Journal of Applied Fluid Mechanics, Vol. 8, No. 3, pp. 473-481, 2015.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.67.222.22873

Numerical Study on MHD Mixed Convection Flow in a Vertical Insulated Square Duct with Strong Transverse Magnetic Field

M. Chutia[1†] and P. N. Deka[2]

[1]*Department of Mathematics, Mariani College, Mariani, Assam, 785634, India*
[2]*Department of Mathematics, Dibrugarh University, Dibrugarh, Assam, 786004, India*

†*Corresponding Author Email: muhimchutia@gmail.com*

(Received April 04, 2014; accepted June 25, 2014)

ABSTRACT

A numerical study on steady laminar magnetohydrodynamics (MHD) mixed convection flow of an electrically conducting fluid in a vertical square duct under the action of transverse magnetic field has been investigated. The walls of the duct are electrically non-conducting. In this study both forced and free convection flows are considered. The viscous dissipation and Joule heat are also considered in the energy equation and moreover the walls of the duct are kept at constant temperature. The governing equations of momentum, induction and energy are first transformed into dimensionless equations by using dimensionless quantities, then these are solved employing finite difference method for velocity, induced magnetic field and temperature distribution. The computed results for velocity, induced magnetic field and temperature distribution are presented graphically for different dimensionless parameters Hartmann number **M**, Prandtl number **Pr**, Grashof number **Gr** and magnetic Reynolds number **Rm**.

Keywords: Buoyancy force; Heat transfer; Insulated walls; Magnetohydrodynamics; Mixed convection, Square duct.

NOMENCLATURE

a	length of cross-section of the square duct
$\vec{B}$	magnetic field
B_0	applied magnetic field
B	magnetic field in the z-direction
B^*	dimensionless magnetic field
c_p	specific heat at constant pressure
$\vec{E}$	electric field
Gr	Grashof number
g	acceleration due to gravity
k	thermal conductivity
M	Hartmann number
$\vec{J}$	current density
Pr	Prandtl number
p	pressure force
Re	Reynolds number
Rm	magnetic Reynolds number
T	fluid temperature
T_0	wall temperature
$\vec{V}$	velocity
W	velocity in the z-direction
W^*	dimensionless velocity
x, y, z	cartesian coordinates
β	coefficient of thermal expansion
ρ	density of liquid
σ	electrical conductivity
μ	coefficient of viscosity
μ_e	magnetic permeability
υ	kinematic viscosity
λ	magnetic diffusivity
Subscripts	
m, n	grid number in computational domain
i	index refers to x
j	index refers to y
$C_1 etc.$	constants

1. INTRODUCTION

The free or natural convection occurs when fluid motion is generated predominantly by the body forces which caused entirely by the buoyancy forces. This natural buoyancy is the force experienced by fluid while flowing under gravitational field and is associated with the density changes that result from temperature variation in the flow. While the forced convection flow is the phenomenon in which the velocity arising from variable density (i.e. due to force of buoyancy) are negligible in comparison with the velocity of the forced flow. Mixed convection flow is the combination of forced and natural convection flows.

Mixed convection flows in a channel or duct are encountered in the Dual-Coolant Lead-Lithium (DDCL) flow for power conversion, tritium breeding and many industrial applications and engineering devices such as cooling system for electronic components and reactors. In such devices liquid-metal flow are strongly affected by a magnetic field and by a volumetric heating caused by the neutrons with the electrically conducting fluid (Smolentsev *et al*., 2008). Analysis on MHD flows associated with very high values of Hartmann number is important in designing of liquid metal tritium breeder coolant blanket for nuclear fusion reactor. Furthermore, because the flow characteristics of coolant with the temperature difference between wall boundary and the coolant are assumed to be different from usual turbulent MHD flow, it becomes necessary to investigate buoyancy flow at high values of Grashof number under a magnetic field for the design of an actual working system.

Sposito and Ciafalo (2006) studied fully developed flow of an electrically conducting fluid between parallel walls under the simultaneous influence of a driving pressure head, buoyancy and MHD forces, where the fluid was assumed to be internally heated and the flow was modeled as one-dimensional and incompressible. Al-Khawaja *et al*. (1994) solved numerically the problem of fully developed, laminar, steady, forced convection heat transfer in an electrically conducting fluid flowing in an electrically insulated, horizontal circular pipe in a vertical uniform transverse magnetic. Al-Khawaja *et al*. (1999) also studied the same problem for free-and-forced convection flow numerically using finite difference schemes for Grashof numbers 0 to 106 and Hartmaan numbers 0 to 500. Umavathi and Malashetty (2005) solved the problem of combined free and forced convective MHD flow in a vertical channel by taking into account the effect of viscous and ohmic dissipations, analytically by perturbation series method and numerically by finite difference technique. Umavathi and Chamkha (2011) analyzed the effect of heat and mass transfer on mixed convective flow of a viscous incompressible fluid past a vertical infinite plate in the presence of heat source or sink. Garandet and Alboussiere (1992) proposed analytical solutions to the equations of MHD that was used to model the effect of a transverse magnetic field on buoyancy driven convection in a two-dimensional cavity, in the case of high Hartmaan number limit. Blosseville *et al*. (2007) investigated analytically a fully developed buoyant flow in a straight, horizontal rectangular duct with an axial temperature gradient in an arbitrary oriented, transverse magnetic field with insulated walls. Aruna *et al*. (2011) studied the developed MHD mixed convection flow in a vertical channel, where the problem was described by means of partial differential equations and the solutions were obtained by an implicit finite difference technique coupled with a marching procedure. Taghikhani and Najafkhani (2013) investigated the laminar steady MHD natural convection in a filled square enclosure with internal heat generation by two-dimensional numerical simulation. Reddy *et al*. (2012) studied numerically the fully developed MHD laminar mixed convection in a vertical channel with isothermal wall temperature. Chamkha (2002) studied the problem of hydromagnetic fully developed laminar mixed convection flow in a vertical channel with symmetric and asymmetric wall heating conditions in the presence of heat generation or absorption effect. Tomarchio (2009) studied the 2D problem of the mixed convective flow in a vertical rectangular duct in the presence of transverse magnetic field with the steady periodic regime induced by the oscillating wall temperature of the four walls, numerically by finite element approach. Recently, Chutia and Deka (2014) studied numerically steady MHD flow and heat transfer in a rectangular electrically insulated duct in the presence of strong oblique magnetic field. They considered both viscous and Joule dissipation in the energy equation.

