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On the Onset of Convection in a Dusty Couple-Stress Fluid with Variable Gravity through a Porous Medium in Hydromagnetics

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ABSTRACT

In the present note, we have considered the problem of the onset of convection in a couple-stress fluid with variable gravity to include the effects of suspended particles and vertical magnetic field through a porous medium. Following the normal mode method, dispersion relation is obtained in the presence of various parameters like porosity, permeability, suspended particle, couple-stress and magnetic field. For the case of stationary convection, it is found that the parameters like porosity, permeability and suspended particles have a destabilizing effect on the system whereas couple-stress and magnetic field have a stabilizing effect on the onset of convection. The dispersion relation is analyzed numerically and the results are also shown graphically. The necessary condition for the onset of instability and the sufficient condition for the non-existence of convection at the marginal state in the absence and presence of couple-stress parameter have also been obtained by using Rayleigh-Ritz and Cauchy-Schwartz inequality.

Keywords: Couple-Stress fluid, Suspended particles, Magnetic field, Porous medium.

1. INTRODUCTION

Thermal convective instability of a layer of fluid heated from below has extensive applications in many branches of fluid dynamics like geophysics, earth's science, oceanography and heat transfer mechanism. The problem of thermal instability in a horizontal layer of Newtonian fluid heated from below has been discussed in detail by Chandrasekhar (1981). Rayleigh (1916) laid the foundation of the linear theory of the hydrodynamic stability and was the first to apply the method of small perturbations. The growing importance of non-Newtonian fluids in several scientific and engineering problems has attracted researchers for the study on such fluids. Stokes (1966) proposed and formulated the theory of couple-stresses in fluids. One of the important application of couple-stresses in fluid is its use in the study of the mechanism of lubrication of synovial joints, which has become the objective of the scientific research because of its importance in human locomotion. Normal synovial fluid is clear or yellowish and is a viscous non-Newtonian fluid. The two important parts of synovial joints are cartilage and rheological nature of synovial fluid. The cartilage forms the covering on the bone ends in synovial joints and plays a significant role in normal joints functioning. The most important aspect of synovial joints is to understand different lubrication mechanisms during human locomotion. Any degenerative changes in the synovial joints can directly affect the normal physiological functioning of an individual. These degenerative changes can cause arthritis. Lin (1997) theoretically investigated the rheological effects of couple stress fluids on the static and dynamic behaviours of pure squeeze films in journal-bearing systems and concluded that the effects of couple stresses significantly improves the squeeze film characteristics and also results in a longer bearing life. Sunil et al. (2004) have studied the effect of suspended particles in a layer of couple-stress fluid heated and soluted from below in a porous medium and depicted the stabilizing effect of couple-stress and solute gradient parameters and destabilizing effects of suspended particles and medium permeability for the case of stationary convection. Sharma and Sharma (2001) have studied the problem of thermal convection of a couple-stress fluid heated from below saturating a porous medium.

In many branches of sanitary work, notably in the study of factory conditions, the enumeration of the actual number of dust particles present are quite important for the determination of the total weight of dust. Dust is a generic term used to describe fine particles that are suspended in the atmosphere. Dust comes from a wide variety of sources, including soil, vegetation (pollens and fungi), sea salt, fossil fuel combustion, burning of biomass, and industrial activities. It is formed when fine particles are taken up into the atmosphere by the action of wind or other physical disturbances or through the release of particulate-rich gaseous emissions. In geophysical context, the fluid is often not pure but may instead be permeated with dust particles. The effects of suspended particles on the stability of superposed fluids have industrial and scientific importance in geophysics, chemical engineering and astrophysics. Scanlon and Segel (1973) have considered the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure fluid was supplemented by the particles. Since the earth's gravity field varies with the vertical distance from its surface, so it is necessary to take into account the gravity as a variable quantity with height from the earth's centre. This variation of gravity plays an important role for large scale flows in ocean, atmosphere and mantle but for laboratory purposes, we usually neglect these variations. Thermal instability of a fluid layer under varying gravity field heated from above and below has been investigated by Pradhan and Samal (1987).

Magneto-hydrodynamics theory of electrically conducting fluids has several scientific and practical applications in astrophysics, geophysics, space sciences etc. Transformers, microphones, advertising displays, memory storages, and so on, which play a key role in everyday life, could not have been developed without an understanding of magnetic phenomena. Magnetic field is also used in several clinical areas such as in neurology and orthopedics for probing and curing the internal organs of the body in several diseases like tumours detection, heart and brain diseases, stroke damage etc. The problem of thermal instability of an electrically conducting couple-stress fluid heated from below through a porous medium in the presence of a uniform magnetic field has been investigated by Sharma and Thakur (2000). Sharma and Sharma (2004) have investigated the effect of suspended particles on couple-stress fluid heated from below in the presence of rotation and magnetic field and concluded that the effects of magnetic field, rotation and couple stress parameter is to stabilize the system whereas suspended particles have destabilizing effects.

The flow through a porous medium has been of fundamental importance in solidification, chemical processing industry, geophysical fluid dynamics, petroleum industry, recovery of crude oil from earth's interior etc. A detailed study of convection through porous medium has been covered extensively by Nield and Bejan (2006) in his historical monograph. A porous medium is defined as a solid with inter-connected voids. Liquid saturated porous material are often present on and below the surface of the earth in the form of dust particles, limestone and other sediments permeated by groundwater or oil. Recently, the development of geothermal power resources has increased considerable attention in convection through porous medium. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in astrophysical situations (McDonnel [1978]). The problem of both linear and nonlinear thermal convection in a couple stresses fluid-saturated rotating rigid porous layer has been studied by Shivakumara et al. (2011) and found that both couple stress parameter and Taylor number dampen the oscillations of Nusselt number.

