Journal of Applied Fluid Mechanics, Vol. 8, No. 4, pp. 661-665, 2015.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.67.223.19480

Demixing of a Binary Fluid Mixture in Case of MHD Flow with Heat and Mass Transfer due to a Point Sink

B. R. Sharma and K. Nath†

Department of Mathematics, Dibrugarh University, Dibrugarh, Assam-786004, India

†*Corresponding Author Email: kabitanath56@gmail.com*

(Received February 22, 2013; accepted September 1, 2014)

ABSTRACT

The present problem concerns with the effects of the magnetic field, mass flux diffusion and heat transfer on demixing of a binary mixture of incompressible viscous electrically conducting fluids in steady, laminar boundary layer flow in presence of a point sink at the vertex of a cone. The momentum, energy and concentration equations are reduced to non-linear coupled ordinary differential equations by similarity transformations and are solved numerically by using MATLAB's built in solver bvp4c. The local skin friction, the Nusselt number and the Sherwood number are tabulated for various values of the parameters. These numerical results have been demonstrated graphically from which it is observed that the effects of various parameters are to separate the components of the binary mixture by collecting the rarer and lighter component near the surface of the cone and throwing the heavier one away from it.

Keywords: Binary fluid mixture; Incompressible; Mass flux diffusion; Magnetic field, Heat transfer.

NOMENCLATURE

B_0	magnetic field
C_f	local skin friction coefficient
c and T	concentration and temperature
D	binary diffusion coefficient
E_c	Eckert number
f_w	mass transfer parameter
g and G	dimensionless temperature and concentration respectively
k	thermal conductivity
m	strength of point sink
M	magnetic parameter
$\overline{m}$	mass flux of diffusing species
Nu	Nusselt number
p	static pressure respectively
Pr	Prandtl number
R	radius of the cone (R= r sinφ)
Re_r	Local Reynolds number
r and z	distances along and perpendicular to the cone
S_c	Schmidt number

Sh	Sherwood number
t_d	Thermal diffusion number
u and w	velocity components along r and z directions respectively
U	inviscid flow velocity
ν	kinematic viscosity
w and ∞	subscripts at the wall and in the free stream respectively
φ	semi-vertical angle of the cone
ψ and f	dimensional and dimensionless stream functions respectively
η	similarity variable
ρ	density
σ	electrical conductivity
α	thermal diffusivity
μ	viscosity
τ_w and q_w	shear stress and heat transfer rate at the wall

1. INTRODUCTION

This article deals with steady, laminar, heat and mass transfer MHD flow problem and has applications in vortex chambers, power generators, nuclear reactions, evolution of rotating magnetic stars, geophysical fluid dynamics etc. This flow and heat transfer situation is of considerable interest because it can occur in many geothermal, geophysical, technological, and engineering applications such as nuclear reactors, migration of moisture through air contained in fibrous insulations, grain storage, nuclear waste disposal, dispersion of chemical pollutants through water-saturated soil, and others. The study of demixing of the boundary layer flow of an electrically conducting binary mixture of incompressible viscous fluids on a cone due to a point sink with an applied magnetic field is relevant in the study of conical nozzle or diffuser flow problems. The effect of heat transfer in axi-symmetric flow, inside a cone due to a point sink, in absence of magnetic field, has been studied by Rosenhead (1963) using similarity transformations. A series solution for the converging motion of the viscous flow inside a cone under some restricted conditions on the potential flow has been studied by Ackerberg (1965). The steady MHD laminar axi-symmetric boundary layer flow in a cone due to a point sink with an applied magnetic field, heat and mass transfer have been investigated by Takhar (1986). The unsteady MHD forced axi-symmetric flow inside a cone due to a point sink has been studied by Eswara and Roy (2000). Eswara and Bommaiah (2004) investigated the influence of variation of viscosity with temperature on axi-symmetric flow inside a cone due to a point sink. In the previous studies with axi-symmetric flow inside a cone due to a point sink, the thermal conductivity of fluid was assumed to be constant. However, it is known that thermal conductivity of fluid may also be change with temperature. Hence, like other thermo-physical properties, temperature-dependent thermal conductivity also plays a vital role in surface friction and heat transfer rate near the wall. The effect of variable thermal conductivity along a stretching sheet with MHD flow and in the presence of heat source or sink has been studied by Sharma and Singh (2008). Seddeek and Salem (2005) investigated the effects of heat and mass transfer on stretching surface with variable viscosity and variable thermal diffusivity. The effects of variable thermal conductivity and variable viscosity on steady free convective heat transfer flow process along an isothermal vertical plate in the presence of heat sink has been presented by Mahanti and Gaur (2009). The effect of chemical reaction in a heat and mass transfer flow process along a vertical surface has been discussed by Muthucumaraswamy (2002). Ibrahim, Elaiw and Bakr (2008) found analytical solutions for heat and mass transfer flow of Newtonian fluid along a vertical permeable surface in the presence of radiation and also with homogeneous first order chemical reaction. Sharma and Singh (2008, 2009, 2010), Sharma and Nath (2012a, b) and Sharma et al. (2011, 2012) have studied the effect of magnetic field on demixing of a binary fluid mixture. Sharma and Singh (2004, 2007) have studied the effect of temperature gradient on demixing of species in hydromagnetic flow of a binary mixture of incompressible viscous fluids between two parallel plates, first taking the plates horizontal and second by taking the plates vertical. They found that the effect of temperature gradient is to separate the components of the binary mixture and the magnetic field increases the effect of species demixing.

