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Unsteady Hydromagnetic Natural Convection Flow past an Impulsively Moving Vertical Plate with Newtonian Heating in a Rotating System

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ABSTRACT

An investigation of unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid past an impulsively moving vertical plate with Newtonian heating embedded in a porous medium in a rotating system is carried out. The governing partial differential equations are first subjected to Laplace transformation and then inverted numerically using INVLAP routine of Matlab. The governing partial differential equations are also solved numerically by Crank-Nicolson implicit finite difference scheme and a comparison has been provided between the two solutions. The numerical solution for fluid velocity and fluid temperature are depicted graphically whereas the numerical values of skin friction and Nusselt number are presented in tabular form for various values of pertinent flow parameters. Present solution in special case is compared with previously obtained solution and is found to be in excellent agreement.

Keywords: Unsteady hydromagnetic natural convection; Coriolis force; Newtonian heating; Heat absorption; Porous medium.

NOMECLATURE

B_0	uniform mahnetic field
c_p	specific heat at constant pressure
G_r	thermal Grashof number
g	acceleration due to gravity
h_s	heat trancfer coefficient
K_1	permeability parameter
K^2	rotation parameter
k_1	thermal conductivity
M^2	Magnetic parameter
P_r	Prandtl number
Q_0	heat absorption coefficient
T'	fluid temperature
u'	primary fluid velocity
w'	Secondary fluid velocity
β'	coefficient of thermal expansion
ν	kinematic coefficient of velocity
ρ	fluid density
τ_x	primary skin friction
τ_z	secondary skin friction
σ	electrical conductivity
ϕ	heat absorption parameter
Ω	uniform angular velocity

1. INTRODUCTION

Theoretical/experimental investigation of natural convection flow past bodies with different geometries embedded in a fluid saturated porous medium has received considerable attention during the past several decades due to its varied and wide industrial applications. Significant applications include chemical catalytic reactors, porous insulation, nuclear waste disposal, use of porous conical bearings in lubrication technology, fibrous and granular insulation systems, grain storage, food processing, energy efficient drying processes, enhanced recovery of oil and gas, coal combustors, underground energy transport etc. The basic problem of natural convection in porous medium is boundary layer flow along a heated vertical flat plate embedded in a fluid-saturated porous medium, which was investigated by Cheng and Minkowycz (1977). They obtained similarity solutions for the case when wall temperature varies as a power function of the distance from the leading edge. Nakayama and Koyama (1987) analyzed combined free and forced convection flow in Darcian and non-Darcian porous medium. Lai and Kulacki (1991) studied non-Darcy mixed convection flow along a vertical wall in a fluid saturated porous medium. Hsieh *et al.* (1993) obtained non-similar solution for free and forced convection flow from a vertical surface in a porous medium. Rees (1999) analyzed free convection boundary layer flow from an isothermal vertical flat plate embedded in a fluid saturated layered porous medium. Jana *et al.* (2012) studied natural convection boundary layer flow from an inclined flat plate with finite dimensions embedded in a porous medium in a rotating environment. Khan and Pop (2013) investigated the Cheng and Minkowycz problem for triple diffusive natural convection boundary layer flow past a vertical plate in a porous medium. Reddy *et al.* (2013) studied unsteady hydromagnetic natural convection flow past a moving vertical plate in a porous medium in the presence of radiation and chemical reaction. Comprehensive reviews of convective flow in porous media are candidly presented in the form of books and monographs by Ingham and Pop (2002), Ingham *et al.* (2004), Vafai (2005) and Nield and Bejan (2006).

It is well known that the characteristics of heat transfer are dependent on the thermal boundary conditions. Here a conjugate convective type flow or Newtonian heating is considered. Newtonian heating is a kind of wall-to-ambient heating process where the rate of heat transfer from the bounding surface with a finite heat capacity is proportional to the local surface temperature. This type of situation occurs in many important engineering devices such as in heat exchangers, gas turbines and also in seasonal thermal energy storage systems. Therefore, the interaction of conduction-convection coupled effects is of much significance from practical point of view and it must be considered when evaluating the conjugate heat transfer processes in many engineering applications. Merkin (1994) initiated the study of free convection boundary layer flow over a vertical surface with Newtonian heating while Lesnic *et al.* (1999, 2000) analyzed free convection boundary layer flow past vertical and horizontal surfaces in a porous medium generated by Newtonian heating. Chaudhary and Jain (2006) investigated unsteady free convection flow past an impulsively started vertical plate with Newtonian heating. Salleh *et al.* (2009) discussed forced convection boundary layer flow at a forward stagnation point with Newtonian heating. Narahari and Ishak (2011) investigated the effects of thermal radiation on unsteady free convection flow of an optically thick fluid past a moving vertical plate with Newtonian heating. They considered three cases of interest, namely, (i) impulsive movement of the plate; (ii) uniformly accelerated movement of the plate and (iii) exponentially accelerated movement of the plate. Olanrewaju and Makinde (2013) investigated boundary layer stagnation point flow of a nanofluid over a permeable flat surface with Newtonian Heating. Recently, Das *et al.* (2014a) studied unsteady mixed convection flow past a vertical plate with Newtonian heating.