The aim of the present paper is to investigate numerically on laminar steady MHD mixed convection flow of an electrically conducting fluid in a vertical square duct in the presence of strong transverse magnetic field. The walls of duct are assumed as insulating and walls have constant temperature. The local balance equations of momentum, magnetic induction and energy are solved numerically by finite difference method using the Hartmann number M, Grashof number Gr, Prandtl number Pr and magnetic Reynolds number Rm as the parameters. The computed numerical results are presented graphically for velocity, induced field and temperature for different flow parameters.

2. MATHEMATICAL FORMULATION

The steady laminar flow of a Newtonian fluid in a vertical square duct with uniform horizontal magnetic field of constant intensity B_0 applied transverse to the duct walls is considered. The length of the cross section of the duct is a. It is assumed that the fluid occupies an area between $x = 0$, $x = a$ and $y = 0$, $y = a$. The flow is to be driven by a constant pressure gradient $\partial p/\partial z$ and is constrained to move in the z- direction. A uniform temperature T_0 at walls of the duct is assumed. In accordance with the hypothesis of fully developed flow, the velocity and temperature field are parallel and the only non-vanishing components of velocity and temperature $W(x, y)$ and $T(x, y)$ are parallel to the duct axis and independent of the vertical coordinate. The uniform horizontal applied magnetic field of intensity B_0 acts along y-direction and it induces a magnetic field $B(x, y)$ in the flow direction.

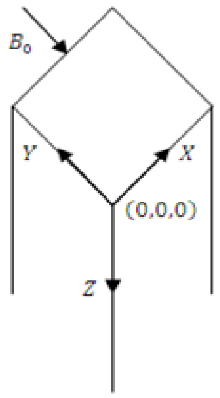


Fig. 1. Geometrical sketch of the problem

In this study an electrically conducting fluid flows along the axis of the duct under the influence of an externally imposed driving pressure gradient and the Lorentz forces caused by the interaction of the flow with the uniform magnetic induction field, directed along y. Buoyancy forces, caused by density changes that results from temperature variation, can easily be included thus giving rise to mixed MHD convection.

Following assumptions are made in this study:

(i) The flow is steady, fully developed and fluid is Newtonian.

(ii) The fluid is finitely conducting.

(iii) The duct is considered to be infinite so that all the fluid properties except pressure gradient are independent of the variable z.

(iv) Displacement currents are negligible and there is no net of current in the z-direction.

Under above assumptions, the velocity $\vec{V}$, magnetic field $\vec{B}$ and temperature T will be of the form

$$\vec{V} = \{0, 0, W(x, y)\}$$

$$\vec{B} = \{0, B_0, B(x, y)\}$$

$$T = T(x, y)$$

3. GOVERNING EQUATIONS

The complete set of governing equations under above assumptions is obtained assembling together the local balance equations for momentum using the Boussinesq approximation for buoyancy, induction and energy as follows:

$$\mu\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right) + \frac{B_0}{\mu_e}\frac{\partial B}{\partial y} + \rho g\beta(T - T_0) - \frac{\partial p}{\partial z} = 0 \quad (1)$$

$$\lambda\left(\frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2}\right) + B_0\frac{\partial W}{\partial y} = 0 \quad (2)$$

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\left\{\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right\} + \frac{1}{\sigma\mu_e^2}\left\{\left(\frac{\partial B}{\partial x}\right)^2 + \left(\frac{\partial B}{\partial y}\right)^2\right\} = 0 \quad (3)$$

Where μ is the coefficient of viscosity, ρ is the density of fluid, g is the acceleration due to gravity, β is the coefficient of thermal expansion, λ is the magnetic diffusivity, k is the thermal conductivity of the fluid, σ is the electrical conductivity and μ_e is the magnetic permeability.

The boundary conditions of the problem (i) no-slip boundary condition at the wall, (ii) the condition of electrically insulating wall and (iii) constant wall temperature at the boundary yield:

At the Hartmann walls

$$\left.\begin{array}{l} W = 0, B = 0, T = T_0 \ \ at \ \ y = 0 \\ W = 0, B = 0, T = T_0 \ \ at \ y = a \end{array}\right\} \quad 4(a)$$

At the side walls

$$\left.\begin{array}{l} W = 0, B = 0, T = T_0 \ \ at \ \ x = 0 \\ W = 0, B = 0, T = T_0 \ \ at \ x = a \end{array}\right\} \quad 4(b)$$

The system of governing Eqs. (1), (2) and (3) can be rewritten in terms of following dimensionless groups:

$$x^* = \frac{x}{a}, y^* = \frac{y}{a}, W^* = \frac{W}{W_0}, B^* = \frac{B}{B_0}, T^* = \frac{T - T_0}{\Delta T} \quad (5)$$