The problem of thermosolutal convection in a couple-stress fluid layer through a porous medium to include the effects of vertical magnetic field and vertical rotation has been discussed by Kumar (2012). Recently Banyal (2013) has investigated thermal instability of a couple-stress fluid heated from below and derive the necessary condition for the onset of instability as a stationary convection. The intent of the present note is to study the onset of convection in a couple-stress fluid saturated porous medium under the influences of varying gravity, suspended particles and uniform magnetic field.

2. FORMULATION OF PROBLEM AND PERTURBATION EQUATIONS

Here we consider an infinite horizontal layer of a couple-stress fluid permeated with suspended particles and bounded by the planes z=0 and z=d in a porous medium of porosity $\in$ and medium permeability k_1. The fluid layer is acted on by a uniform vertical magnetic field **H** (0, 0, H) and a variable gravity field $\mathbf{g} = f(z)g_0$, where $f(z)$ can be positive or negative according as the gravity increases or decreases upward from its value g_0 (>0). The layer is heated from below so that a uniform temperature gradient $\beta = |dT/dz|$ is maintained.

The governing equations of motion and continuity for a couple-stress fluid are

$$\frac{\rho_0}{\in}\left[\frac{\partial q}{\partial t}+\frac{1}{\in}(q.\nabla)q\right]=\left[\begin{array}{l}-\nabla p+\rho_0\mathbf{X_i}-\dfrac{1}{k_1}\left(\mu-\mu'\nabla^2\right)q+\\ \dfrac{K'N_0}{\in}\left(q_d-q\right)+\dfrac{\mu_e}{4\pi}\left(\nabla\times H\right)\times H\end{array}\right] \tag{1}$$

$$\nabla.q = 0 \tag{2}$$

where

$\rho_0, \mu, \mu', \mu_e, q, q_d, N_0(\bar{x}, t)\, and\, \mathbf{X_i} = -\mathbf{g}\lambda_i$ denote, respectively, the density of fluid, viscosity, couple-stress viscosity, magnetic permeability, velocity of pure fluid, velocity of suspended particles, number density of the suspended particles and the gravitational acceleration term. $\bar{x} = (x, y, z)$ and $K' = 6\pi\rho\upsilon\delta$, where δ being particle radius, is the Stokes's drag coefficient.

The presence of suspended particles adds an extra force term, in equation of motion, proportional to velocity difference between particles and fluid. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. Inter-particle reactions are ignored as the distances between the particles are assumed to be quite large compared with their diameters. The effects of pressure, magnetic field and gravity on the particles are very small and hence ignored.

If mN_0 is the mass of particles per unit volume, then the equations of motion and continuity for the particles are:

$$mN_0\left[\frac{\partial q_d}{\partial t}+\frac{1}{\in}\left(q_d.\nabla\right)q_d\right]=K'N_0\left(q-q_d\right) \tag{3}$$

$$\in\frac{\partial N_0}{\partial t}+\nabla.\left(N_0 q_d\right)=0 \tag{4}$$

The equation for temperature balance is:

$$\left[\in\rho_0 c_v+\rho_s c_s\left(1-\in\right)\right]\frac{\partial T}{\partial t}+\rho_0 c_v\left(q.\nabla\right)T+mN_0 c_{pt}\left(\in\frac{\partial}{\partial t}+q_d.\nabla\right)T=k\nabla^2 T \tag{5}$$

Where, in the above equation, $\rho_s, c_s, c_v, c_{pt}, T\ and\ k$ denote, respectively, the density of solid material, heat capacity of solid material, the specific heat at constant volume, heat capacity of suspended particles, the temperature and the co-efficient of heat conduction.

The Maxwell's equations of electromagnetism are:

$$\in\frac{\partial H}{\partial t}=\nabla\times\left(q\times H\right)+\in\eta\nabla^2 H \tag{6}$$

and

$$\nabla.H=0, \tag{7}$$

Where η denote the electrical resistivity.

The density equation of state is

$$\rho=\rho_0\left[1+\alpha\left(T_0-T\right)\right] \tag{8}$$

The steady state corresponding to the system of Eqs. (1) to (8) is defined as:

$$q=(0,0,0),\ T=T_0-\beta z,\ \rho=\rho_0\left[1+\alpha\beta z\right],$$
$$p=p_0-g\rho_0\left[1+\alpha\beta z^2/2\right], H=\left[0,0,H\right] \tag{9}$$