However, in all these investigations, the effects of magnetic field are ignored. The interaction between electrically conducting fluid and a magnetic field has profound applications in various technical systems employing liquid metal and plasma flows (Liron and Wilhelm, 1974). Therefore, the study of unsteady hydromagnetic convective boundary layer flow of electrically conducting fluids in porous and non-porous media has become a subject of great interest and is widely investigated due to its significant applications in boundary layer flow control, plasma studies, geothermal energy extraction, solar energy collection, cooling of an infinite metallic plate in a cooling bath, magnetohydrodynamic (MHD) stirring of molten metal, magnetic levitation and casting, MHD marine propulsion and on the performance of many engineering devices, namely, MHD power generators, MHD pumps, MHD accelerators, MHD flow-meters, controlled thermonuclear reactors etc. Keeping in view the importance of such study, Raptis (1986) investigated unsteady two-dimensional natural convection flow of an electrically conducting, viscous and incompressible fluid along an infinite vertical plate embedded in a porous medium in the presence of magnetic field. Jha (1991) considered unsteady hydromagnetic free convection and mass transfer flow past a uniformly accelerated moving vertical plate through a porous medium when magnetic field is fixed with the moving plate. Chamkha (1997) analyzed unsteady MHD free convection flow through a porous medium supported by a surface. Kim (2000) studied unsteady MHD free convection flow past a moving semi-infinite vertical porous plate embedded in a porous medium with variable suction. Hayat *et al.* (2008) investigated the effects of magnetic field and porous medium on some unidirectional flows of a second grade fluid. In their study MHD flows are induced by the application of periodic pressure gradient or by the impulsive motion of one or two boundaries or by an oscillating plate. Ogulu and Makinde (2008) considered unsteady hydromagnetic free convection flow of a dissipative and radiative fluid past a vertical plate with constant heat flux. Recently, Seth and Sarkar (2014) investigated unsteady hydromagnetic free convection flow of a viscous, incompressible and electrically conducting fluid past an impulsively moving vertical plate with Newtonian surface heating, embedded in a uniform porous medium.

It is noticed that there may be significant temperature difference between ambient fluid and surface of the solid in a number of fluid flow problems of physical interest. Therefore, it is appropriate to consider temperature dependent heat source and/or sink which may have strong influence on heat transfer characteristics. Sparrow and Cess (1961) were one of the initial investigators to study temperature dependent heat absorption on steady stagnation point flow and heat transfer. Several physical problems exist for possible application in industry where heat generation and absorption take place, namely, fire and combustion modeling, fluids undergoing exothermic and/or endothermic chemical reaction, development of metal waste from spent nuclear fuel, nuclear thermal power generation etc. Keeping in view the importance of such study, Moalem (1976) considered steady state heat transfer in a porous medium with temperature-dependent heat generation. Ramadan and Chamkha (2000) investigated hydromagnetic natural convection of a particulate suspension from an inclined plate with heat absorption. Kamel (2001) considered unsteady hydromagnetic natural convection flow due to heat and mass transfer through a porous medium bounded by an infinite vertical porous plate with temperature-dependent heat sources/sinks. Chamkha (2004) analyzed unsteady hydromagnetic natural convection heat and mass transfer flow past a semi-infinite vertical moving plate with heat absorption. Singh and Makinde (2012) investigated steady hydromagnetic natural convection flow along an inclined plate with Newtonian heating in the presence of volumetric heat generation. Prasad *et al.* (2013) considered the effects of heat generation/absorption, thermal radiation, magnetic field, and temperature-dependent thermal conductivity on the flow and heat transfer characteristics of a non-Newtonian Maxwell fluid over a stretching sheet. Kumar (2013) studied MHD free convection flow over a stretching porous sheet in the presence of heat source and radiation. Das *et al.* (2014b) investigated unsteady hydromagnetic flow of a heat absorbing dusty fluid past a permeable vertical plate with ramped temperature.

Investigation of hydromagnetic natural convection flow in a rotating medium is of considerable importance due to its application in various areas of geophysics, astrophysics and fluid engineering viz. maintenance and secular variations of Earth's magnetic field due to motion of Earth's liquid core, internal rotation rate of the Sun, structure of the magnetic stars, solar and planetary dynamo problems, turbo machines, rotating MHD generators, rotating drum type separators for liquid metal MHD applications etc. It may be noted that Coriolis and magnetic forces are comparable in magnitude and Coriolis force induces secondary flow in the flow-field. Taking into consideration the importance of such study, unsteady hydromagnetic natural convection flow past an infinite moving plate in a rotating medium has been studied by a number of researchers. Mention may be made of research studies of Singh (1984), Raptis and Singh (1985), Kythe and Puri (1987), Singh *et al.* (2010) and Seth *et al.* (2011). Seth *et al.* (2013) considered effects of rotation on unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and heat radiating fluid past an impulsively moving vertical plate with ramped temperature in a porous medium. Recently, Seth *et al*. (2015) investigated effects of Hall current and rotation on hydromagnetic natural convection flow with heat and mass transfer of a heat absorbing fluid past an impulsively moving vertical plate with ramped temperature. To the best of our knowledge no researcher has yet considered the effects of rotation and heat absorption on unsteady hydromagnetic natural convection flow past a flat plate embedded in a porous medium when natural convection is induced due to Newtonian heating of the plate.

