)\right],$$

$$F_1(y,t)=\frac{1}{2}\left[\left(t+\frac{y}{2\sqrt{\lambda}}\right)e^{y\sqrt{\lambda}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)\right.$$
$$+\left(t-\frac{y}{2\sqrt{\lambda}}\right)e^{-y\sqrt{\lambda}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{\lambda t}\right)$$
$$-\left(t+\frac{y}{2\sqrt{\phi}}\right)e^{y\sqrt{\phi}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{\phi t}\right)$$
$$\left.-\left(t-\frac{y}{2\sqrt{\phi}}\right)e^{-y\sqrt{\phi}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{\phi t}\right)\right].$$

It is noticed from the solutions (20) and (21) that the solutions (23) and (24) for fluid temperature and species concentration can also be deduced directly by setting $P_r=1$ and $S_c=1$ in the solutions (20) and (21).

2.2 Solution in case of Isothermal plate

Solutions (20) and (22) represent the analytical solution for fluid temperature and fluid velocity for the flow of a viscous, incompressible, electrically conducting and heat absorbing fluid past an impulsively moving vertical plate with ramped temperature taking Hall current and rotation into account. In order to highlight the influence of ramped temperature distribution within the plate on the flow-field, it may be justified to compare such a flow with the one past an impulsively moving vertical plate with uniform temperature. Keeping in view the assumptions made in this paper, solution for the fluid temperature and fluid velocity for the flow past an impulsively moving isothermal vertical plate is obtained and is presented in the following form

$$T(y,t)=\frac{1}{2}\left[e^{y\sqrt{\phi P_r}}erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}+\sqrt{\phi t}\right)\right.$$
$$\left.+e^{-y\sqrt{\phi P_r}}erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}-\sqrt{\phi t}\right)\right], \tag{26}$$

$$F(y,t)=\frac{(1+\alpha)}{2}\left[e^{y\sqrt{\lambda}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{\lambda t}\right)\right.$$
$$\left.+e^{-y\sqrt{\lambda}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{\lambda t}\right)\right]$$
$$-\frac{\alpha e^{\beta_1 t}}{2}\left[\left\{e^{y\sqrt{(\lambda+\beta_1)}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(\lambda+\beta_1)t}\right)\right.\right.$$
$$\left.+e^{-y\sqrt{(\lambda+\beta_1)}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(\lambda+\beta_1)t}\right)\right\}$$

$$-\left\{e^{y\sqrt{P_r(\phi+\beta_1)}}erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}+\sqrt{(\phi+\beta_1)t}\right)\right.$$
$$\left.\left.+e^{-y\sqrt{P_r(\phi+\beta_1)}}erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}-\sqrt{(\phi+\beta_1)t}\right)\right\}\right]$$
$$-\frac{\alpha}{2}\left[e^{y\sqrt{\phi P_r}}erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}+\sqrt{\phi t}\right)\right.$$
$$\left.+e^{-y\sqrt{\phi P_r}}erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}-\sqrt{\phi t}\right)\right]-G_2C^*, \qquad (27)$$

where

$\alpha = G_r/(1-P_r)\beta_1$.

The solutions for fluid temperature presented by (20), (23) and (26) are already obtained by Seth *et al*. (2013).

2.3 Skin Friction and Nusselt Number

The expressions for primary skin friction τ_x , secondary skin friction τ_z and Nusselt number Nu , which are measures of shear stress at the plate due to primary flow, shear stress at the plate due to secondary flow and rate of heat transfer at the plate respectively, are presented in the following form for ramped temperature and isothermal plates.

(i) For ramped temperature plate

$$\tau_x + i\tau_z = \sqrt{\lambda}\left(erfc\left(\sqrt{\lambda t}\right)-1\right)-\frac{1}{\sqrt{\pi t}}e^{-\lambda t}$$
$$-G_1\left[F_2(0,t)-H(t-1)F_2(0,t-1)\right]$$
$$-G_2C_2(0,t), \qquad (28)$$

$$Nu = N^*(0,t)-H(t-1)N^*(0,t-1), \qquad (29)$$

Where

$$F_2(0,t)=\frac{e^{\beta t}}{\beta_1^2}\left[\sqrt{(\lambda+\beta_1)}\left(erfc\left(\sqrt{(\lambda+\beta_1)t}\right)-1\right)\right.$$
$$\left.-\sqrt{P_r(\phi+\beta_1)}\left(erfc\left(\sqrt{(\phi+\beta_1)t}\right)-1\right)\right]$$
$$-\frac{1}{\beta}\left[\frac{1}{2}\left\{\frac{1}{\sqrt{\lambda}}\left(erfc\left(\sqrt{\lambda t}\right)-1\right)-\sqrt{\frac{P_r}{\phi}}\left(erfc\left(\sqrt{\phi t}\right)-1\right)\right\}\right.$$
$$+\left(t+\frac{1}{\beta_1}\right)\left\{\sqrt{\lambda}\left(erfc\left(\sqrt{\lambda t}\right)-1\right)\right.$$
$$\left.\left.-\sqrt{\phi P_r}\left(erfc\left(\sqrt{\phi t}\right)-1\right)\right\}-\frac{t}{\sqrt{\pi t}}\left\{e^{-\lambda t}-\sqrt{P_r}\,e^{-\phi t}\right\}\right],$$