Now, we analyze the stability of the basic state using perturbation technique. Let the initial state solutions described by Eq. (9) be slightly perturbed. We assume that **q** (u,v,w), $\mathbf{q_d}$(l,r,s), N, θ, δp, $\delta\rho$ and $h\left(h_x,h_y,h_z\right)$ denote, respectively, the perturbation in fluid velocity **q**(0,0,0), perturbation in particle velocity $\mathbf{q_d}$(0,0,0), perturbation in particle number density N_0, temperature T, pressure p, density ρ and magnetic field H. The change in density $\delta\rho$ caused by perturbation θ in temperature, is given by

$$\delta\rho=-\alpha\theta\rho_0 \tag{10}$$

Now, assuming the perturbation quantities to be very small, the governing perturbation equations (using Oberbeck-Boussinesq approximation) due to linearization procedure are:

$$\frac{1}{\in}\frac{\partial q}{\partial t}=\left[\begin{array}{l}-\frac{1}{\rho_0}\nabla\left(\delta p\right)+\mathbf{g}\alpha\theta\lambda_i-\frac{1}{k_1}\left(\upsilon-\upsilon'\nabla^2\right)q\\+\frac{K'N}{\rho_0\in}\left(q_d-q\right)+\frac{\mu_e}{4\pi\rho_0}\left\{\left(\nabla\times h\right)\times H\right\}\end{array}\right] \tag{11}$$

$$\nabla.q=0 \tag{12}$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)q_d=q \tag{13}$$

$$\in\frac{\partial N}{\partial t}+\nabla.\left(N q_d\right)=0 \tag{14}$$

$$\left(E+b\in\right)\frac{\partial\theta}{\partial t}=\beta\left(w+bs\right)+\kappa\nabla^2\theta \tag{15}$$

$$\nabla.h=0 \tag{16}$$

$$\in\frac{\partial h}{\partial t}=\left(\nabla H\right)q+\in\eta\nabla^2 h \tag{17}$$

Where $\upsilon=\frac{\mu}{\rho_0},\ \kappa=\frac{k}{\rho_0 C_v},\ \lambda_i=\left(0,0,1\right), w$ and s denote, respectively, the kinematic viscosity, the co-efficient of thermometric conductivity, unit vertical vector, vertical fluid velocity and suspended particles velocity and $E=\in+\left(1-\in\right)\left(\frac{\rho_s C_s}{\rho_0 C_v}\right)\ and\ b=\frac{mN_0 C_{pt}}{\rho_0 C_v}$.

Eliminating δp between the three component equations of Eq. (11) and using Eqs. (12) and (13), we obtain:

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}\left(\nabla^2 w\right)=\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left[\begin{array}{l}\mathbf{g}\alpha\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\theta+\frac{\mu_e H}{4\pi\rho_0}\frac{\partial}{\partial z}\\\left(\nabla^2 h_z\right)-\frac{1}{k_1}\left(\upsilon-\upsilon'\nabla^2\right)\nabla^2 w\end{array}\right]-\frac{mN}{\in\rho_0}\frac{\partial}{\partial t}\left(\nabla^2 w\right) \tag{18}$$

and

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial\zeta}{\partial t}=\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left[\frac{\mu_e H}{4\pi\rho_0}\frac{\partial\xi}{\partial z}-\frac{1}{k_1}\left(\upsilon-\upsilon'\nabla^2\right)\zeta\right]-\frac{mN}{\in\rho_0}\frac{\partial\zeta}{\partial t} \tag{19}$$

Where $\zeta = \left(\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}\right)$ and $\xi = \dfrac{\partial h_y}{\partial x} - \dfrac{\partial h_x}{\partial y}$ stand for the z-component of vorticity and z-component of current density, respectively.

From Eq. (15), we obtain:

$$\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left[\left(E+b\in\right)\frac{\partial}{\partial t}-\kappa\nabla^2\right]\theta = \beta\left[\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)+b\right]w \tag{20}$$

The x and y component equations of Eq. (17) along with Eq. (16) yields:

$$\in\frac{\partial \xi}{\partial t} = H\frac{\partial \zeta}{\partial z} + \in\eta\nabla^2\xi \tag{21}$$

and the z- component equation of Eq. (17) is

$$\in\frac{\partial h_z}{\partial t} = H\frac{\partial w}{\partial z} + \in\eta\nabla^2 h_z \tag{22}$$

Where

$\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$ is the three dimensional Laplacian operator.

3. DISPERSION RELATION AND DISCUSSION

We now use normal mode method by decomposing the disturbances with a dependence on x, y and t of the form:

$$\left[w,\theta,\zeta,h_z,\xi\right] = \left[W(z),\Theta(z),Z(z),K(z),X(z)\right] \exp\left(ik_x x + ik_y y + nt\right) \tag{23}$$

Where k_x, k_y are the wave numbers along x and y directions, respectively and $k^2 = \left(k_x{}^2 + k_y{}^2\right)$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex quantity.

Using expression (23) and making the substitutions of the non-dimensional quantities of the form

$z = z^* d,\ a = kd,\ \sigma = \dfrac{nd^2}{\upsilon},\ \tau_1 = \dfrac{m\upsilon}{K'd^2},\ N = \dfrac{\rho_0 f}{m},$
$E_1 = E + b\in,\ \mathrm{B} = \mathrm{b}+1.$

$P_l = \dfrac{k_1}{d^2}$, is the dimensionless medium permeability, $p_1 = \dfrac{\upsilon}{\kappa}$, is the thermal Prandtl number, $p_2 = \dfrac{\upsilon}{\eta}$, is the magnetic Prandtl number and $\Upsilon = \dfrac{\upsilon'}{\upsilon d^2}$, is the dimensionless couple-stress parameter.