Objective of the present investigation is to study unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid past an impulsively moving infinite vertical plate embedded in a uniform porous medium in a rotating system when the natural convection is induced due to Newtonian heating of the plate. According to the best of authors' knowledge this problem has not yet received attention of researchers although being significantly important in science and engineering.

2. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Consider unsteady natural convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid past an infinite vertical plate embedded in a uniform porous medium with Newtonian heating at the surface of the plate. Coordinate system is chosen in such a way that x'- axis is taken along the plate in the upward direction, y' - axis is taken normal to the plane of plate in the fluid and z' - axis is taken normal to the $x'y'$ plane. Fluid is permeated by a uniform transverse magnetic field B_0 which is applied in a direction parallel to y' - axis. The fluid and plate rotate in unison with uniform angular velocity Ω about y' - axis. Initially, i.e. at time $t' \leq 0$, both the fluid and plate are at rest and at a uniform temperature T'_∞. At time $t' > 0,$ the plate is given an impulsive motion in x' - direction against the gravitational field such that it attains a uniform velocity U_0. It is assumed that natural convection is generated by Newtonian heating i.e. rate of heat transfer from the plate is proportional to the local surface temperature. The geometry of the problem is presented in Fig. 1. Since plate is of infinite extent in x' and z' directions and is electrically non-conducting, all physical quantities depend on y' and t' only. Also no applied or polarized voltages exist so the effect of polarization of fluid is negligible. This corresponds to the case where no energy is added or extracted from the fluid by electrical means (Cramer and Pai, 1973). It is assumed that the induced magnetic field generated by fluid motion is negligible in comparison to the applied one. This assumption is valid because magnetic Reynolds number is very small for liquid metals and partially ionized fluids which are commonly used in industrial applications (Cramer and Pai, 1973).

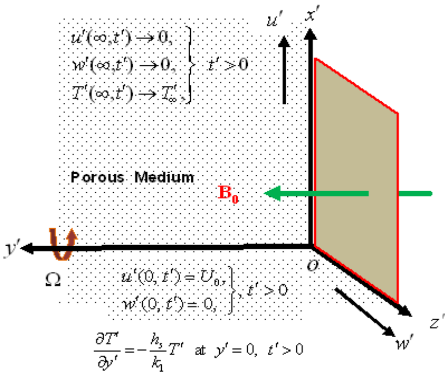

Fig. 1. Geometry of the Problem

Taking into consideration the assumptions made above, the governing equations for unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid through a uniform porous medium in a rotating frame of reference, under Boussinesq approximation, are given by

$$\frac{\partial u'}{\partial t'}+2\Omega w'=\nu\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma B_0^{\,2}}{\rho}u'-\frac{\nu}{K_1'}u'+g\beta'\left(T'-T_\infty'\right), \quad (1)$$

$$\frac{\partial w'}{\partial t'}-2\Omega u'=\nu\frac{\partial^2 w'}{\partial y'^2}-\frac{\sigma B_0^{\,2}}{\rho}w'-\frac{\nu}{K_1'}w', \quad (2)$$

$$\frac{\partial T'}{\partial t'}=\frac{k_1}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2}-\frac{Q_0}{\rho c_p}\left(T'-T_\infty'\right), \quad (3)$$

where $u', w', \nu, \sigma, \rho, K_1', g, \beta', T', k_1, c_p$ and Q_0 are, respectively, fluid velocity in x'- direction, fluid velocity in z'- direction, kinematic coefficient of viscosity, electrical conductivity, fluid density, permeability of porous medium, acceleration due to gravity, coefficient of thermal expansion, fluid temperature, thermal conductivity, specific heat at constant pressure and heat absorption coefficient.

The initial and boundary conditions for the problem are specified as

$$t'\le 0:\ u'=0,\ w'=0,\ T'=T_\infty' \quad \text{for all} \quad y'\ge 0, \quad (4a)$$

$$t'>0: u'=U_0,\ w'=0,\ \frac{\partial T'}{\partial y'}=-\frac{h_s}{k_1}T' \text{ at } y'=0, \quad (4b)$$

$$u'\to 0,\ \ w'\to 0,\ T'\to T_\infty' \quad \text{as} \quad y'\to\infty. \quad (4c)$$

where h_s is heat transfer coefficient and $U_0=\frac{h_s\nu}{k_1}$.