$$C_2(0,t)=\frac{1}{\beta_2}\left[\sqrt{\lambda}\left(erfc\left(\sqrt{\lambda t}\right)-1\right)+\frac{1}{\sqrt{\pi t}}\left(\sqrt{S_c}-1\right)\right.$$
$$-e^{-\beta_2 t}\left\{\sqrt{\lambda-\beta_2}\left(erfc\sqrt{(\lambda-\beta_2)t}-1\right)\right.$$
$$\left.\left.-i\sqrt{S_c\beta_2}\left(erfc\sqrt{i\beta_2 t}-1\right)\right\}\right],$$

$$N^*(0,t)=\frac{1}{2}\sqrt{\frac{P_r}{\phi}}\left(erfc\left(\sqrt{\phi t}\right)-1\right)-t\sqrt{\phi P_r}$$
$$+t\left\{\sqrt{\phi P_r}\,erfc\left(\sqrt{\phi t}\right)-\sqrt{\frac{P_r}{\pi t}}e^{-\phi t}\right\}.$$

(ii) For isothermal plate

$$\tau_x + i\tau_z = (1+\alpha)\sqrt{\lambda}\left(erfc\left(\sqrt{\lambda t}\right)-1\right)-\frac{1}{\sqrt{\pi t}}e^{-\lambda t}$$
$$-\alpha e^{\beta t}\left\{\sqrt{(\lambda+\beta_1)}\left(erfc\left(\sqrt{(\lambda+\beta_1)t}\right)-1\right)\right.$$
$$\left.-\sqrt{P_r(\phi+\beta_1)}\left(erfc\left(\sqrt{P_r(\phi+\beta_1)t}\right)-1\right)\right\}$$
$$-\alpha\left\{\sqrt{\phi P_r}\left(erfc\left(\sqrt{\phi t}\right)-1\right)\right\}-G_2C_2, \qquad (30)$$

$$Nu=\sqrt{\phi P_r}\left(erfc\left(\sqrt{\phi t}\right)-1\right)-\sqrt{\frac{P_r}{\pi t}}e^{-\phi t}. \qquad (31)$$

2.4 Sherwood Number

The expression for Sherwood number *Sh*, which is a measure of rate of mass transfer at the plate, is given by

$$Sh=-\sqrt{\frac{S_c}{\pi t}}. \qquad (32)$$

It may be noted from (32) that Sherwood number *Sh* increases on increasing Schmidt number S_c and decreases on increasing time *t*. Since Schmidt number S_c is a measure of relative strength of viscosity and molecular (mass) diffusivity of fluid, S_c decreases on increasing molecular (mass) diffusivity of fluid. Thus we conclude from (32) that mass diffusion tends to reduce the rate of mass transfer at the plate and there is reduction in rate of mass transfer at the plate with the progress of time.

3. RESULTS AND DISCUSSION

In order to analyze the effects of Hall current, rotation, thermal buoyancy force, concentration buoyancy force, mass diffusion and time on the flow-field, numerical values of the primary and secondary fluid velocities in the boundary layer region, computed from the analytical solutions (22) and (27), are displayed graphically versus boundary layer coordinate *y* in Figs. 2 to 13 for various values of Hall current parameter *m*, rotation parameter K^2, thermal Grashof number G_r , solutal Grashof number G_c , Schmidt number S_c and time *t* taking magnetic parameter $M^2=15$, permeability parameter $K_1=0.4$, heat absorption coefficient $\phi=5$ and Prandtl number $P_r=0.71$. It is revealed from Figs. 2 to 13 that, for both ramped temperature and isothermal plates, primary velocity *u* and secondary velocity *w* attain a distinctive maximum value near surface of the plate and then decrease properly on increasing boundary layer coordinate *y* to approach free stream value. It is also noticed that the primary and secondary fluid velocities are slower in the case of ramped temperature plate than that of isothermal plate. It is evident from Figs. 2 and 3 that, for both ramped temperature and isothermal plates, *u* increases on increasing *m* in the region near the plate and it decreases on increasing *m* in the region away from the plate whereas *w* increases on increasing *m* throughout the boundary layer region. This implies that, for both

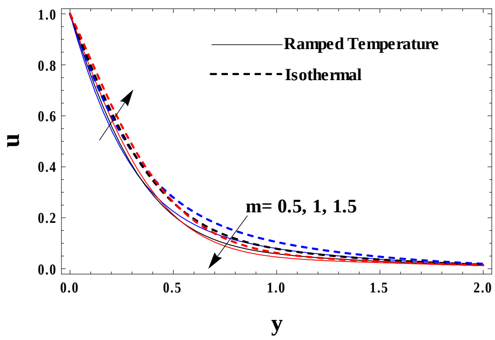

Fig. 2. Primary velocity profiles when $K^2 = 5, G_r = 6,$ $G_c = 5, S_c = 0.6$ and $t = 0.5$