Where $W_0 = -\frac{a^2}{\rho\upsilon}\frac{\partial p}{\partial z}$

$$B_0 = -a^2\mu_e\left(\frac{\sigma}{\rho\upsilon}\right)^{1/2}\frac{\partial p}{\partial z}$$

$$\Delta T = \frac{W_0^2}{c_p}$$

Using dimensionless group (5), the system of governing Eqs. (1), (2) and (3) dropping asterisks can be rewritten as:

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} + M\frac{\partial B}{\partial y} + \frac{Gr}{Re}T + 1 = 0 \quad (6)$$

$$\frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2} + M\frac{\partial W}{\partial y} = 0 \quad (7)$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + Pr\left\{\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2\right\} + \frac{M^2 Pr}{Rm^2}\left\{\left(\frac{\partial B}{\partial x}\right)^2 + \left(\frac{\partial B}{\partial y}\right)^2\right\} = 0 \quad (8)$$

Where,

$M = B_0 a\left(\frac{\sigma}{\rho\upsilon}\right)^{1/2}$, is the Hartmann number

$Gr = \frac{g\beta a^3(T - T_0)}{\upsilon^2}$, is the Grashof number

$Re = \frac{W_0 a}{\upsilon}$, is the Reynolds number

$Pr = \frac{\mu c_p}{k}$, is the Prandtl number

$Rm = \sigma\mu_e a W_0$, is the magnetic Reynolds number

The corresponding boundary condition (4) become

At the Hartmann walls

$$\left.\begin{array}{l} W = 0, B = 0, T = 0 \ \ at \ \ y = 0 \\ W = 0, B = 0, T = 0 \ \ at \ y = 1 \end{array}\right\} \quad 9(a)$$

At the side walls

$$\left.\begin{array}{l} W = 0, B = 0, T = 0 \ \ at \ \ x = 0 \\ W = 0, B = 0, T = 0 \ \ at \ x = 1 \end{array}\right\} \quad 9(b)$$

The system of coupled non-linear governing Eqs. (6), (7) and (8) are to be solved with the boundary condition (9). In view of complexities it is not convenient to obtain a closed form solution particularly in the case of high Hartmann number flow. Here an attempt has been made to solve the governing equations numerically using the finite difference method. In this numerical technique, we have descretized the system of governing equations along with the prescribed boundary condition into a system of algebraic equations. Central difference scheme of second order accuracy (Jain *et al.*, 1994) is used to descretize these equations, since it is more accurate than forward and backward differences. The resultant difference equations of the system of Eqs. (6), (7) and (8) are as follows:

$$W_{i,j} = C_1\left(W_{i+1,j} + W_{i-1,j}\right) + C_2\left(W_{i,J+1} + W_{i,j-1}\right) + C_3\left(B_{i,J+1} - B_{i,j-1}\right) + C_4 T_{i,j} + C_5 \quad (10)$$

$$B_{i,j} = C_1\left(B_{i+1,j} + B_{i-1,j}\right) + C_2\left(B_{i,J+1} + B_{i,j-1}\right) + C_3\left(W_{i,J+1} - W_{i,j-1}\right) \quad (11)$$

$$T_{i,j} = C_1\left(T_{i+1,j} + T_{i-1,j}\right) + C_2\left(T_{i,J+1} + T_{i,j-1}\right) + C_6\left(W_{i+1,j} - W_{i-1,j}\right)^2 + C_7\left(W_{i,j+1} - W_{i,j-1}\right)^2 + C_8\left(B_{i+1,j} - B_{i-1,j}\right)^2 + C_9\left(B_{i,j+1} - B_{i,j-1}\right)^2 \quad (12)$$

Where,

$$C_1 = \frac{k^2}{2(h^2+k^2)},\ C_2 = \frac{h^2}{2(h^2+k^2)},\ C_3 = \frac{h^2 k}{4(h^2+k^2)}M\ ,$$

$$C_4 = \frac{h^2k^2}{2(h^2+k^2)}\frac{Gr}{Re},\ C_5 = \frac{h^2k^2}{2(h^2+k^2)}\ ,\ C_6 = \frac{k^2}{8(h^2+k^2)}Pr,$$

$$C_7 = \frac{h^2}{8(h^2+k^2)}Pr\ ,\ C_8 = \frac{k^2}{8(h^2+k^2)}\frac{M^2Pr}{Rm^2}\ ,$$

$$C_9 = \frac{h^2}{8(h^2+k^2)}\frac{M^2Pr}{Rm^2}$$

Are constants.

The corresponding descretized numerical boundary conditions are as follows

At the Hartmann walls

$$\left.\begin{array}{l} W_{i,1} = 0, \quad B_{i,1} = 0, \quad T_{i,1} = 0 \\ W_{i,n+1} = 0, B_{i,n+1} = 0, T_{i,n+1} = 0 \\ \quad for\ 1 \le i \le m+1 \end{array}\right\} \quad (13a)$$

At the side walls

$$\left.\begin{array}{l} W_{1,j} = 0, \quad B_{1,j} = 0, \quad T_{1,j} = 0 \\ W_{m+1,j} = 0, B_{1,m+1} = 0, T_{1,m+1} = 0 \\ \quad for\ 1 \le j \le n+1 \end{array}\right\} \quad (13b)$$