We obtain the non-dimensional form of Eqs. (18) to (22) (after dropping the asterisk for convenience) as:

$$\left[\frac{\sigma}{\in}\left(1+\frac{f}{1+\tau_1\sigma}\right)+\frac{1}{P_l}\left\{1-\Upsilon\left(D^2-a^2\right)\right\}\right]\left(D^2-a^2\right) W(z) + \frac{f(z)g_0\alpha a^2 d^2\Theta}{\upsilon} - \frac{\mu_e H d}{4\pi\rho_0\upsilon}\left(D^2-a^2\right)DK = 0 \tag{24}$$

$$\left[\frac{\sigma}{\in}\left(1+\frac{f}{\left(1+\tau_1\sigma\right)}\right)+\frac{1}{P_l}\left\{1-\Upsilon\left(D^2-a^2\right)\right\}\right]Z = \frac{\mu_e H d}{4\pi\rho_0\upsilon}DX \tag{25}$$

$$\left[\left(D^2-a^2\right)-E_1 p_1\sigma\right]\Theta = -\left(\frac{\beta d^2}{\kappa}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W \tag{26}$$

$$\left[p_2\sigma-\left(D^2-a^2\right)\right]\in K = \left(\frac{Hd}{\eta}\right)DW \tag{27}$$

$$\left[p_2\sigma-\left(D^2-a^2\right)\right]\in X = \left(\frac{Hd}{\eta}\right)DZ \tag{28}$$

Eliminating Θ *and* K from Eqs.(24), (26) and (27), we obtain:

$$\left[\frac{\sigma}{\in}\left(1+\frac{f}{1+\tau_1\sigma}\right)+\frac{1}{P_l}\left\{1-\Upsilon\left(D^2-a^2\right)\right\}\right] \left[\left(D^2-a^2\right)-E_1p_1\sigma\right]\left[p_2\sigma\mathbf{D}\left(D^2-a^2\right)\right]\left(D^2-a^2\right)\in W(z) - Q\left[\left(D^2-a^2\right)-E_1p_1\sigma\right]\left(D^2-a^2\right)D^2W(z) = Ra^2 f(z)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\left[p_2\sigma-\left(D^2-a^2\right)\right]\in W(z) \tag{29}$$

Where $R = \dfrac{g_0\alpha\beta d^4}{\upsilon\kappa}$, is the thermal Rayleigh number and $Q = \dfrac{\mu_e H^2 d^2}{4\pi\rho_0\upsilon\eta}$, is the Chandrasekhar number.

From Eqs. (25) and (28), we obtain:

$$\left[\frac{\sigma}{\in}\left(1+\frac{f}{1+\tau_1\sigma}\right)+\frac{1}{P_l}\left\{1-\Upsilon\left(D^2-a^2\right)\right\}\right] \left[p_2\sigma-\left(D^2-a^2\right)\in - QD^2\right](Z,X) = 0 \tag{30}$$

It is apparent from Eq. (30) that for the problem under consideration

$$X = 0 \ \mathit{and}\ Z = 0 \tag{31}$$

i.e. the z- components of vorticity and current density vanish identically for the problem on hand.

The boundary conditions appropriate to two free and perfectly conducting boundaries are defined as:

W = D^2W = DZ = DK = h_z= $\Theta = 0$
on z = 0 and d. (32)

Equation (29) together with the boundary conditions (31) and (32) constitute an eigen-value problem for the present problem. It is also evident that when $\Upsilon = 0$, the system reduces to that for an ordinary viscous fluid.

4. SOLUTION OF THE EIGEN-VALUE PROBLEM

The boundary conditions (31) and (32) suggest that a proper solution for W belonging to the lowest mode is defined by

$$W = W_0 \sin \pi z \tag{33}$$

Where W_0 is a constant.

Substituting solution Eq. (33) in Eq. (29) and letting $R_1 = \frac{R}{\pi^4}$, $x = \frac{a^2}{\pi^2}$, $i\sigma_1 = \frac{\sigma}{\pi^2}$, $P = \pi^2 P_l$, $Q_1 = \frac{Q}{\pi^2}$, $\Upsilon = \frac{\Upsilon_1}{\pi^2}$, we get an eigen-value expression of the form:

$$\left[\frac{i\sigma_1}{\in}\left(1+\frac{f}{1+i\sigma_1\pi^2\tau_1}\right)+\frac{1}{P}\left\{1+\Upsilon_1(1+x)\right\}\right]\left[(1+x)+i\sigma_1 E_1 p_1\right]$$
$$\left[i\sigma_1 p_2+(1+x)\right](1+x)\in +Q_1(1+x)\left[(1+x)+i\sigma_1 E_1 p_1\right] =$$
$$R_1 x f(z)\left(\frac{B+i\sigma_1\pi^2\tau_1}{1+i\sigma_1\pi^2\tau_1}\right)\left[i\sigma_1 p_2+(1+x)\right]\in \tag{34}$$

Equation (34) is required dispersion relation accounting the effects of magnetic field, suspended particles, medium permeability and medium porosity on thermal instability of a couple-stress fluid through a varying gravity field in a porous medium.

5. THE STATIONARY CONVECTION

When instability sets in as stationary convection, the marginal state will be characterized by putting $\sigma = 0$ in Eq. (34) and we get an eigen-value relationship for a stationary instability of the form:

$$R_1 = \frac{1}{xBf(z)}\left[\left\{\frac{(1+x)^2}{P}+\frac{\Upsilon_1(1+x)^3}{P}\right\}+\frac{Q_1}{\in}(1+x)\right] \tag{35}$$

Minimizing Eq. (35) with respect to x, yields the third order equation in x as

$$2\Upsilon_1 \in x^3 + (3\Upsilon_1+1)\in x^2 - \left\{(\Upsilon_1+1)\in + Q_1 P\right\} = 0 \tag{36}$$

In order to investigate the effects of suspended particles, magnetic field, medium permeability, couple-stress parameter and medium porosity, we examine the behaviour of $\frac{dR_1}{dB}, \frac{dR_1}{dQ_1}, \frac{dR_1}{dP}, \frac{dR_1}{d\Upsilon_1}$ *and* $\frac{dR_1}{d\in}$ analytically.