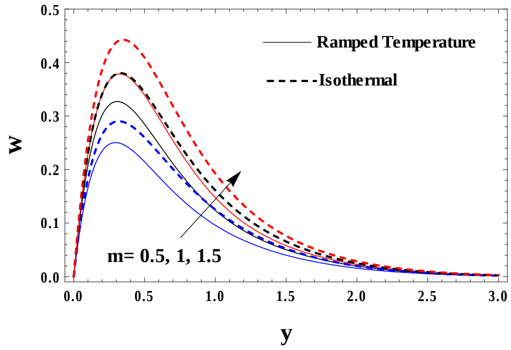

Fig. 3. Secondary velocity profiles when $K^2 = 5, G_r = 6,$ $G_c = 5, S_c = 0.6$ and $t = 0.5$

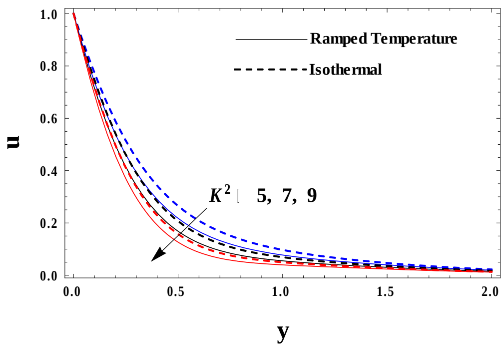

Fig. 4. Primary velocity profiles when $m = 0.5, G_r = 6,$ $G_c = 5, S_c = 0.6$ and $t = 0.5$

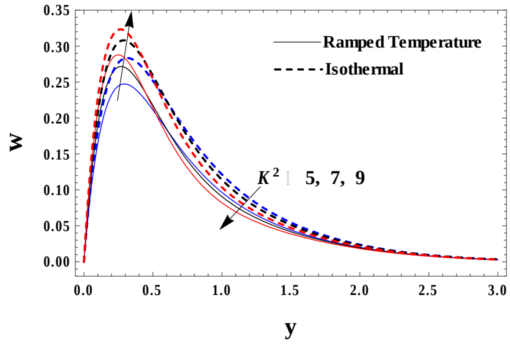

Fig. 5. Secondary velocity profiles when $m = 0.5, G_r = 6,$ $G_c = 5, S_c = 0.6$ and $t = 0.5$

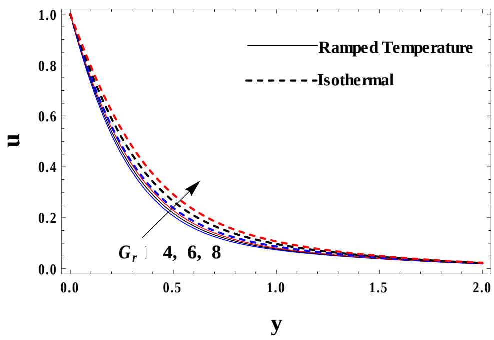

Fig. 6. Primary velocity profiles when $m = 0.5, K^2 = 5,$ $G_c = 5, S_c = 0.6$ and $t = 0.5$

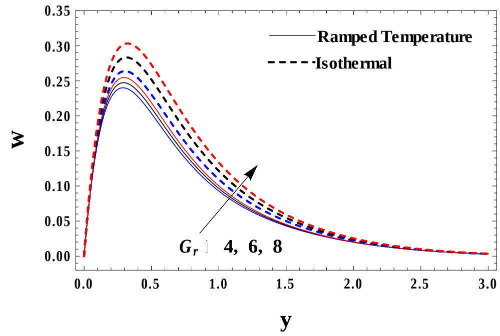

Fig. 7. Secondary velocity profiles when $m = 0.5, K^2 = 5,$ $G_c = 5, S_c = 0.6$ and $t = 0.5$

ramped temperature and isothermal plates, Hall current tends to accelerate secondary fluid velocity throughout the boundary layer region. Hall current tends to accelerate primary fluid velocity in the region near the plate whereas it has a reverse effect on primary fluid velocity in the region away from the plate. It is perceived from Figs. 4 and 5 that, for both ramped temperature and isothermal plates, u decreases on increasing K^2 whereas w increases on increasing K^2 in the region near the plate and it decreases on increasing K^2 in the region away from the plate. This implies that, for both ramped temperature and isothermal plates, rotation tends to retard primary fluid velocity throughout the boundary layer region whereas it tends to accelerate secondary fluid velocity in the region near the plate. It has a reverse effect on secondary fluid velocity in the region away from the plate. It is revealed from Figs. 6 to 9 that, for both ramped temperature and isothermal plates, u and w increase on increasing G_r and G_c. G_r represents the relative strength of thermal buoyancy force to viscous force and G_c represents the relative strength of concentration buoyancy force to viscous force. Therefore, G_r and G_c increase on increasing the strengths of thermal and concentration buoyancy forces respectively. This implies that thermal and concentration buoyancy forces tend to accelerate primary and secondary fluid velocities for both ramped temperature and isothermal plates It is noticed from Figs. 10 and 11 that, for both ramped temperature and isothermal plates, u and w decrease on increasing S_c.