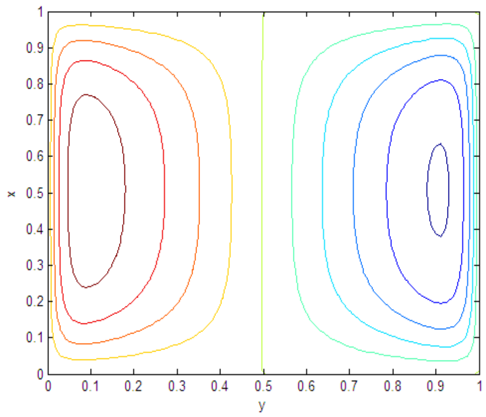
Where index i refers to x and j refers to y, m and n denote the number of grids inside the computational domain in the direction of X and Y respectively. The computed values of $W_{i,j}$, $B_{i,j}$ and $T_{i,j}$ appearing in the descretized Eqs. (10), (11) and (12) with dimensionless parameters M, Gr, Pr and Rm using the boundary conditions (13a, 13b) have been iterated. We repeat the process till the converged solutions for $W_{i,j}$, $B_{i,j}$ and $T_{i,j}$ in the grid points are obtained.

4. Results and Discussion

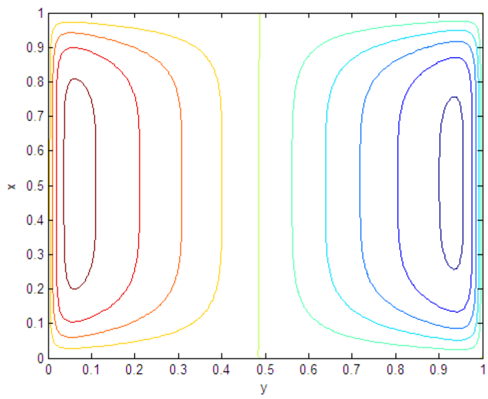
In this study we are considering numerical study on MHD mixed convection flow and heat transfer in a vertical square duct in the presence of horizontal uniform strong transverse magnetic field. The system of algebraic equations have been solved using Matlab programming (Mathews and Fink, 2009; Al-Khawaja and Selmi, 2010). Numerical computations are carried out in the spatial domain $0 \le x \le 1$ and $0 \le y \le 1$. In this numerical computation, we have considered $h = k = 0.01$ so that the system is convergent and stable. Prandtl number Pr gives no effect to the velocity and induced magnetic field, as can be seen from Eqs. (7) and (8). The computed results for velocity, induced magnetic field and temperature are shown and drawn in terms of graphics for different flow parameters. We have presented our computed results for the following two cases:

4.1 Forced Convection with MHD Effects

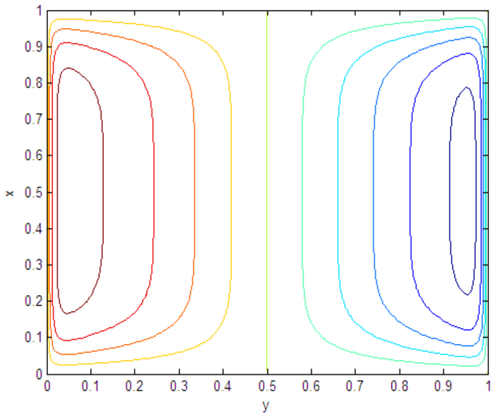
Computed results obtained in the case of forced convection, $(\mathrm{Gr} = 0)$ with a uniform external transverse magnetic field are reported. In this case buoyancy force has no aiding effect on the mixed convection, but the Lorentz force has an opposing effect. The results are obtained for different values of Hartmann number M, Prandtl number Pr and magnetic Reynolds number Rm and presented in Figs. 2 to 6.



(a)



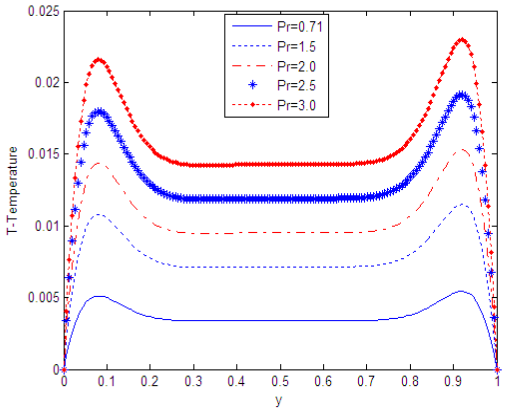
(b)



(c)

Fig. 2. (a) Current density $[J_x, J_y]$ for $M = 30$; (b) Current density $[J_x, J_y]$ for $M = 50$; (c) Current density $[J_x, J_y]$ for $M = 80$

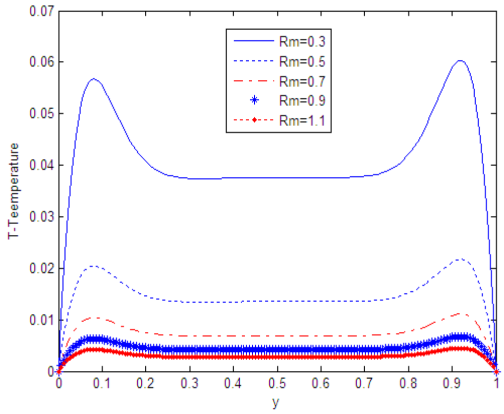
In Figs. 2(a), 2(b) and 2(c) streamline plots of the current density vector $\vec{J}$ across the duct cross section are presented in xy-plane for $M = 30$, $M = 50$ and $M = 80$ respectively for forced convection with transverse magnetic field. It is evident, as presented in figures, that current lines can not enter the insulated walls and close completely inside the duct cross section. Moreover, it can be noted that for increasing values of the Hartmann number, the closure of the current lines are segregated to a narrow region near the Hartmann walls, where consequently the effect of the Lorentz force is more intense and large velocity gradient appear.