Equation (35) yields

$$\frac{dR_1}{dB} = -\frac{1}{xf(z)B^2}\left[\left\{\frac{(1+x)^2}{P}+\frac{\Upsilon_1(1+x)^3}{P}\right\}+\frac{Q_1}{\in}(1+x)\right] \tag{37}$$

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{\in f(z)xB} \tag{38}$$

$$\frac{dR_1}{dP} = -\frac{1}{Bxf(z)}\left[\frac{(1+x)^2}{P^2}+\frac{\Upsilon(1+x)^3}{P^2}\right] \tag{39}$$

$$\frac{dR_1}{d\Upsilon_1} = \frac{(1+x)^3}{BxPf(z)} \tag{40}$$

$$\frac{dR_1}{d\in} = -\frac{1}{xBf(z)\in^2}\left[Q_1(1+x)\right] \tag{41}$$

Eqs. (37) - (41) show that, for the case of stationary convection, magnetic field and couple-stress parameter have stabilizing effects, whereas suspended particle, medium permeability and medium porosity have destabilizing effects for $\left(f(z)>0\right)$.

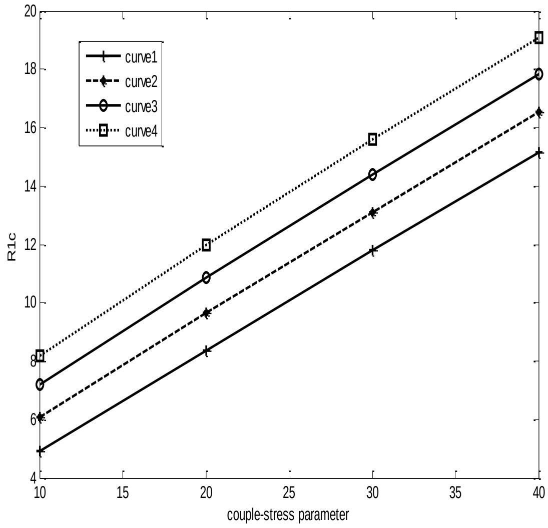

Fig. 1. Variation of critical thermal Rayleigh number R_{1c} with couple-stress parameter Υ_1 for fixed values of $\in = 0.1, B = 2, f(z) = 10, P = 1$ and $Q_1 = 1$ for curve 1, $Q_1 = 2$ for curve 2, $Q_1 = 3$ for curve 3, $Q_1 = 4$ for curve 4.

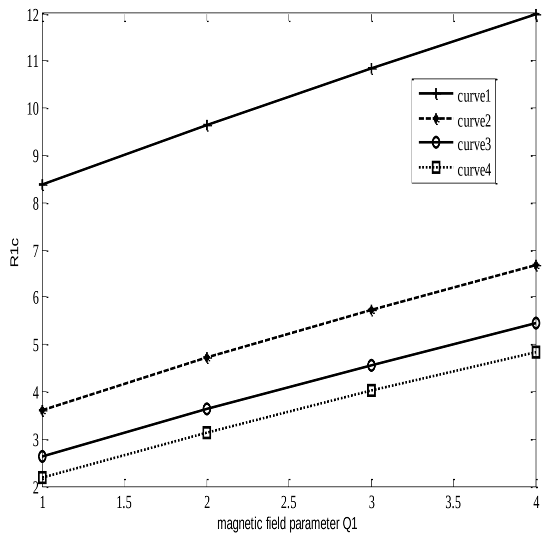

Fig. 2. Variation of critical thermal Rayleigh number R_{1c} with magnetic field parameter Q_1 for fixed values of $\in = 0.1, B = 2, f(z) = 10, \Upsilon_1 = 20$ and $P = 1$ for curve 1, $P = 3$ for curve 2, $P = 5$ for curve 3, $P = 7$ for curve 4.

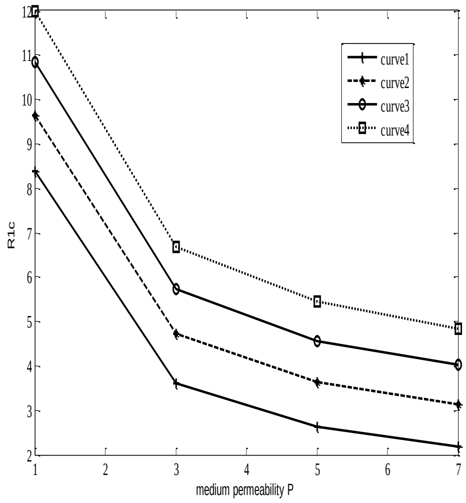

Fig.3. Variation of critical thermal Rayleigh number R_{1c} with medium permeability P for fixed values of $\in = 0.1,\ B = 2,\ f(z) = 10,\ \Upsilon_1 = 20$ and $Q_1 = 1$ for curve 1, $Q_1 = 2$ for curve 2, $Q_1 = 3$ for curve 3, $Q_1 = 4$ for curve 4.