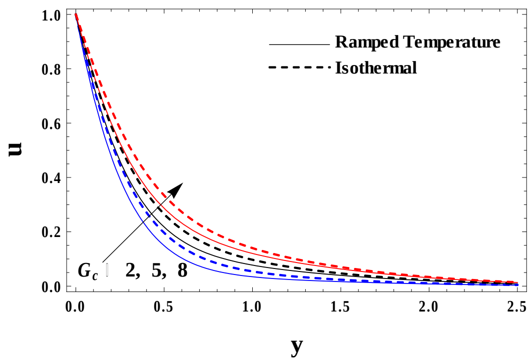

Fig. 8. Primary velocity profiles when $m = 0.5$, $K^2 = 5$, $G_r = 6$, $S_c = 0.6$ and $t = 0.5$

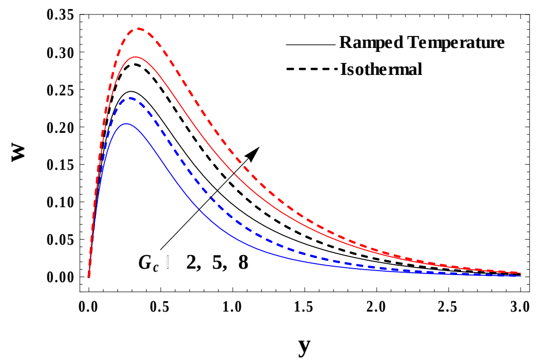

Fig. 9. Secondary velocity profiles when $m = 0.5$, $K^2 = 5$, $G_r = 6$, $S_c = 0.6$ and $t = 0.5$

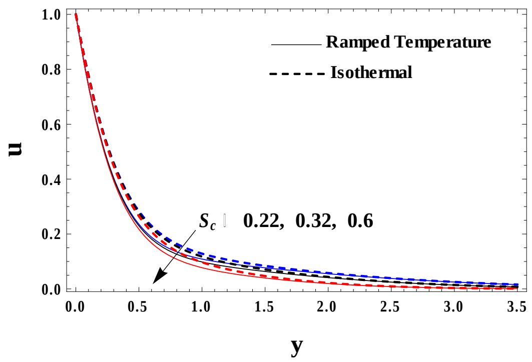

Fig. 10: Primary velocity profiles when $m = 0.5$, $K^2 = 5$, $G_r = 6$, $G_c = 5$ and $t = 0.5$

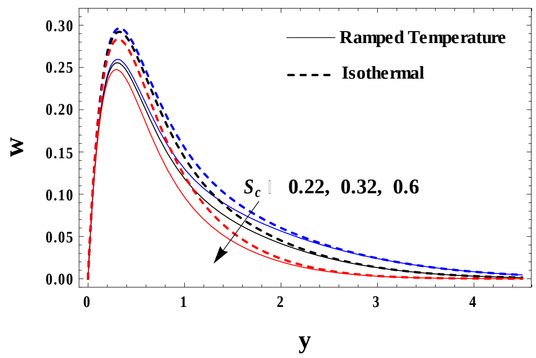

Fig. 11. Secondary velocity profiles when $m = 0.5$, $K^2 = 5$, $G_r = 6$, $G_c = 5$ and $t = 0.5$

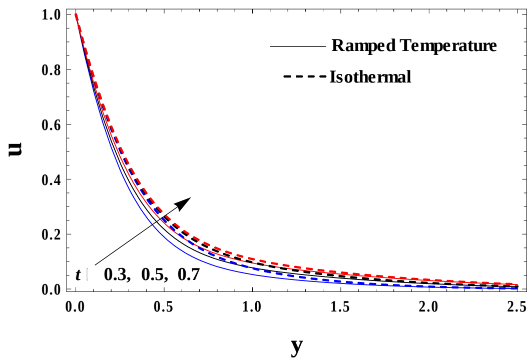

Fig. 12. Primary velocity profiles when $m = 0.5$, $K^2 = 5$, $G_r = 6$, $G_c = 5$ and $S_c = 0.6$