Eqs. (1) to (3), in non-dimensional form, assume the following form

$$\frac{\partial u}{\partial t}+2K^2 w=\frac{\partial^2 u}{\partial y^2}-M^2 u-\frac{u}{K_1}+G_r T, \quad (5)$$

$$\frac{\partial w}{\partial t}-2K^2 u=\frac{\partial^2 w}{\partial y^2}-M^2 w-\frac{w}{K_1}, \quad (6)$$

$$\frac{\partial T}{\partial t}=\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2}-\phi T, \quad (7)$$

where

$$y=y'U_0/\nu,\ u=u'/U_0,\ w=w'/U_0,\ t=t'U_0^{\,2}/\nu,$$
$$M^2=\sigma B_0^{\,2}\nu/\rho U_0^{\,2},\ K^2=\nu\Omega/U_0^{\,2}, K_1=K_1'U_0^{\,2}/\nu^2,$$

$$T=\left(T'-T_\infty'\right)/T_\infty',\ G_r=g\beta'\nu T_\infty'/U_0^{\,3},\quad P_r=\nu\rho c_p/k_1$$

and $\phi=\nu Q_0/\rho c_p U_0^{\,2}$.

M^2, K^2, K_1, G_r, P_r and ϕ are, respectively, magnetic parameter, rotation parameter, permeability parameter, Grashof number, Prandtl number and heat absorption parameter.

The initial and boundary conditions (4), in non-dimensional form, become

$$t\le 0:\ u=0,\quad w=0,\ T=0 \quad \text{for all} \quad y, \quad (8a)$$

$$t>0:\ u=1, w=0,\ \frac{\partial T}{\partial y}=-\left(1+T\right) \text{ at } y=0, \quad (8b)$$

$$u\to 0,\quad w\to 0,\ T\to 0 \quad \text{as} \quad y\to\infty. \quad (8c)$$

Eqs. (5) and (6) are presented, in compact form, as

$$\frac{\partial F}{\partial t}=\frac{\partial^2 F}{\partial y^2}-\lambda F+G_r T, \quad (9)$$

where $F=u+iw$ and $\lambda=M^2+1/K_1-2iK^2$.

Initial and boundary conditions (8a) to (8c), in compact form, become

$$F=0, T=0 \quad \text{for} \quad y\ge 0 \text{ and } t\le 0, \quad (10a)$$

$$F=1,\ \frac{\partial T}{\partial y}=-\left(1+T\right) \quad \text{at} \quad y=0 \quad \text{for} \quad t>0, \quad (10b)$$

$$F\to 0,\ \ T\to 0 \text{ as } y\to\infty \quad \text{for} \quad t>0. \quad (10c)$$

Eqs. (7) and (9), after taking Laplace transform and using initial conditions (10a), reduce to

$$\frac{d^2\bar{T}}{dy^2}-P_r\left(s+\phi\right)\bar{T}=0, \quad (11)$$

$$\frac{d^2\bar{F}}{dy^2}-\left(s+\lambda\right)\bar{F}+G_r\bar{T}=0, \quad (12)$$

where $\bar{T}(y,s)=\int_0^\infty T(y,t)e^{-st}dt$,

$\bar{F}(y,s)=\int_0^\infty F(y,t)e^{-st}dt$ and $s>0$ (s being Laplace transform parameter).

Boundary conditions (10b) and (10c), after taking Laplace transform, become

$$\bar{F} = 1/s,\ \ \frac{d\bar{T}}{dy} = -\left(\frac{1}{s} + \bar{T}\right)\ \text{ at } y = 0, \tag{13a}$$

$$\bar{F} \to 0,\ \ \bar{T} \to 0,\ \text{ as } y \to \infty. \tag{13b}$$

Solution of Eqs. (11) and (12) subject to the boundary conditions (13a) and (13b) are given by

$$\bar{T}(y,s) = \frac{e^{-y\sqrt{P_r(s+\phi)}}}{s\left(\sqrt{P_r(s+\phi)} - 1\right)}, \tag{14}$$

$$\bar{F}(y,s) = \frac{1}{s}e^{-y\sqrt{s+\lambda}} - \frac{G_1}{s(s+\lambda_3)\left(\sqrt{P_r(s+\phi)} - 1\right)} \times \left\{e^{-y\sqrt{s+\lambda}} - e^{-y\sqrt{P_r(s+\phi)}}\right\}, \tag{15}$$

where $G_1 = G_r/(1-P_r)$ and $\lambda_3 = \lambda/(1-P_r)$.