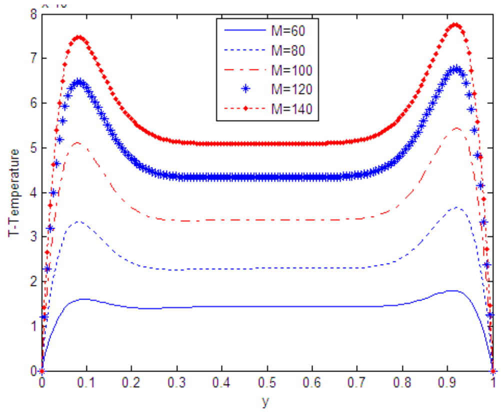
(a)

In Fig. 3 we have presented the variation of temperature distribution for fixed $Re = 1$, for various Pr, Rm and M for forced convection with transverse magnetic field. It is evident from Fig. 3(a) that temperature increases as Pr increases. Figure 3(b) represents that temperature decreases with the increasing values of Rm. It is observed in Fig. 3(c) that temperature distribution increases

for increasing values of Hartmann number M, this is due to fact that internal heating caused by Joule heat increases as M increases.



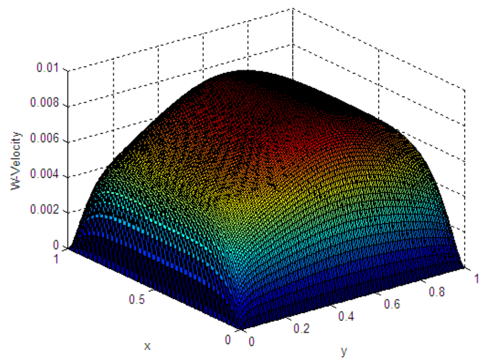
(b)



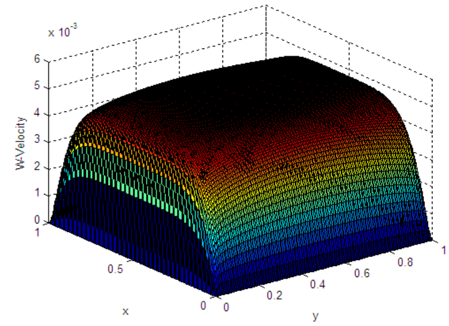
(c)

Fig. 3. (a) Temperature at various Pr; (b) Temperature at various Rm; (c) Temperature at various M

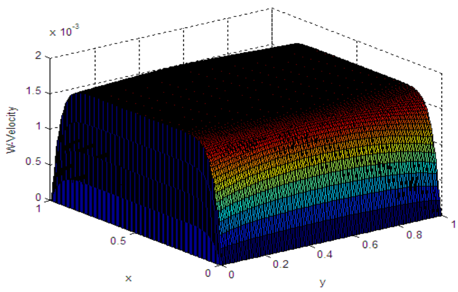
The velocity distributions W along the duct axis are presented in Figs. 4(a), 4(b) and 4(c) for $M = 50, 100$ *and* 300 respectively for forced convection with transverse magnetic field. It is noticed that the velocity gradient is greater at the walls normal to the direction of the magnetic field. This effect is more relevant for increasing values of the Hartmann number M.



(a)

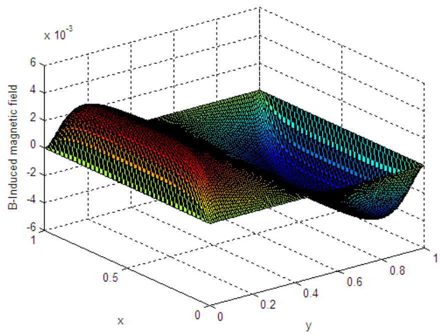


(b)

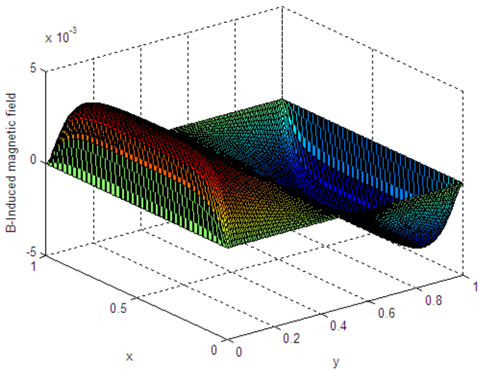


(c)

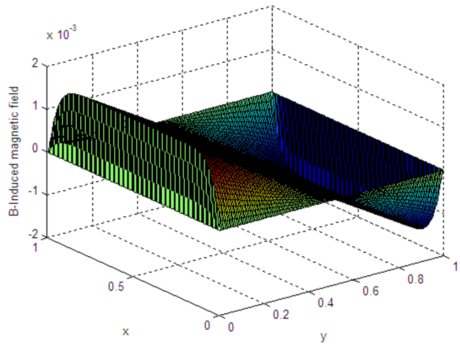
Fig. 4. (a) Velocity for $M = 50$; (b) Velocity for $M = 100$; (c) Velocity for $M = 300$



(a)



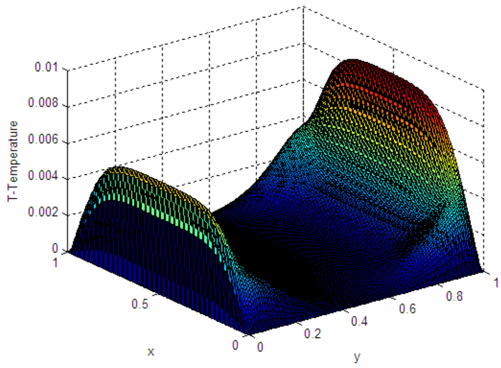
(b)