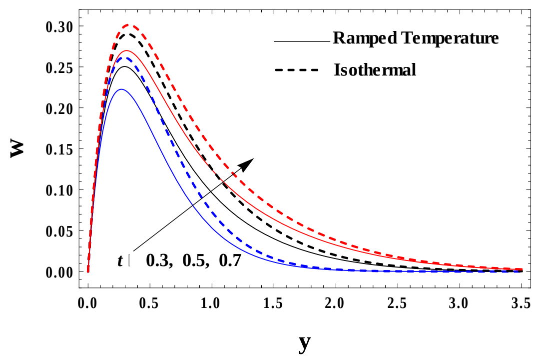

Fig. 13. Secondary velocity profiles when $m = 0.5$, $K^2 = 5$, $G_r = 6$, $G_c = 5$ and $S_c = 0.6$

S_c is the measure of relative strength of viscosity to molecular (mass) diffusivity of the fluid, S_c decreases on increasing mass diffusivity. This implies that mass diffusion tends to accelerate primary and secondary fluid velocities for both ramped temperature and isothermal plates. It is evident from Figs. 12 and 13 that, for both ramped temperature and isothermal plates, u and w increase on increasing t. This implies that primary and secondary fluid velocities are getting accelerated with the passage of time for both ramped temperature and isothermal plates.

The numerical values of species concentration C, computed from analytical solution (21), are presented graphically versus boundary layer coordinate y in Figs.14 and 15 for various values of Schmidt number S_c and time t. It is evident from Figs. 14 and 15 that species concentration C decreases on increasing S_c whereas it increases on increasing t. This implies that mass diffusion tends to enhance species concentration and there is an enhancement in species concentration with passage of time.

The numerical values of primary skin friction τ_x, secondary skin friction τ_z, computed from analytical expressions (28) and (30), are presented in tabular form in Tables 1 to 6 for various values of m, K^2, G_r, G_c, S_c and t taking $M^2 = 15, K_1 = 0.4, \phi = 5$ and $P_r = 0.71$. It is evident from Tables 1 to 6 that primary skin friction τ_x decreases on increasing m, G_r, G_c and t whereas it increases on increasing

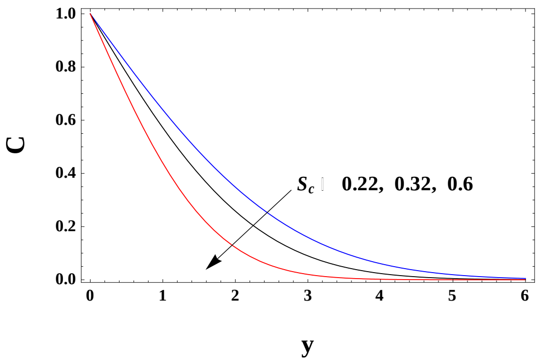

Fig.14. Concentration profiles when $t = 0.5$

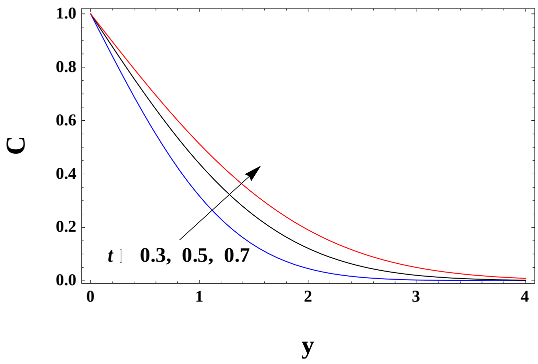

Fig.15. Concentration profiles when $S_c = 0.6$

Table 1 Skin Friction for ramped temperature plate when $G_r = 6, G_c = 5, S_c = 0.6$ and $t = 0.5$

$K^2 \rightarrow$ $m \downarrow$	$-\tau_x$			τ_z		
	5	7	9	5	7	9
0.5	3.0119	3.2593	3.5083	2.2441	2.6637	3.0359
1	2.6220	2.9332	3.2257	2.7228	3.1183	3.4622
1.5	2.2614	2.6265	2.9525	2.9085	3.3013	3.6277

Table 2 Skin Friction for isothermal plate when $G_r = 6, G_c = 5, S_c = 0.6$ and $t = 0.5$

$K^2 \rightarrow$ $m \downarrow$	$-\tau_x$			τ_z		
	5	7	9	5	7	9
0.5	2.5352	2.8141	3.0909	2.4077	2.8406	3.2204
1	2.1446	2.4925	2.8157	2.9329	3.3335	3.6782
1.5	1.7737	2.1804	2.5402	3.149	3.5420	3.865

Table 3 Skin Friction for ramped temperature plate when $m = 0.5, K^2 = 5, S_c = 0.6$ and $t = 0.5$

$G_c \rightarrow$ $G_r \downarrow$	$-\tau_x$			τ_z		
	2	5	8	2	5	8
4	3.6426	3.1507	2.6587	2.0566	2.208	2.359
6	3.5039	3.0120	2.52	2.0927	2.2441	2.3953
8	3.3652	2.8732	2.3813	2.1291	2.2804	2.4316

Table 4 Skin Friction for isothermal plate when $m = 0.5, K^2 = 5, S_c = 0.6$ and $t = 0.5$