The objective of this paper is to investigate numerically the demixing of fluid mixture in case of two dimensional steady, laminar, MHD boundary layer flow of an incompressible viscous electrically conducting binary mixture of fluids in a circular cone due to a point sink at the vertex with an applied weak axial magnetic field, mass flux diffusion and heat transfer. The results are presented as concentration profiles for different values of parameters entering in the problem. The effects of the parameters f_w , t_d and S_c on the local skin friction, the Nusselt number and the Sherwood number are presented numerically in tabular form.

2. MATHEMATICAL FORMULATION

We consider 1species demixing due to pressure gradient and temperature gradient in a steady laminar axi-symmetric boundary layer flow of a binary mixture of an electrically conducting incompressible fluid in a circular cone with a hole at the vertex. The hole is considered as three-dimensional sink. A magnetic field B_0 fixed relative to the fluid is applied in the z-direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The wall and the free stream are maintained at a constant temperature and concentration. The Hall effect term is neglected. The effect of mass transfer (suction and injection) has been included in the analysis. It is assumed that the injected binary fluid mixture possesses the same physical properties as the boundary layer binary fluid mixture and has a static temperature equal to the wall temperature. The boundary layer equations under the foregoing assumptions are:

$$\frac{\partial}{\partial r}(ru)+\frac{\partial}{\partial z}(rw)=0, \tag{1}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\frac{\partial^2 u}{\partial z^2}-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial z}\right)^2, \tag{3}$$

$$u\frac{\partial c}{\partial r}+w\frac{\partial c}{\partial z}=D\begin{bmatrix}\frac{\partial^2 c}{\partial z^2}+S_T\frac{\partial c}{\partial z}\frac{\partial T}{\partial z}+\\ S_T\, c\frac{\partial^2 T}{\partial z^2}\end{bmatrix}, \tag{4}$$

where $-\frac{1}{\rho}\frac{\partial p}{\partial r}=U\frac{\partial U}{\partial r}+\frac{1}{\rho}\sigma B_0^2 U,$

$$U=-{m}/{r^2}\ , m>0. \tag{5}$$

The boundary conditions are given by

$u\ (r,0) = 0$, $w\ (r,0) = -w_w$,
$T(r,0) = T_w$, $c(r,0)=c_w$
$u\ (r,\infty) = U$, $T(r,\infty) = T_\infty$, $c(r,\infty)= c_\infty$ (6)

Applying the following transformations

$$\eta = {m^{\frac{1}{2}}z}\Big/{(2\nu r^3)^{\frac{1}{2}}}$$

$$ru=\frac{\partial\psi}{\partial z}, rw=-\frac{\partial\psi}{\partial r}\ , \psi=-(2\nu m r)^{\frac{1}{2}}f(\eta),,$$

$$u = Uf'(\eta), w = \left(mv/_{2r^3}\right)^{\frac{1}{2}} (f - 3\eta f'),$$
$$g(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, G(\eta) = \frac{c - c_\infty}{c_w - c_\infty},$$
$$M = \frac{2\sigma B_0^2 r^3}{(m\rho)}, \Pr = \frac{\nu}{\alpha}, S_c = \frac{\nu}{D},$$
$$f_w = -w_w \left(2r^3/_{mv}\right)^{\frac{1}{2}}, E_c = U^2/_{c_p(T_w - T_\infty)} \tag{7}$$