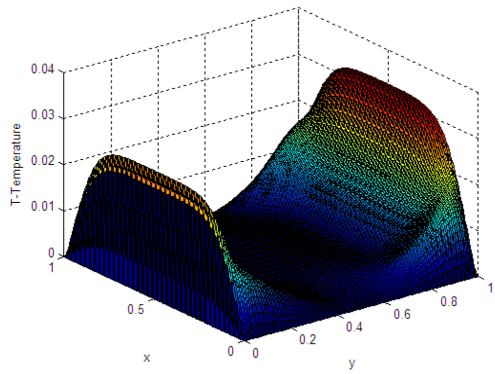
(c)

Fig. 5. (a) Induced magnetic field for $M = 50$; (b) Induced magnetic field for $M = 100$; (c) Induced magnetic field for $M = 300$

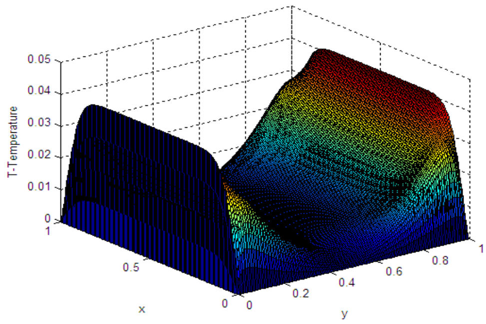
In Figs. 5(a), 5(b) and 5(c), the distribution of induced magnetic field B for forced convection with transverse magnetic field are plotted for $M = 50, 100$ *and* 300 respectively. It is observed that B become flatten for increasing values of Hartmann number M. Moreover, current lines and magnetic field are almost orthogonal in almost all the duct cross section.



(a)

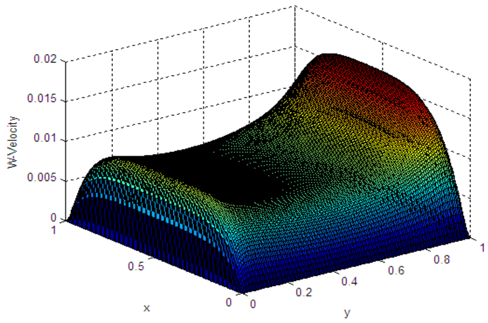


(b)



(c)

Fig. 6. (a) Temperature for $M = 50$; (b) Temperature for $M = 100$; (c) Temperature for $M = 300$



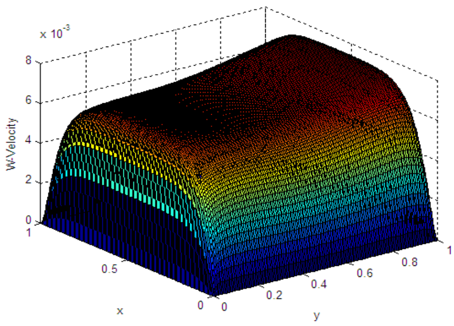
(c)

Fig. 7. (a) Velocity for $M = 100, Gr = 100$; (b) Velocity for $M = 100, Gr = 150$; (c) Velocity for $M = 100, Gr = 200$

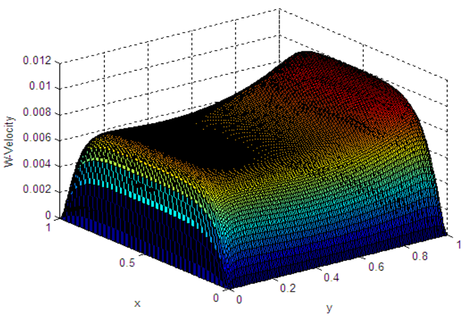
In Fig. 6 temperature distributions are plotted for forced convection flow at various Hartmann number (a) $M = 50$, (b) $M = 100$ and (c) $M = 300$. It is evident from these figures that temperature increases as M increases.

4.2 Mixed Convection with MHD Effects

Computed numerical results are reported in the case of mixed (combined forced and free) convection. Here both buoyancy and Lorentz forces influence on the mixed convection flow. The buoyancy force has an aiding effect on the mixed convection flow, while the Lorentz force has an opposing effect. The computed results for velocity, induced magnetic field and temperature with various flow parameters M, Gr, Pr and Rm are drawn in Figs. 7 to 10.

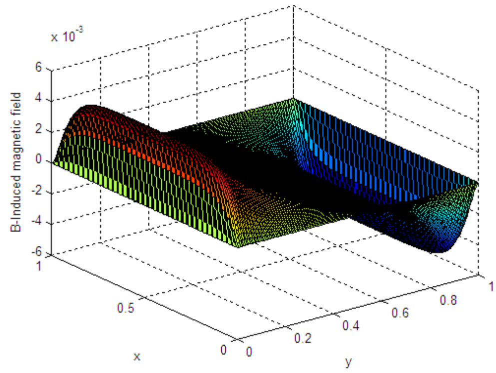


(a)

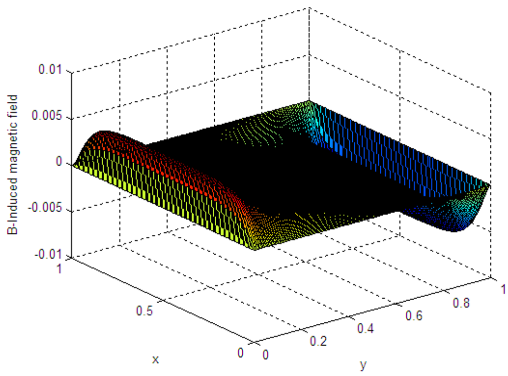


(b)