$G_c \rightarrow$ $G_r \downarrow$	$-\tau_x$			τ_z		
	2	5	8	2	5	8
4	3.3248	2.8327	2.3409	2.1656	2.3168	2.4681
6	3.0272	2.5352	2.0433	2.2564	2.4077	2.5589
8	2.7295	2.2376	1.7456	2.3472	2.4985	2.6497

Table 5 Skin Friction for ramped temperature plate when $m = 0.5, K^2 = 5, G_r = 6$ and $G_c = 5$

$t \rightarrow$ $S_c \downarrow$	$-\tau_x$			τ_z		
	0.3	0.5	0.7	0.3	0.5	0.7
0.22	3.0913	2.898	2.7119	2.2991	2.3689	2.4338
0.32	3.1389	2.934	2.7423	2.2472	2.3291	2.4001
0.6	3.2404	3.012	2.8081	2.1367	2.2441	2.328

Table 6 Skin Friction for isothermal plate when $m = 0.5, K^2 = 5, G_r = 6$ and $G_c = 5$

$t \rightarrow$ $S_c \downarrow$	$-\tau_x$			τ_z		
	0.3	0.5	0.7	0.3	0.5	0.7
0.22	2.4403	2.4213	2.413	2.5081	2.5325	2.5443
0.32	2.4879	2.4572	2.4434	2.4562	2.4927	2.5106
0.6	2.5894	2.5352	2.5092	2.3457	2.4077	2.4385

Table 7 Nusselt number -*Nu* when P_r =0.71

$\phi \rightarrow$ $t \downarrow$	Ramped Temperature Plate			Isothermal Plate		
	1	5	9	1	5	9
0.3	0.5713	0.749	0.8981	1.1160	1.9209	2.5352
0.5	0.7791	1.1294	1.4043	0.9830	1.8915	2.5284
0.7	0.9692	1.507	1.9099	0.9253	1.8859	2.5279

K^2 and S_c for both ramped temperature and isothermal plates. Secondary skin friction τ_z increases on increasing m, K^2, G_r, G_c and t whereas it decreases on increasing S_c for both ramped temperature and isothermal plates. This implies that, for both ramped temperature and isothermal plates, Hall current, thermal buoyancy force, concentration buoyancy force and mass diffusion have tendency to reduce primary skin friction whereas these physical quantities have reverse effect on secondary skin friction. Rotation tends to enhance both the primary and secondary skin frictions for both ramped temperature and isothermal plates. As time progresses, primary skin friction is getting reduced whereas secondary skin friction is getting enhanced for both ramped temperature and isothermal plates.

The numerical values of Nusselt number *Nu*, calculated from analytical expressions (29) and (31), are presented in tabular form in Table 7 for various values of ϕ and t taking $P_r = 0.71$. It is evident from Table 7 that, for both ramped temperature and isothermal plates, Nusselt number *Nu* increases on increasing ϕ. *Nu* increases on increasing t for ramped temperature plate but it decreases on increasing t for isothermal plate. This implies that, for both ramped temperature and isothermal plates, heat absorption tends to enhance rate of heat transfer at the plate. As time progresses, rate of heat transfer at the plate is getting enhanced for ramped temperature plate whereas it is getting reduced for isothermal plate.

4. CONCLUSIONS

An investigation of the effects of Hall current and rotation on unsteady hydromagnetic natural convection flow with heat and mass transfer of an electrically conducting, viscous, incompressible and heat absorbing fluid past an impulsively moving vertical plate with ramped temperature embedded in a porous medium has been carried out. Significant findings are as follows:
For both ramped temperature and isothermal plates:

- Hall current tends to accelerate secondary fluid velocity throughout the boundary layer region. It tends to accelerate primary fluid velocity in the region near the plate whereas it has a reverse effect on primary fluid velocity in the region away from the plate.
- Rotation tends to retard primary fluid velocity throughout the boundary layer region whereas it tends to accelerate secondary fluid velocity in the region near the plate. It has a reverse effect on secondary fluid velocity in the region away from the plate.
- Thermal and concentration buoyancy forces and mass diffusion tend to accelerate primary and secondary fluid velocities throughout the boundary layer region.
- Primary and secondary fluid velocities are getting accelerated with the passage of time throughout the boundary layer region.
- Hall current, thermal buoyancy force, concentration buoyancy force and mass diffusion have tendency to reduce primary skin friction whereas these physical quantities have reverse effect on secondary skin friction.
- Rotation tends to enhance both the primary and secondary skin frictions.
- As time progresses, primary skin friction is getting reduced whereas secondary skin friction is getting enhanced.

Rate of heat transfer at the plate is getting enhanced for ramped temperature plate whereas it is getting reduced for isothermal plate with the progress of time.

ACKNOWLEDGEMENT

Authors are grateful to the reviewers for their valuable comments and suggestions which helped them to improve the quality of the research paper.