and $t_d = S_T(T_w - T_\infty)$

it can easily be verified that the balance of mass given by equation (1) is identically satisfied. The set of equations (2) – (4) are coupled non – linear partial differential equations. Introducing the relation (7) into the equations (2) – (4) we obtain the following non-linear coupled ordinary differential equations

$$f''' - ff'' + 4\left(1 - f'^2\right) + M\left(1 - f'\right) = 0, \tag{8}$$
$$\frac{1}{Pr} g'' + Ec\, f'' - f g' = 0, \tag{9}$$
$$G'' - S_c\, f G' + t_d\left(G'g' + Gg''\right) = 0. \tag{10}$$

The boundary conditions (6) reduce to

$$f = f_w, f' = 0, g = G = 1 \quad at\ \eta = 0$$
$$f' \to 1, g \to 0, G \to 0 \quad as\ \eta \to \infty. \tag{11}$$

It may be remarked that the boundary layer approximation is not valid in the immediate neighbourhood of the hole. Also the mass transfer parameter f_w will be a constant if the velocity normal to the wall w_w varies as $r^{-\frac{3}{2}}$ as mv is a constant. Also $f_w \gtrless 0$ according to whether it is injection or suction.

The local skin-friction coefficient can be expressed in the form

$$c_f = \frac{2\tau_w}{\rho U^2} = 2^{\frac{1}{2}} Re_r^{-\frac{1}{2}} f_w'' \tag{12}$$

where $\tau_w = -\mu(u_z)_w, Re_r = m/(rv)$

The local heat-transfer coefficient in terms of Nusselt number is given by

$$\text{Nu} = rq_w / [k\,(T_w - T_\infty)] = -2^{-\frac{1}{2}} Re_r^{\frac{1}{2}} g_w' \tag{13}$$

where $q_w = -k(T_z)_w$.

Similarly, the local mass flux of the diffusing species in terms of Sherwood number can be expressed as

$$Sh = r\overline{m}_w / [\rho D(c_w - c_\infty)] = -2^{-\frac{1}{2}} Re_r^{\frac{1}{2}} G_w' \tag{14}$$

where $\overline{m}_w = -\rho D(c_z)_w$.

3. RESULTS AND DISCUSSION

The set of equations (8) to (10) under boundary conditions (11) are non-linear coupled ordinary differential equations so their solutions cannot be obtained in closed form therefore these equations are solved numerically with MATLAB's built – in solver bvp4c.

Numerical calculations have been carried out for concentration of the rarer component of the binary fluid mixture for various values of the parameters f_w, t_d and S_c and are plotted against η in Figures 1-3.

Concentration distribution of the rarer and lighter component of the binary fluid mixture is plotted against η in Figure 1 for various values of the mass transfer parameter f_w (= 4, 5, 6) by taking Ec=1, M=1, Pr=0.7, S_c=0.7 and t_d =0.001 to exhibit the effect of the mass transfer represented by f_w on the species separation. It is found that the concentration of the rarer and lighter component of the binary mixture is more near the surface of the cone and decreases exponentially as η increases to 2.5. Thereafter in the region $\eta > 2.5$ no variation in G is observed. Thus we conclude that the separation of the binary mixture takes place mostly in the region 0< η < 2.5 and thereafter separation is found to be negligible. It is evident from Fig. 1 that the rate of separation can be enhanced by increasing the values of f_w.

Figure 2 displays concentration distribution of the rarer and lighter component of the binary fluid mixture against η for various values of the thermal diffusion number t_d(= 0.001, 0.068, 0.45) by taking Ec=1, M=1, Pr=0.7, S_c=0.7 and $f_w = 4$. It is found from the graph that the concentration of the rarer and lighter component of the binary mixture is more near the surface of the cone and decreases exponentially as η increases to 3.Thereafter in the region $\eta > 3$ no variation in G is observed. Thus we conclude that the separation of the binary mixture