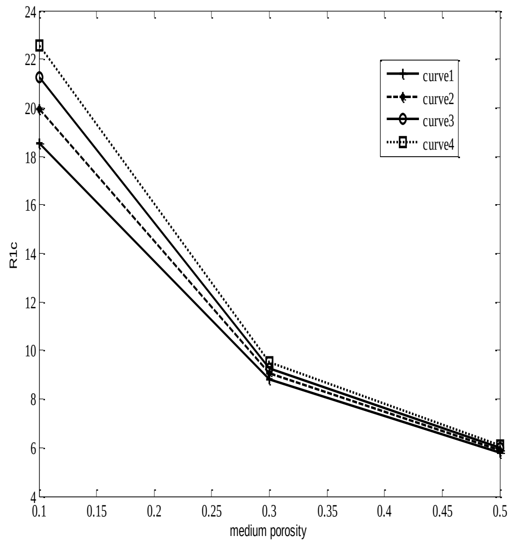

Fig.4. Variation of critical thermal Rayleigh number R_{1c} with medium porosity $\in$ for fixed values of $P = 1,\ \Upsilon_1 = 50,\ f(z) = 10$; $Q_1 = 1$ for curve 1, $Q_1 = 2$ for curve 2, $Q_1 = 3$ for curve 3, $Q_1 = 4$ for curve 4 and $B = 2, 4\, and\ 6$ for all curves.

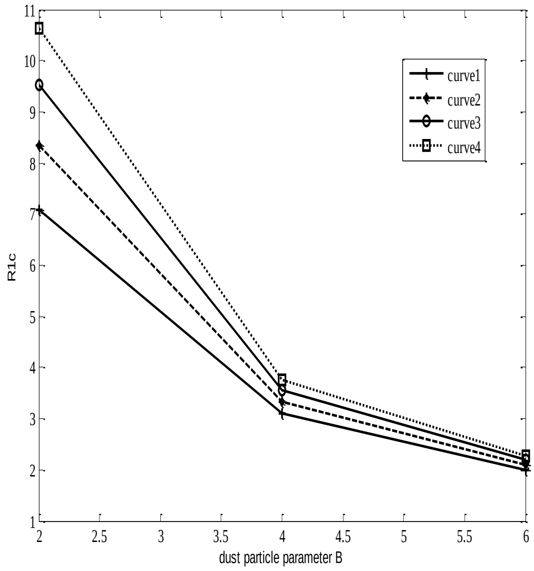

Fig.5. Variation of critical thermal Rayleigh number R_{1c} with dust particle parameter B for fixed values of $P = 3,\ \Upsilon_1 = 50,\ f(z) = 10$; $Q_1 = 1$ for curve 1, $Q_1 = 2$ for curve 2, $Q_1 = 3$ for curve 3, $Q_1 = 4$ for curve 4 and $\in = 0.1, 0.3\, and\ 0.5$ for all curves.

Equation (36) will give the critical wave numbers x_c using Newton-Raphson method by assigning various values to physical parameters and then the critical thermal Rayleigh number R_{1c} for stationary stability/instability can be deduced from Eq. (35).

The variation in numerical values of critical thermal Rayleigh number R_{1c} against various values of physical parameters and wave numbers are given in Tables 1-2 and also shown graphically by the Figs. 1-5.

Table 1 Critical thermal Rayleigh numbers and wave numbers of the unstable modes at marginal stability for the onset of stationary convection for various values of couple-stress parameter, magnetic field and permeability

		$\in = 0.1,\ B = 2,\ f(z) = 10$							
		$P = 1$		$P = 3$		$P = 5$		$P = 7$	
Υ_1	Q_1	x_c	R_{1c}	x_c	R_{1c}	x_c	R_{1c}	x_c	R_{1c}
10	1	0.6854	4.9292	0.9124	2.3924	1.0774	1.8363	1.2111	1.5792
	2	0.8104	6.0971	1.1472	3.3769	1.3770	2.7426	1.5585	2.4392
	3	0.9124	7.1773	1.3253	4.2810	1.5989	3.5782	1.8127	3.2355
	4	1.0000	8.2000	1.4722	5.1382	1.7799	4.3740	2.0190	3.9963
20	1	0.6034	8.3732	0.7489	3.6167	0.8623	2.6181	0.9573	2.1699
	2	0.6816	9.6511	0.9116	4.7180	1.0780	3.6324	1.2126	3.1296
	3	0.7489	10.8500	1.0401	5.7300	1.2428	4.5629	1.4041	4.0114
	4	0.8084	11.9920	1.1483	6.6868	1.3793	5.4441	1.5614	4.8484
30	1	0.5720	11.7773	0.6803	4.7910	0.7686	3.3505	0.8444	2.7132
	2	0.6294	13.1095	0.8078	5.9623	0.9423	4.4341	1.0531	3.7391
	3	0.6803	14.3729	0.9114	7.0437	1.0782	5.4286	1.2130	4.6800
	4	0.7263	15.5837	1.0000	8.0667	1.1922	6.3689	1.3457	5.5707
40	1	0.5552	15.1684	0.6418	5.9460	0.7147	4.0624	0.7783	3.2359
	2	0.6007	16.5333	0.7474	7.1652	0.8616	5.1960	0.9571	4.3112
	3	0.6418	17.8380	0.8352	8.2968	0.9789	6.2389	1.0967	5.2977
	4	0.6796	19.0947	0.9112	9.3693	1.0783	7.2248	1.2133	6.2304

Table 2 Critical thermal Rayleigh numbers and wave numbers of the unstable modes at marginal stability for the onset of stationary convection for various values of porosity and dust particle parameter