REFERENCES

Abdallah, M. Si. and B. Zeghmati (2011). Natural convection heat and mass transfer in the boundary layer along a vertical cylinder with opposing buoyancies. *J. Appl. Fluid Mech.* 4(4), 15–21.

Abo-Eldahab, E. M. and E. M. E. Elbarbary (2001). Hall current effect on magnetohydrodynamic free-convection flow past a semi-infinite vertical plate with mass transfer. *Int. J. Engg. Sci*. 39, 1641–1652.

Bejan, A. (1993). *Convection Heat Transfer*. Second Ed., Wiley, NY, pp. 176.

Bejan, A. and K. R. Khair (1985). Heat and mass transfer by natural convection in a porous medium. *Int. J. Heat Mass Transf*. 28, 909–918.

Bisht, V., M. Kumar and Z. Uddin (2011). Effects of variable thermal conductivity and chemical reaction on steady mixed convection boundary layer flow with heat and mass transfer inside a cone due to a point sink. *J. Appl. Fluid Mech.* 4(4), 59–63.

Chamkha, A. J. (2004). Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. *Int. J. Engng. Sci*. 42, 217–230.

Chamkha, A. J. and A. A. Khaled (2000). Hydromagnetic combined heat and mass transfer by natural convection from a permeable surface embedded in a fluid saturated porous medium. *Int. J. Numer. Methods Heat Fluid Flow* 10(5), 455–476.

Chamkha, A. J., H. S. Takhar and V. M. Soundalgekar (2001). Radiation effects on free convection flow past a semi-infinite vertical plate with mass transfer. *Chem. Eng. Journal* 84, 335–342.

Chandran, P., N. C. Sacheti and A. K. Singh (2005). Natural convection near a vertical plate with ramped wall temperature. *Heat Mass Transf*. 41, 459–464.

Chen, C. H. (2004). Combined heat and mass transfer in mhd free convection from a vertical surface with Ohmic heating and viscous dissipation. *Int. J. of Eng. Sci.* 42, 699–713.

Cramer, K. R. and S. I. Pai (1973). *Magnetofluid Dynamics for Engineers and Applied Physicists*, McGraw Hill Book Company, New York.

Eckert, E. R. and R. M. Drake (1972). *Analysis of Heat and Mass Transfer*. Mc-Graw Hill, NY.

Elbashbeshy E. M. A. (1997). Heat and mass transfer along a vertical plate with variable surface tension and concentration in the presence of the magnetic field. *Int. J. Eng. Sci*. 35, 515–522.

Eldabe, N. T. M., E. M. A. Elbashbeshy, W. S. A. Hasanin and E. M. Elsaid (2011). Unsteady motion of MHD viscous incompressible fluid with heat and mass transfer through porous medium near a moving vertical plate. *Int. J. Energy and Tech*. 3 (35), 1–11.

Fossa, M. and G. Tanda (2010). Frost formation in vertical channels under natural convection. *Int. J. Multiph. Flow* 36, 210–220

Gebhart, B., Y. Jaluria, R. L. Mahajan and B. Sammakia (1998). *Buoyancy Induced Flow and Transport*. New York Hemisphere.

Ghosh, S. K., O. Anwar Bég and M. Narahari (2013). A study of unsteady rotating hydromagnetic free and forced convection in a channel subject to forced oscillation under an oblique magnetic field. *J. Appl. Fluid Mech.* 6(2), 213–227.

Hayday, A. A., D. A. Bowlus and R. A. McGraw (1967). Free convection from a vertical plate with step discontinuities in surface temperature. *ASME J. Heat Transf.* 89, 244–250.

Hossain, M. A. and A. C. Mandal (1985). Mass transfer effects on the unsteady hydromagnetic free convection flow past an accelerated vertical porous plate. *J. Phys. D: Appl. Phys*. 18, 163–169.

Ibrahim, F. S., I. A. Hassanien and A. A. Bakr (2004). Unsteady magnetohydrodynamic micropolar fluid flow and heat transfer over a vertical porous plate through a porous medium in the presence of thermal and mass diffusion with a constant heat source. *Canad. J. Phys*. 82, 775–790.

Jang, J. Y. and W. J. Chang (1988). Buoyancy-induced inclined boundary layer flow in a porous medium resulting from combined heat and mass buoyancy effects. *Int. Commun. Heat Mass Transf*. 15, 17–30.

Jha, B. K. (1991) MHD free convection and mass transform flow through a porous medium. *Astrophys. Space Sci*. 175, 283–289.

Kamel, M. H. (2001). Unsteady MHD convection through porous medium with combined heat and mass transfer with heat source/sink, *Energy Conv. Management* 42, 393–405.

Kao, T. T. (1975). Laminar free convective heat transfer response along a vertical flat plate with step jump in surface temperature. *Lett. Heat Mass Transf.* 2, 419–428.

Kelleher, M. (1971). Free convection from a vertical plate with discontinuous wall temperature. *ASME J. Heat Transf.* 93, 349–356.