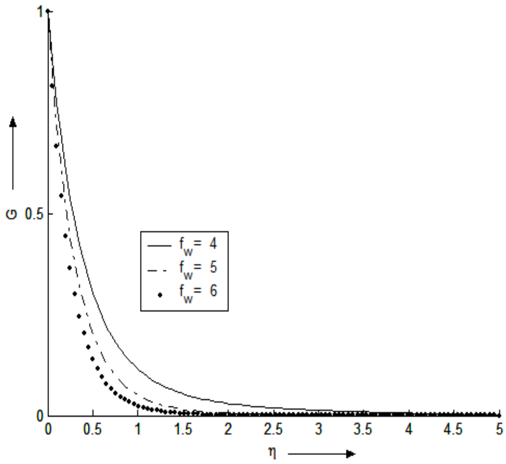

Fig. 1. The graph of G against η Ec=1, M=1, Pr=0.7, S_c = 0.7 and t_d =0.001 for various values of f_w.

takes place mostly in the region 0< η < 2.5 and thereafter separation is found to be negligible. It is evident from Figure 1 that the rate of separation can be enhanced by decreasing the values of t_d .

Figure 3 displaying concentration distribution of the rarer and lighter component of the binary fluid mixture against η for various values of the Schmidt number S_c(= 0.6, 0.7, 0.9) by taking Ec =1, Pr =0.7, M =1, t_d=0.001 and f_w=4 exhibits the effect of the Schmidt number S_c on the species separation. The effect of the parameter S_c is found to be similar to the mass transfer parameter f_w.

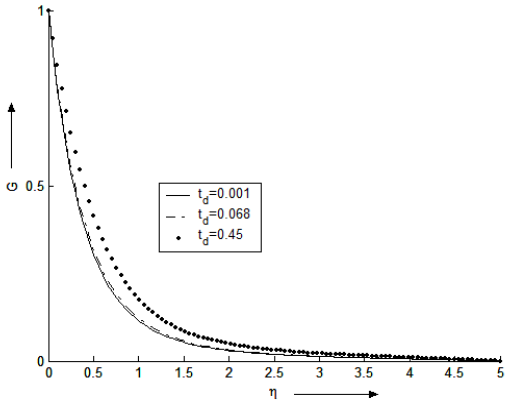

Fig. 2. The graph of G against η Ec=1, M=1, Pr = 0.7, S_c =0.7 and f_w = 4 for various values of t_d.

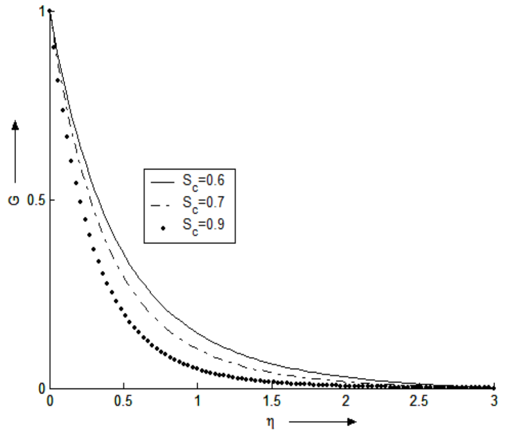

Fig. 3. The graph of G against η Ec=1, Pr = 0.7,M = 1, t_d = 0.001 and f_w =4 for various values of S_c .

From the process of numerical computation, the local skin friction, the Nusselt number and the Sherwood number, which are respectively proportional to $f''(0), -g'(0)$ and $-G'(0)$, are also worked out and their numerical values are presented in a tabular form in Table 1.

Table 1 Numerical values of $f''(0)$, $-g'(0)$ and $-G'(0)$ for Pr = 0.7, Ec =1 and M =1.

f_w	t_d	S_c	$f''(0)$	$-g'(0)$	$-G'(0)$
4	0.001	0.7	5.2292	1.8210	2.5631
5	0.001	0.7	6.0634	2.6248	3.3458
6	0.001	0.7	6.9322	3.3687	4.0810
4	0.001	0.7	5.2292	1.8210	2.5631
4	0.068	0.7	5.2292	1.8210	2.4299
4	0.45	0.7	5.2292	1.8210	1.6567
4	0.001	0.6	5.2291	1.8635	2.1911
4	0.001	0.7	5.2291	1.8635	2.5968
4	0.001	0.9	5.2291	1.8635	3.4177

4. CONCLUSION

In the present investigation the effects of all these parameters are to demix the binary mixture by collecting the rarer and lighter component of the binary fluid mixture near the surface of the cone and throwing the heavier component away from it.