			$\Upsilon_1 = 50,\ f(z) = 10$							
			$P = 1$		$P = 3$		$P = 5$		$P = 7$	
$\in$	B	Q_1	x_c	R_{1c}	x_c	R_{1c}	x_c	R_{1c}	x_c	R_{1c}
0.1	2	1	0.5448	18.5536	0.6171	7.0914	0.6792	4.7633	0.7343	3.7473
		2	0.5825	19.9406	0.7075	8.3458	0.8072	5.9353	0.8918	4.8614
		3	0.6171	21.2741	0.7840	9.5161	0.9111	7.0169	1.0164	5.8851
		4	0.6492	22.5637	0.8510	10.6275	1.0000	8.0400	1.1215	6.8527
0.3	4	1	0.5176	8.7965	0.5448	3.0923	0.5703	1.9485	0.5943	1.4565
		2	0.5315	9.0386	0.5825	3.3234	0.6280	2.1708	0.6694	1.6716
		3	0.5448	9.2768	0.6171	3.5457	0.6792	2.3816	0.7343	1.8736
		4	0.5578	9.5113	0.6492	3.7606	0.7255	2.5836	0.7918	2.0662
0.5	6	1	0.5120	5.7989	0.5288	1.9979	0.5448	1.2369	0.5603	0.9103
		2	0.5205	5.8968	0.5527	2.0929	0.5825	1.3294	0.6104	1.0005
		3	0.5288	5.9937	0.5752	2.1853	0.6171	1.4183	0.6554	1.0865
		4	0.5369	6.0896	0.5967	2.2755	0.6492	1.5042	0.6964	1.1692
0.7	8	1	0.5095	4.3281	0.5216	1.4777	0.5334	0.9073	0.5448	0.6626
		2	0.5156	4.3808	0.5392	1.5292	0.5614	0.9578	0.5825	0.7122
		3	0.5216	4.4330	0.5560	1.5797	0.5876	1.0067	0.6171	0.7598
		4	0.5276	4.4849	0.5721	1.6292	0.6123	1.0543	0.6492	0.8058

6. PRINCIPLE OF EXCHANGE OF STABILITIES

THEOREM: If $R \succ 0, B \succ 0, \mu_e \succ 0, \in \succ 0, P_l \succ 0, \Upsilon \succ 0$ *and* $\sigma = 0$ then the necessary condition for the onset of convection at the marginal state for the existence of non-trivial solution of Eqs. (24), (26) and (27) together with the boundary conditions (31) and (32) is that the inequality

$$R \succ \frac{\left[\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}+\left(3\pi^2\Upsilon-1\right)\right]^2\left[\Upsilon\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}+3\Upsilon\left(1+\pi^2\Upsilon\right)\right]}{16P_l\Upsilon^2\left[\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}-\left(\pi^2\Upsilon+1\right)\right]}$$

must be satisfied and in the absence of couple-stress parameter (i.e. $\Upsilon = 0$), the necessary condition for the onset of stationary convection is that $R \succ \frac{4\pi^2}{P_l}$.

Proof: When the principle of exchange of stabilities is satisfied then the marginal state will be characterized by $\sigma = 0$, for this, we multiply Eq. (24) by W^* (the complex conjugate of W), integrating over the range of z and making use of Eqs. (26) and (27) with the help of boundary conditions (31) and (32), we obtain

$$\frac{1}{P_l}I_1 + \frac{\Upsilon}{P_l}I_2 - \frac{\mathbf{g}\alpha a^2\kappa}{B\beta\upsilon}I_3 + \frac{\mu_e \in}{4\pi\rho_0 p_2}I_4 = 0 \tag{42}$$

Where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,$$

$$I_2=\int_0^1\left(|D^2W|^2+a^4|W|^2+2a^2|DW|^2\right)dz,$$

$$I_3=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,$$

$$I_4=\int_0^1\left(|D^2K|^2+a^4|K|^2+2a^2|DK|^2\right)dz$$

Where $W\,and\,\Theta$ satisfies the boundary condition (32).

Now, multiplying Eq. (26) by Θ^* (the complex conjugate of Θ), with $\sigma=0$ and integrating over the range of z using the boundary condition (32), we obtain

$$\int_0^1\Theta^*\left(D^2-a^2\right)\Theta\,dz=-\left(\frac{B\beta d^2}{\kappa}\right)\int_0^1\Theta^*W\,dz$$

$$\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz=\left(\frac{B\beta d^2}{\kappa}\right)\int_0^1\Theta^*W\,dz\le\left(\frac{B\beta d^2}{\kappa}\right)\left|\int_0^1\Theta^*W\,dz\right|$$

$$\le\left(\frac{B\beta d^2}{\kappa}\right)\int_0^1|\Theta^*||W|\,dz=\left(\frac{B\beta d^2}{\kappa}\right)\int_0^1|\Theta||W|\,dz$$

$$\le\left(\frac{B\beta d^2}{\kappa}\right)\left[\left\{\int_0^1|\Theta|^2\,dz\right\}^{\frac{1}{2}}\left\{\int_0^1|W|^2\,dz\right\}^{\frac{1}{2}}\right] \tag{43}$$

Where Cauchy-Schwartz inequality has been used in the right hand side of the above expression.