Kyth, P. K. and P. Puri (1988). Unsteady MHD free-convection flows on a porous plate with time-dependent heating in a rotating medium. *Astrophys. Space Sci*. 143 (1), 51–62.

Lee, S. and M. M. Yovanovich (1991). Laminar natural convection from a vertical plate with a step change in wall temperature. *ASME J. Heat Transf.* 113, 501–504.

Makinde, O. D. (2009). On MHD boundary-layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux. *Int. J. Numer. Methods Heat Fluid Flow* 19, 546–554.

Makinde, O. D. and P. Sibanda (2008). Magnetohydrodynamic mixed convective flow and heat and mass transfer past a vertical plate in a porous medium with constant wall suction. *J. Heat Transf.*, 130, Article No. 112602, 8 pages.

Nakayama, A. and M. A. Hossain (1995). An integral treatment for combined heat and mass transfer by natural convection in a porous medium. *Int. J. Heat and Mass Transf*. 38, 761–765.

Nanousis, N. (1992). Thermal-diffusion effects on MHD free-convective and mass-transfer flow past a moving infinite vertical plate in a rotating fluid. *Astrophys. Space Sci*. 191 (2), 313–322.

Narahari, M. (2012). Transient free convection flow between long vertical parallel plates with ramped wall temperature at one boundary in the presence of thermal radiation and constant mass diffusion. *Meccanica* 47, 1961–1976

Raptis, A. A. (1982). Free convection and mass transfer effects on the oscillatory flow past an infinite moving vertical isothermal plate with constant suction and heat sources. *Astrophy. Space Sci*. 86, 43–53.

Raptis, A. A. and A. K. Singh (1985). Rotation effects on MHD free-convection flow past an accelerated vertical plate. *Mech. Res. Comm.* 12 (1), 31–40.

Reddy, M.G. and N.B. Reddy (2011). Mass transfer and heat generation effects on mhd free convection flow past an inclined vertical surface in a porous medium. *J. Appl. Fluid Mech.* 4(2), 7–11.

Reddy, P. B. A., N. Bhaskar Reddy and S. Suneetha (2012). Radiation effects on MHD flow past an exponentially accelerated isothermal vertical plate with uniform mass diffusion in the presence of heat source. *J. Appl. Fluid Mech.* 5 (3), 119–126.

Saha, S. C., J. C. Patterson and C. Lei (2010). Scaling of natural convection of an inclined flat plate: ramp cooling condition. *Int J Heat Mass Transf*. 53, 5156–5166.

Schetz, J. A. (1963). On the approximate solution of viscous flow problems. *ASME J. Appl. Mech*. 30, 263–268.

Schetz, J. A. and R. Eichhorn (1964). Natural convection with discontinuous wall-temperature variations. *J Fluid Mech* 18, 167–176.

Seth, G. S. and Md. S. Ansari (2010). MHD natural convection flow past an impulsively moving vertical plate with ramped wall temperature in the presence of thermal diffusion with heat absorption. *Int. J. Appl. Mech. Eng.* 15, 199–215.

Seth, G. S., G. K. Mahato, S. Sarkar (2013a). Effects of Hall current and rotation on MHD natural convection flow past an impulsively moving vertical plate with ramped temperature in the presence of thermal diffusion with heat absorption. *Int. J. Energy Tech.* 5(16): 1–12.

Seth, G.S., R. Nandkeolyar and M. S. Ansari (2013b). Effects of thermal radiation and rotation on unsteady hydromagnetic free convection flow past an impulsively moving vertical plate with ramped temperature in a porous medium. *J. Appl. Fluid Mech.* 6(1), 27–38.

Sharma, P. R., G. Singh, A. J. Chamkha (2012). Steady mixed convection flow of water at $4^{o}C$ along a non-isothermal vertical moving plate with transverse magnetic field. *Int. J. Ind. Math*. 4(3), Article ID: IJIM-00260, (16 pages).

Singh, A. K., N. P. Singh, U. Singh and H. Singh (2009). Convective flow past an accelerated porous plate in rotating system in presence of magnetic field. *Int. J. Heat Mass Transfer*, 52 (13-14), 3390–3395.

Sparrow, E. M. and R. D. Cess (1961). Temperature dependent heat sources or sinks in a stagnation point flow, *Appl. Sci. Res*., 10, 185-197.

Takhar, H. S., S. Roy and G. Nath (2003). Unsteady free convection flow over an infinite vertical porous plate due to the combined effects of thermal and mass diffusion, magnetic field and Hall currents. *Heat Mass Transf*. 39:825–834.

Tokis, J. N. (1988). Free convection and mass transfer effects on the magnetohydrodynamic flows near a moving plate in a rotating medium. *Astrophys. Space Sci*. 144 (1-2), 291–301.

Yih, K. A. (1997). The effect of transpiration on coupled heat and mass transfer in mixed convection over a vertical plate embedded in a saturated porous medium, *Int. Commun. Heat Mass Transf*. 24, 265–275.