An exact inverse Laplace transform of Eq. (14) can be obtained when $\phi = 0$ i.e. in the absence of heat absorption (Chaudhary and Jain, 2006). Moreover, inverse Laplace transform of the second term in Eq. (15) can be obtained only when $\lambda = 0$ and $\phi = 0$ i.e. in the absence of magnetic field, porous medium, rotation and heat absorption (Chaudhary and Jain, 2006). Therefore, the presence of either magnetic field or Coriolis force or permeability of medium or heat absorption in Eq. (15) and the presence of heat absorption in Eq. (14) causes the task to obtain analytical solution of the governing equations impossible and no researcher has yet obtained a closed form analytical solution taking into account any of the above entities to the best of our knowledge. Thus, the Laplace transform inversion of Eqs. (14) and (15) is obtained numerically using INVLAP routine in Matlab. Exact inversion of Eq. (14) can be obtained in the absence of heat absorption i.e. when $\phi = 0$ which agrees with that of Chaudhary and Jain (2006) and is given by

$$T(y,t) = e^{-y+t/P_r} erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}} - \sqrt{\frac{t}{P_r}}\right) - erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}\right), \tag{16}$$

3. NUMERICAL SOLUTION

Eqs. (5) to (7) subject to the initial and boundary conditions (8) cannot be solved analytically due to the reasons mentioned in the previous section and hence we resorted to INVLAP routine of Matlab. However, Eqs. (5) to (7) under the initial and boundary conditions (8) can be solved numerically using Crank-Nicolson implicit finite difference scheme. Therefore, we have also obtained numerical solution of this problem using Crank-Nicolson implicit finite difference scheme.

For this purpose, the region under consideration is restricted to a rectangle of finite dimensions with $y_{\max}$=6 (corresponding to $y \to \infty$) and $t_{\max}$=2. Assumption of $y_{\max} = 6$ was finalized when boundary condition (10c) was satisfied within tolerance limit of 10^{-4}. Computational domain is divided into 241×801 grid points and the grid refinement check is performed by comparing results in this case (with mesh size $\Delta y \times \Delta t$ where $\Delta y = 1/40$ and $\Delta t = 1/400$) with the results obtained when mesh size is reduced to 50% of the present case and it is noticed that the difference between these two results is less than half a unity in the fourth decimal place. The finite difference equations for each time step constitute a tridiagonal system of equations which are solved by Thomas algorithm as given in Carnahan *et al.* (1969). Numerical solution for fluid temperature and fluid velocity is obtained corresponding to desired degree of accuracy for required time by performing computations for a number of time steps. It was found that the absolute difference between the numerical values of fluid temperature and fluid velocity obtained for two consecutive time steps is less than 10^{-4}. Hence the scheme designed is stable. Moreover, Crank-Nicolson scheme has local truncation error of $O\left\{(\Delta y)^2 + (\Delta t)^2\right\}$ which tends to zero as Δy and Δt tends to zero which justifies consistency (Antia, 1991, pp. 643-644). Stability and consistency together ensure convergence of the scheme.

Skin friction τ and Nusselt number N_u are given by

$$\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0}, \tag{17}$$

$$N_u = \left.\frac{\partial T}{\partial y}\right|_{y=0}. \tag{18}$$

The numerical values of skin friction and Nusselt number are obtained using computed values of fluid velocity and fluid temperature respectively. It may be noted that the derivatives involved in Eqs. (17) and (18) are evaluated using five point forward difference formula for the first order derivative (Antia 1991, page 161).

3.1 Validation of Numerical Solution

In order to validate our numerical scheme we have presented in Fig. 2 a comparison between the exact values of fluid temperature computed from exact solution (16) with the numerical values of fluid temperature obtained by Crank-Nicolson implicit finite difference scheme and by INVLAP routine of Matlab for the case when $\phi = 0$ (absence of heat absorption). It is seen that there is an excellent agreement between these solutions.

Expression for Nusselt number N_u when $\phi = 0$ is obtained using solution (16) which is given by

$$N_u = e^{t/\mathrm{Pr}} erfc\left(-\sqrt{\frac{t}{P_r}}\right). \tag{19}$$

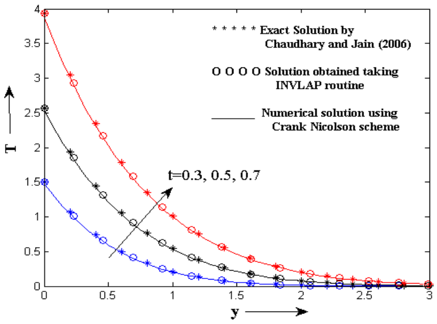

Fig.2. Temperature profiles when $\phi = 0$ and $P_r = 0.71$

We have presented in Table 1 a comparison between the numerical values of Nusselt number obtained using the INVLAP routine of Matlab and finite difference scheme mentioned above with the exact value obtained from expression (19). It is evident from Table 1 that the numerical values of Nusselt number obtained through finite difference scheme are in good agreement with the values of Nusselt number obtained by INVLAP routine of Matlab. Moreover, it is also noticed from Table 1 that numerical values for Nusselt number obtained by INVLAP routine of Matlab are in excellent agreement with the exact values of Nusselt number obtained from (19). This justifies the correctness of the results presented in the manuscript.