Now, using the Rayleigh-Ritz inequality (1973),

$$\int_0^1|DW|^2\,dz\ge\pi^2\int_0^1|W|^2\,dz\;and\;\int_0^1|D\Theta|^2\,dz\ge\pi^2\int_0^1|\Theta|^2\,dz \tag{44}$$

in the left hand side of inequality (43), we obtain

$$\left(\pi^2+a^2\right)\left\{\int_0^1|\Theta|^2\,dz\right\}^{\frac{1}{2}}\le\left(\frac{B\beta d^2}{\kappa}\right)\left\{\int_0^1|W|^2\,dz\right\}^{\frac{1}{2}} \tag{45}$$

So the combination of inequalities (43) and (45) yields

$$\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz\le\left(\frac{B\beta d^2}{\kappa}\right)^2\frac{1}{\left(\pi^2+a^2\right)}\left|\int_0^1|W|^2\,dz\right| \tag{46}$$

Now, utilizing the inequalities (44), (46) and Banerjee et al. (1992) inequality $\int_0^1|D^2W|^2\,dz\ge\pi^4\int_0^1|W|^2\,dz,\;\int_0^1|D^2K|^2\,dz\ge\pi^4\int_0^1|K|^2\,dz$ in Eq. (42), we obtain

$$\frac{\mu_e\in}{4\pi\rho_0 p_2}\left(\pi^2+a^2\right)^2 I_5+\left[\begin{array}{c}\dfrac{\left(\pi^2+a^2\right)}{P_l}\\ \left\{1+\Upsilon\left(\pi^2+a^2\right)\right\}\\ -\dfrac{Ra^2B}{\left(\pi^2+a^2\right)}\end{array}\right]\int_0^1|W|^2\,dz\prec 0 \tag{47}$$

Where in the above inequality (47), $I_5=\int_0^1|K|^2\,dz$ is positive definite.

Therefore, when $R\succ 0,\,B\succ 0,\,\mu_e\succ 0,\in\succ 0,P_l\succ 0,and\,\Upsilon\succ 0$ inequality (47) yields that

$$R\succ\left[\frac{\left(\pi^2+a^2\right)^2}{P_l a^2 B}\left\{1+\Upsilon\left(\pi^2+a^2\right)\right\}\right] \tag{48}$$

Since the minimum value of $\left[\frac{\left(\pi^2+a^2\right)^2}{P_l a^2 B}\left\{1+\Upsilon\left(\pi^2+a^2\right)\right\}\right]$ is

$$\frac{\left[\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}+\left(3\pi^2\Upsilon-1\right)\right]^2\left[\Upsilon\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}+3\Upsilon\left(1+\pi^2\Upsilon\right)\right]}{16P_l\Upsilon^2\left[\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}-\left(\pi^2\Upsilon+1\right)\right]} \tag{49}$$

for $a^2=\dfrac{-\left(1+\pi^2\Upsilon\right)+\left[\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}\right]}{4\Upsilon}$ (50)

So the inequality

$$R\succ\frac{\left[\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}+\left(3\pi^2\Upsilon-1\right)\right]^2\left[\Upsilon\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}+3\Upsilon\left(1+\pi^2\Upsilon\right)\right]}{16P_l\Upsilon^2\left[\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}-\left(\pi^2\Upsilon+1\right)\right]}$$

gives the necessary condition for the onset of instability when the principle of exchange of stabilities is satisfied. Further, in the absence of couple-stress parameter, inequality (48) gives

$$R\succ\frac{4\pi^2}{P_l} \tag{51}$$

For

$$a^2=\pi^2 \tag{52}$$

It is evident from Eqs. (51) and (52) that when $\Upsilon=0$ *and* $P_l=1$ then the results agree well with those for Newtonian fluids at the onset of convection through a porous medium and this validates the proof of the theorem, when both the boundaries are dynamically free.

7. CONCLUSION

The effect of a vertical magnetic field and suspended particles on the onset of thermal convection in a couple-stress fluid in the presence of variable gravity field through a porous medium is taken into account in the present paper treated here. The following results are drawn while investigating the problem:

(a). For the case of stationary convection:

- The suspended particles, medium porosity and medium permeability are found to hasten the onset of thermal instability when the gravity field increases upward from its value g_0 i.e. $\left(f(z)>0\right)$.
- The effects of magnetic field and couple-stress parameter is to stabilize the system, as such their effect is to postpone the onset of thermal instability when the gravity field increases upward from its value g_0 i.e. $\left(f(z)>0\right)$.

(b). At the marginal state (i.e. when the principle of exchange of stabilities is satisfied), the necessary condition for the onset of instability is that the inequality

$$R \succ \frac{\left[\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}+\left(3\pi^2\Upsilon-1\right)\right]^2 \left[\Upsilon\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}+3\Upsilon\left(1+\pi^2\Upsilon\right)\right]}{16P_l\Upsilon^2\left[\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}-\left(\pi^2\Upsilon+1\right)\right]}$$

must be satisfied. Thus the sufficient condition for the non-existence of stationary convection is that

$$R \leq \frac{\left[\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}+\left(3\pi^2\Upsilon-1\right)\right]^2 \left[\Upsilon\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}+3\Upsilon\left(1+\pi^2\Upsilon\right)\right]}{16P_l\Upsilon^2\left[\sqrt{\left(1+\pi^2\Upsilon\right)\left(1+9\pi^2\Upsilon\right)}-\left(\pi^2\Upsilon+1\right)\right]},$$

for the problem under consideration.

(c). In the absence of couple-stress parameter (i.e. $\Upsilon = 0$), the necessary condition for the onset of instability is that the inequality $R \succ \frac{4\pi^2}{P_l}$ is satisfied and thus the sufficient condition for the non-existence of stability is that $R \leq \frac{4\pi^2}{P_l}$.

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