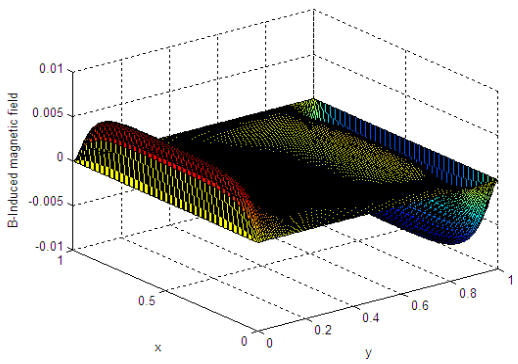
In Fig. 7, velocity distributions are shown at fixed Hartmann number M for various Grashof number (a) $Gr = 100$, (b) $Gr = 150$ and (c) $Gr = 200$, it is observed that velocity distribution increases for increasing values of Gr in one half of the line $y = 0.5$, such type of behavior of velocity gradient is caused by both Lorentz and Buoyancy forces in the dynamics. Buoyancy force acts along the flow direction and has aiding effect on the fluid flow, but Lorentz force acts against the flow direction. In Figs. 8 and 9 induced magnetic field and temperature are presented for various Gr, it is observed that the nature of induced magnetic field is different from the forced convection flow.



(a)



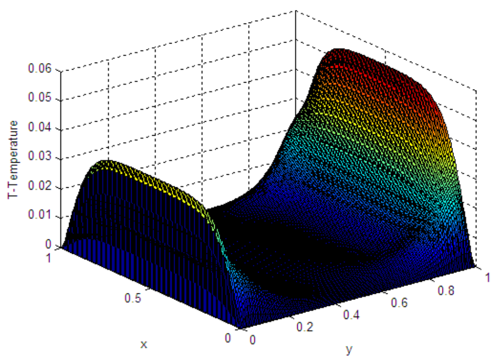
(b)



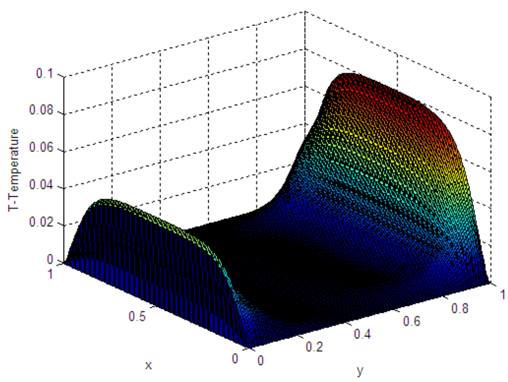
(c)

Fig. 8. (a) Induced magnetic field for $M = 100$, $Gr = 100$; (b) Induced magnetic field for $M = 100$, $Gr = 150$; (c) Induced magnetic field for $M = 100$, $Gr = 200$

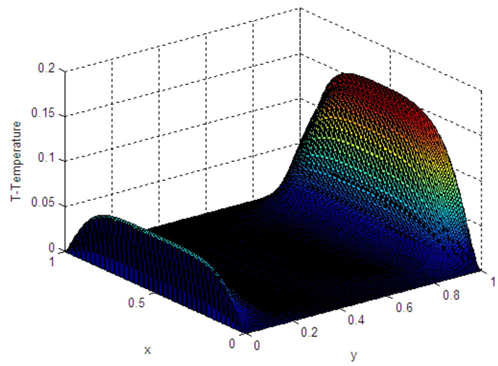
Variation of axial velocity and temperature distribution for fixed values of $M = 50$ and $Gr = 100$ at various Rm are plotted in Figs. 10(a) and 10(b) respectively for mixed convection with transverse magnetic field. It is observed from Fig. 10(a) that velocity decreases with the increasing values of Rm. It is also evident from Fig. 10(b) that temperature distribution decreases as Rm increases. Figure 10(c) represents the temperature distribution at various Pr, it is observed that temperature increases for increasing values of Pr.



(a)

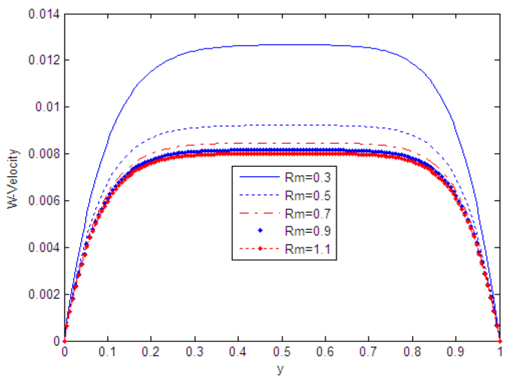


(b)

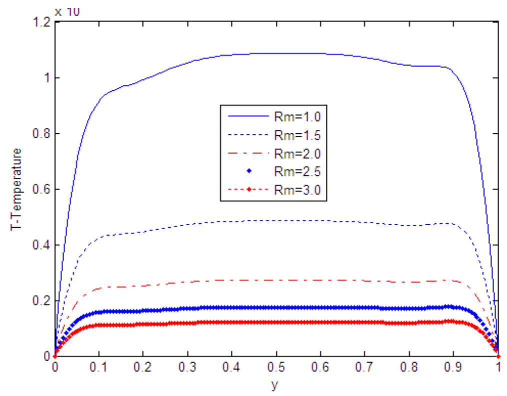


(c)