Table 1 Nusselt number –Nu when $\phi = 0$

$P_r \rightarrow$ $t \downarrow$	Result by Finite Difference			Result by INVLAP routine			Exact Result		
	0.3	0.5	0.71	0.3	0.5	0.71	0.3	0.5	0.71
0.3	5.0084	3.1459	2.5052	5.009	3.1462	2.5055	5.009	3.1462	2.5055
0.5	10.2251	5.0084	3.5683	10.2295	5.0090	3.5687	10.2295	5.0089	3.5686
0.7	20.268	7.7264	4.9304	20.3074	7.7281	4.9310	20.3074	7.7281	4.9310

4. RESULTS AND DISCUSSION

In order to analyze the effects of magnetic field, rotation, permeability of the medium, thermal buoyancy force, heat absorption, thermal diffusion and time on the flow-field, the numerical solution of primary fluid velocity u and secondary fluid velocity w is depicted graphically versus boundary layer coordinate y in Figs. 3 to 9 for various values of magnetic parameter M^2, rotation parameter K^2, permeability parameter K_1, Grashof number G_r, heat absorption parameter ϕ, Prandtl number P_r and time t. It is revealed from the Figs. 3 to 9 that, secondary fluid velocity attains maximum value near surface of the plate and then decrease properly on increasing boundary layer coordinate y to approach free stream value. This is due to the fact that Coriolis force is dominant in the region near the axis of rotation.

Figure 3 illustrates the influence of magnetic field on the primary fluid velocity u and secondary fluid velocity w. It is revealed from Fig. 3 that both u and w decrease on increasing M^2. Since M^2 signifies the relative strength of magnetic force to viscous force, M^2 increases on increasing the strength of magnetic force. This implies that, magnetic field has a tendency to retard fluid flow in both the primary and secondary flow directions throughout the boundary layer region. This phenomenon is attributed to the Lorentz force, induced due to the movement of an electrically conducting fluid in the presence of magnetic field, which has a tendency to resist fluid motion.

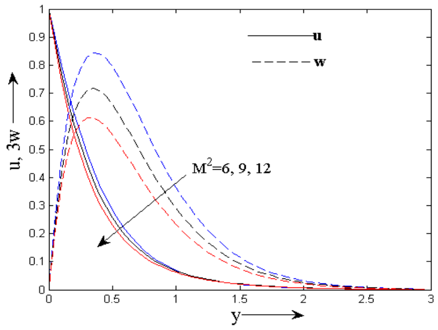

Fig. 3. Primary and Secondary velocity profiles when
$K^2 = 5, K_1 = 0.4, G_r = 4, \phi = 2, \mathrm{P}_r = 0.71 \text{and } t = 0.5$

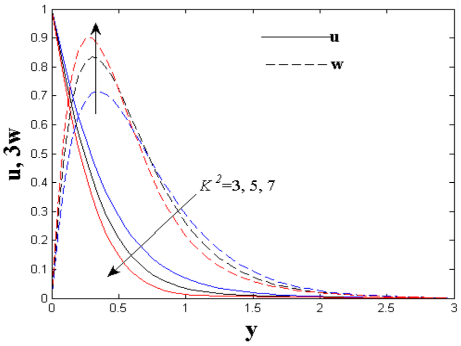

Fig. 4. Primary and Secondary velocity profiles when
$M^2 = 6, K_1 = 0.4, G_r = 4, \phi = 2, \mathrm{P}_r = 0.71$ and $t = 0.5$

Figure 4 demonstrates the effects of rotation on the primary and secondary fluid velocities. It is perceived from Fig. 4 that *u* decreases on increasing K^2 throughout the boundary layer region whereas *w* increases on increasing K^2 in the region near the plate and it decreases on increasing K^2 in the region away from the plate. This implies that, rotation tends to retard fluid flow in the primary flow direction throughout the boundary layer region whereas it has a reverse effect on fluid flow in the secondary flow direction in the region near the plate. Although rotation is known to induce secondary flow in the flow-field by suppressing primary flow, its accelerating effect on the fluid flow in secondary flow direction is prevalent only in the region near the plate.

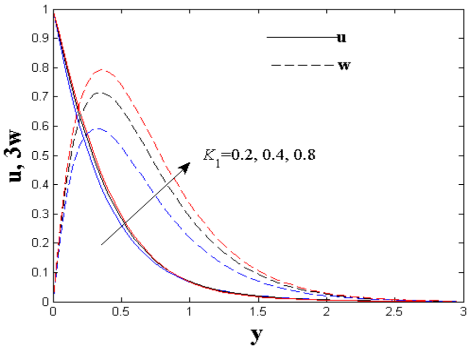

Fig. 5. Primary and Secondary velocity profiles when

$M^2 = 6, K^2 = 5, G_r = 4, \phi = 2, \mathrm{P}_r = 0.71$ and $t = 0.5$

Figure 5 presents the influence of permeability of the medium on the primary and secondary fluid velocities. It is evident from Fig. 5 that both *u* and *w* increase on increasing K_1. It may be noted that an increase in K_1 implies that there is a decrease in the resistance of the porous medium. Due to this reason permeability of the medium tends to accelerate fluid flow in both the primary and secondary flow directions throughout the boundary layer region. Figure 6 depicts the effects of thermal buoyancy force on the primary and secondary fluid velocities. It is noticed from Fig. 6 that both *u* and *w* increase on increasing G_r. Since G_r presents the relative strength of thermal buoyancy force to viscous force, G_r increases on increasing the strength of thermal buoyancy force. This implies that, thermal buoyancy force tends to accelerate fluid flow in both the primary and secondary flow directions throughout the boundary layer region.