ACKNOWLEDGEMENT

This research work is funded by grants from the UGC, New Delhi, India (File No. 39-43/2010 (SR)) as a Major Research Project awarded to Dr. B. R. Sharma. Kabita Nath is associated with the project as a Project Fellow. The authors are grateful to UGC for providing financial support during the research work.

REFERENCES

Ackerberg, R.C. (1965). The viscous incompressible flow inside a cone, J. *Fluid Mech.* 21, 47-81.

Eswara, A. T and. B. C. Bommaiah (2004). The effect of variable viscosity on laminar flow due to a point sink, *Indian J. Pure Applied Math*. 35(6), 811-815.

Eswara, A. T. , S. Roy and G. Nath (2000). Unsteady MHD forced flow due to a point sink, *Acta Mechanica* 145, 159-172.

Ibrahim, F. S., A. M Elaiw and A. A. Bakr (2008). Effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source and suction, *Communication S in Nonlinear Science and Numerical simulation* 13, 1056-1066.

Mahanti, N. C. and P. Gaur (2009). Effects of varying viscosity and thermal conductivity on steady free convective flow and heat transfer along an isothermal vertical plate in the presence of heat sink. *Journal of Applied Fluid Mechanics* 2(1), 23-28.

Muthucumaraswamy, R. (2002). Effects of chemical reaction on a moving vertical isothermal surface with suction, *Acta Mechanica* 155, 65-70.

Rosenhead, L. (1963). Laminar Boundary Layer, *Oxford University Press*. Oxford.

Sharma, P. R. and G. Singh (2008). Effects of variable thermal conductivity and heat source / sink on MHD flow near a stagnation point on a linearly stretching sheet, *Journal of Applied fluid mechanics* 2(1), 13-21.

Seddeek, M. A. and A. M. Salem (2005). Laminar mixed convection adjacent to vertical continuously stretching sheet with variable viscosity and variable thermal diffusivity, *Heat and Mass transfer* 41, 1048-1055.

Sharma, B. R. and R. N. Singh (2008).

Barodiffusion and thermal diffusion a binary fluid mixture confined between two parallel discs in presence of a small axial magnetic field, *Latin American Applied Research* 38, 313-320.

Sharma, B. R. and. R. N. Singh (2009). Thermal diffusion in a binary fluid mixture confined between two concentric circular cylinders in presence of radial magnetic field, *J. Energy Heat Mass Transfer* 3, 127-38.

Sharma, B. R. and R. N. Singh (2010). Separation of species of a binary fluid mixture confined between two concentric rotating circular cylinders in presence of a strong radial magnetic field, *Heat Mass Transfer* 46, 769-777.

Sharma, B. R. and K. Nath. (2012). The effect of magnetic field on separation of binary mixture of viscous fluids by barodiffusion and thermal diffusion near a stagnation point- a numerical study, *Int. Jour. Mathematical Archive* 3(3), 1118-1124.

Sharma, B. R. and K. Nath (2012). Effect of axial magnetic field on demixing of a binary fluid mixture due to the rotation of a heated sphere, *Int. Journal of Computer Applications* (0975-8887), 53(10), 47-52.

Sharma, B. R., R. N. Singh and Kr. Gogoi, Rupam (2011). Effect of a Strong Transverse Magnetic Field on Separation of Species of a Binary Fluid Mixture in Generalized Couette Flow, *Int. Journal of Applied Engineering Research* 6, 2223-2235.

Sharma, B. R., R. N. Singh, Gogoi, Kr. Rupam and K. Nath. (2012). Separation of species of a binary fluid mixture confined in a channel in presence of a strong transverse magnetic field, *Hem. Ind*. 66(2), 171-180.

Sharma, B. R. and R. N. Singh (2004). Soret effect in generalized MHD Couette flow of a binary mixture, *Bull Cal Math Soc*. 96, 367-374.

Sharma, B. R. and R. N. Singh (2007). Soret effect due to natural convection between heated vertical plates in a horizontal small magnetic field, *Ultra Science* 19, 97-106.

Takhar, H. S., C. D. Surma Devi and G. Nath (1986). MHD flow with heat and mass transfer due to a point sink, *Indian J. Pure Applied Math* 7(10), 1242-1247.