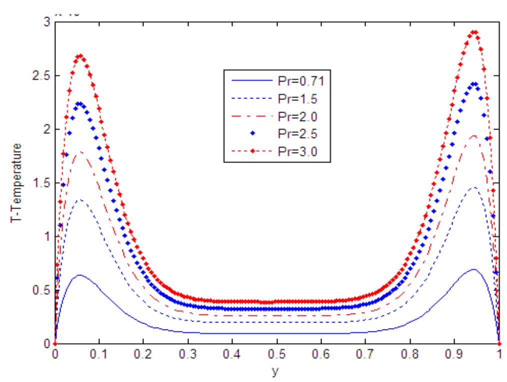
Fig. 9. (a) Temperature for $M = 100$, $Gr = 100$; (b). Temperature for $M = 100$, $Gr = 150$; (c). Temperature for $M = 100$, $Gr = 200$



(a)



(b)



(c)

Fig. 10. (a) Velocity at various ; (b) Temperature at various Rm ; (c). Temperature at various Pr

5. CONCLUSION

As mentioned above, mixed convection flow of an electrically conducting fluid in the square duct driven by the constant pressure gradient along the axis of the duct is affected by buoyancy and Lorentz forces. Since the walls of the duct are kept at constant temperature, so the buoyancy force only has an aiding effect on mixed convection, there is no reversal effect of buoyancy force but the Lorentz force has an opposing effect. It is observed from the Figs. 4 and 5 that the buoyancy force has no effect on mixed convection flow when $Gr = 0$ and the dynamic system is strongly affected by Lorentz force only. But, when $Gr \neq 0$ and $M \neq 0$ then both buoyancy and Lorentz forces are important and both affect the mixed convection. The results obtained have shown that the effect of magnetic field has great influence on the mixed convection heat exchange and fluid flow in steady case, both in terms of flow pattern modification due to Lorentz forces and heat generation due to Joule effect.

ACKNOWLEDGMENTS

The Authors want to acknowledge the continuous help and support received from Prof. Prasanta Sarmah and Dipjyoti Sarma for preparation and completion throughout this work.

REFERENCES

Al-khawaja, M. J., R. A. Gardner and R. Agarwal (1994). Numerical study of magneto-fluid-mechanics forced convection pipe flow. *Engineering Journal of Qatar University*, 7, 115-134.

Al-Khawaja, M. J., R. K. Agarwal and R. A. Gardner (1999). Numerical study of magneto-fluid-mechanics combined free-and-forced convection heat transfer. *International Journal of Heat and Mass Transfer* 42(3), 467-475.

Al-Khawaja, M. J. and M. Selmi (2010). Finite difference solutions of MFM square duct flow with heat transfer using Matlab Program. *Matlab-Modelling, Programming and Simulations, Sciyo*, 365-388.

Aruna, G., V. R. B. Reddy, S. V. K. Varma and N. CH. S. N. Iyengar (2011). A numerical study on developed MHD laminar mixed convection in vertical channels. *Journal of Computer and Mathematical Sciences* 2(2), 336-351.

Blosseville, P. M., S. Aleksandrova and S. Molokov (2007). Buoyancy driven MHD flows in electrically insulated rectangular ducts. *Magnetohydrodynamics*, 43(3), 315-321.

Chamkha Ali, J. (2002). On laminar hydromagnetic mixed convection flow in a vertical channel with symmetric and asymmetric wall heating conditions. *International Journal of Heat and Mass Transfer*, 45, 2509-2525.

Chutia, M. and P. N. Deka (2014). Numerical study on steady magnetohydrodynamics (MHD) flow and heat transfer in a heated rectangular electrically insulated duct under the action of strong oblique transverse magnetic field. *International Journal of Computer Applications* 86(15), 27-34.

Garandet, J. P. and T. Alboussiere (1992). Buoyancy driven convection in a rectangular enclosure with a transverse magnetic field. *International Journal of Heat and Mass Transfer* 35(4), 741-748.

Jain, M. K., S. R. K. Iyenger and R. K. Jain (1994). Compuational Method for Partial Differential Equations. *Wiley Eastern Limited*.

Mathews, J. H. and K. D. Fink (2009). Numerical Method using Matlab, *PHI Learning Private Limited*.

Reddy, D. V. R., M. R. C. Reddy and B. R. B. Reddy (2012). A numerical study on developed MHD laminar mixed convection with vertical channel in down flow. *Journal of Computational and Mathematical Sciences* 3(6), 615-629.

Smolentsev, S., R. Moreau and M. Abdou (2008). Characteristic of key magnetohydrodynamics phenomena in PbLi flows for US DCLL blanket. *Fusion Engineering and Design*, 83, 771-783.

Sposito, G. and M. Ciafalo (2006). One-dimensional MHD convection. *International journal of Heat and Mass Transfer*, 49, 2939-2949.

Taghikhani, M. A. and H. Najafkhani (2013). Investigation of magnetic field effect on natural convection flow using fast $\Psi - \Omega$ method. *International Research Journal of Applied and Basic Sciences* 4(10), 2939-2949.

Tomaricho, V. (2009). Magnetohydrodynamics effects on mixed convection flows in channels and ducts. *Ph. D. thesis, Department of Energetic, Nuclear and Environmental Control Engineering, Amla Mater Studiorum-University of Bolongana, Italy*, 68-90.

Umavathi, J. C. and M. S. Malashetty (2005). Magnetohydrodynamic mixed convection in a vertical channel. *International Journal of Non-Linear Mechanics* 40(1), 91-101.

Umavathi, J. C. and A. J. Chamkha (2011). Fully developed mixed convection in a vertical channel in the presence of heat source and heat sink. *International journal of Energy and Technology* 3(24), 1-9.