Figure 7 illustrates the influence of heat absorption on the primary and secondary fluid velocities. It is perceived from Fig. 7 that both *u* and *w* decrease on increasing ϕ. This implies that, heat absorption tends to retard fluid flow in both the primary and secondary flow directions throughout the boundary layer region.

Figures 8 and 9 depict the effects of thermal diffusion and time on the primary and secondary fluid velocities. It is noticed from Figs. 8 and 9 that both *u* and *w* decrease on increasing P_r whereas both *u* and *w* increase on increasing *t*. Since P_r is a measure of relative strength of viscosity to thermal diffusivity of the fluid, P_r decreases on increasing thermal diffusivity. This implies that, thermal diffusion tends to accelerate fluid flow in both the primary and secondary flow directions throughout the boundary layer region. As time progresses, fluid flow is getting accelerated in both the primary and secondary flow directions throughout the boundary layer region.

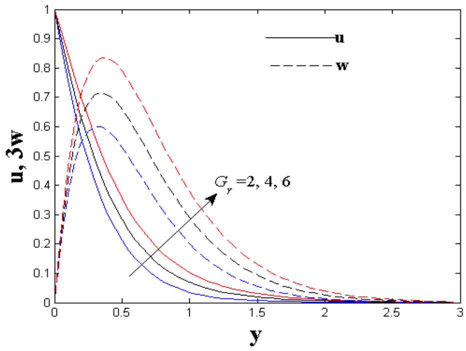

Fig. 6. Primary and Secondary velocity profiles when

$M^2 = 6, K^2 = 5, K_1 = 0.4, \phi = 2, \mathrm{P}_r = 0.71$ and $t = 0.5$

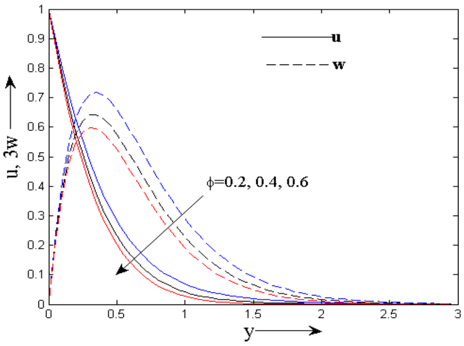

Fig. 7. Primary and Secondary velocity profiles when

$M^2 = 6, K^2 = 5, K_1 = 0.4, G_r = 4, P_r = 0.71$ and $t = 0.5$

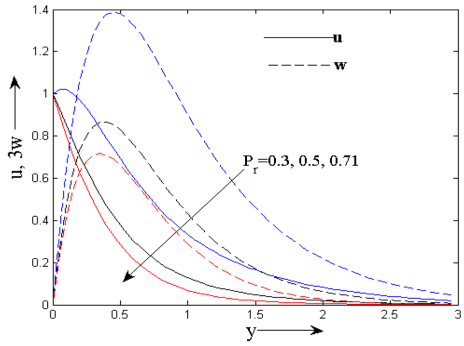

Fig. 8. Primary and Secondary velocity profiles when

$M^2 = 6, K^2 = 5, K_1 = 0.4, G_r = 4, \phi = 2$ and $t = 0.5$

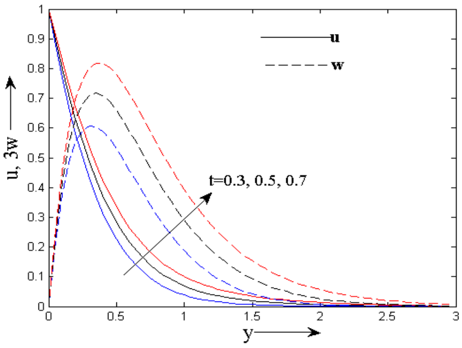

Fig. 9. Primary and Secondary velocity profiles when
$M^2 = 6, K^2 = 5, K_1 = 0.4, G_r = 4, P_r = 0.71$ and $\phi = 2$

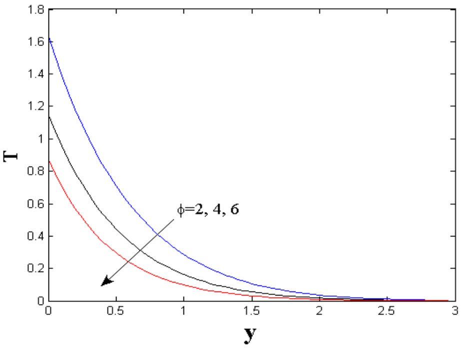

Fig. 10. Temperature profiles when
$P_r = 0.71$ and $t = 0.5$

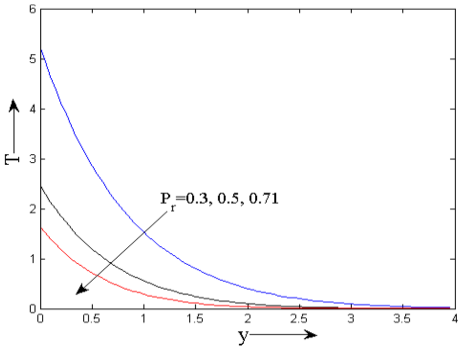

Fig. 11. Temperature profiles when
$\phi = 2$ and $t = 0.5$

The numerical solution of fluid temperature T is depicted graphically versus boundary layer coordinate y in Figs. 2, 10 and 11 for various values of P_r, ϕ and t. Figures 2, 10 and 11 reveal that fluid temperature T decreases on increasing P_r and ϕ whereas it increases on increasing t. This implies that, throughout the boundary layer region, thermal diffusion tends to enhance fluid temperature whereas heat absorption has a reverse effect on it. Fluid temperature is getting enhanced with the progress of time.

The numerical values of primary skin friction τ_x and secondary skin friction τ_z are presented in tabular form in Tables 2 to 4 for various values of M^2, K^2, G_r, K_1, ϕ and t taking $P_r = 0.71$ (ionized air). It is perceived from Table 2 that both τ_x and τ_z increase on increasing K^2. This implies that rotation tends to enhance both the primary and secondary skin frictions. It is noticed from Tables 2 to 4 that τ_x increases on increasing M^2 and ϕ whereas it decreases on increasing G_r, K_1 and t. τ_z decreases on increasing M^2 and ϕ whereas it increases on increasing G_r, K_1 and t. This implies that magnetic field and heat absorption tend to enhance primary skin friction whereas these agencies have reverse effect on secondary skin friction. Thermal buoyancy force and permeability of the medium tend to reduce primary skin friction whereas these agencies have reverse effect on secondary skin friction. As time progresses, primary skin friction is getting reduced whereas secondary skin friction is getting enhanced.

Table 2 Primary and Secondary Skin Frictions when $G_r = 4$, $K_1 = 0.4$, $\phi = 2$ and $t = 0.5$

$K^2 \rightarrow$	$-\tau_x$			τ_z		
$M^2 \downarrow$	5	7	9	5	7	9
6	2.1031	2.4341	2.7605	1.8333	2.3551	2.7905
9	2.5266	2.7891	3.0642	1.6198	2.1294	2.5673
12	2.9278	3.1382	3.3696	1.4556	1.9440	2.3756

Table 3 Primary and Secondary Skin Frictions when $M^2 = 6$, $K^2 = 5$, $\phi = 2$ and $t = 0.5$

$K_1 \rightarrow$	$-\tau_x$			τ_z		
$G_r \downarrow$	0.2	0.4	0.8	0.2	0.4	0.8
2	3.0256	2.6954	2.5268	1.5179	1.6745	1.7672
4	2.4574	2.1031	1.9219	1.6515	1.8333	1.9414
6	1.8892	1.5107	1.3170	1.7850	1.9921	2.1157

Table 4 Primary and Secondary Skin Frictions when
$M^2 = 6$, $K^2 = 5$, $G_r = 4$ and $K_1 = 0.4$

$t \rightarrow$	$-\tau_x$			τ_z		
$\phi \downarrow$	0.3	0.5	0.7	0.3	0.5	0.7
2	2.4968	2.1031	1.7600	1.6930	1.8333	1.9586
4	2.6602	2.4789	2.3713	1.6589	1.7335	1.7785
6	2.7773	2.6958	2.6619	1.6335	1.6732	1.6894

5. CONCLUSIONS

An investigation of unsteady hydromagnetic natural convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid past an impulsively moving infinite vertical plate embedded in a uniform porous medium in a rotating system when the natural convection is induced due to Newtonian heating of the plate is carried out. Significant findings are as follows:

(i) Magnetic field has a tendency to retard fluid flow in both the primary and secondary flow directions throughout the boundary layer region. Rotation tends to retard fluid flow in the primary flow direction throughout the boundary layer region whereas it has a reverse effect on fluid flow in the secondary flow direction in the region near the plate. Permeability of the medium, thermal buoyancy force and thermal diffusion tend to accelerate fluid flow in both the primary and secondary flow directions throughout the boundary layer region whereas heat absorption has a reverse effect on it. As time progresses, fluid flow is getting accelerated in both the primary and secondary flow directions throughout the boundary layer region.

(ii) Thermal diffusion tends to enhance fluid temperature whereas heat absorption has a reverse effect on it.

(iii) Rotation tends to enhance both the primary and secondary skin frictions. Magnetic field and heat absorption tend to enhance primary skin friction whereas these agencies have reverse effect on secondary skin friction. Thermal buoyancy force and permeability of the medium tend to reduce primary skin friction whereas these agencies have reverse effect on secondary skin friction. As time progresses, primary skin friction is getting reduced whereas secondary skin friction is getting enhanced.

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