

Weak Nonlinear Double Diffusive Magneto-Convection in a Newtonian Liquid under Gravity Modulation

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(Received January 26, 2014; accepted August 19, 2014)

ABSTRACT

A theoretical analysis of thermo-convective instability in an electrically conducting two component fluid layer is carried out when the gravity field vary with time in a sinusoidal manner. Newtonian liquid is considered between two horizontal surfaces, under a constant vertical magnetic field. The disturbance is expanded in terms of power series of amplitude of convection, which is assumed to be small. We use the linear matrix differential operator method to find the Ginzburg–Landau amplitude equation for the modulated problem. Use the solution of the Ginzburg–Landau equation in quantifying the amount of heat and mass transports in terms of Nusselt and Sherwood numbers. It is found that, the effect of magnetic field is to stabilize the system. Effect of various parameters on the heat and mass transport is also discussed. Further, it is found that the heat and mass transports can be controlled by suitably adjusting frequency and amplitude of gravity modulation.

Keywords: Double diffusive magnetoconvection; Gravity modulation; Weak nonlinear theory.

NOMENCLATURE

A	amplitude of convection	ΔT	temperature difference
d	depth of the fluid layer	ΔS	concentration difference
\vec{g}	acceleration due to gravity	t	time
Q	Chandersekhar number $\mu_m H_b^2 d^2 / \rho_0 \nu v_m$	\vec{q}	fluid velocity
k_c	critical wave number	(x, z)	horizontal, vertical space co ordinates
Nu	Nusselt number	α	coefficient of thermal expansion
Sh	Sherwood number	β	coefficient of solute expansion
p	reduced pressure	ε	perturbation parameter
Pr	Prandtl number ν / κ_T	κ_S	solutal diffusivity
Pm	magnetic Prandtl number, ν_m / κ_T	ω	frequency of modulation
Le	Lewis number, κ_T / κ_S	δ	amplitude of modulation
Ra_T	thermal Rayleigh number	μ	dynamic co-efficient of viscosity of the fluid
$Ra_T = \frac{\alpha g \Delta T d^3}{\nu \kappa_T}$		μ_m	magnetic permeability
Ras	Solutal Rayleigh number,	ν	kinematic viscosity
$Ras = \frac{\beta g \Delta S d^3}{\nu \kappa_S}$		ν_m	magnetic viscosity
S	solute concentration	ρ	fluid density
Ro_c	Critical Rayleigh number,	ψ	Stream function
\vec{H}	magnetic field	ψ^*	dimensionless stream function

Φ	magnetic potential
κ_T	thermal diffusivity
γ	heat capacity ratio
Φ^*	dimensionless magnetic potential
τ	slow time, $\tau = \varepsilon^2 t$

Other symbols

$$\nabla^2 \quad \partial/\partial x^2 + \partial/\partial y^2 + \partial/\partial z^2$$

Subscripts

b	basic state
c	critical
0	reference value

Superscripts

/	perturbed quantity
*	dimensionless quantity

1. INTRODUCTION

Double diffusive convection is an important fluid dynamics phenomenon that involves motions driven by two different density gradients diffusing at different rates. In double-diffusive convection, the buoyancy force is affected not only by the difference of temperature, but also by the difference of concentration of the fluid. Some of examples of double diffusive convection can be seen in oceanography, lakes and underground water, atmospheric pollution, chemical processes, laboratory experiments, modeling of solar ponds, electrochemistry, magma chambers and Sparks, formation of microstructure during the cooling of molten metals, fluid flows around shrouded heat-dissipation fins, migration of moisture through air contained in fibrous insulations, grain storage system, the dispersion of contaminants through water saturated soil, crystal P growth, solidification of binary mixtures, and the underground disposal of nuclear wastes. The study of double-diffusive magneto convection has recently drawn the attention of astrophysicists, geophysicists, oceanographers, engineers and a host of others. The study of magneto-convection in an electrically conducted horizontal fluid layer motivated by astrophysical and geophysical, relate in some way or the other to problems concerning the external constraints like magnetic field, rotation and/or gravity field operative on double-diffusive systems, in particular by observation of sunspots Thomas and Weiss (1992). Convection in planetary cores, stellar interiors and Earth's metallic core occurs in the presence of strong magnetic field. Thompson (1951) and Chandrasekhar (1961) were the first to study the magneto-convection in horizontal fluid layer.

Lortz (1965) was the first to study the effect of magnetic field on double-diffusive convection. His object was to clarify some of the mathematical aspects of stability criterion (Malkus and Veronis 1958) but, his analysis is silent about the detailed study of stability analysis. Stommel et al. (1956) in their experiments explained the diffusion is generally a stabilizing factor in a single-component fluid. But in the case of two-component system it can act to release the potential energy in the component that is heaviest at the top and make the system unstable. Rudraiah and Shivakumara (1984) and Shivakumara (1985) have investigated both linear and nonlinear theory of this problem in detail. They have shown that the magnetic field, under certain conditions makes the system unstable.

They have also investigated the effect of magnetic field on the physically preferred cell pattern. Rudraiah (1986) investigated the interaction between double-diffusive convection and an externally imposed vertical magnetic field in a Boussinesq fluid. Both linear and nonlinear theories have been discussed. The effects of cross-diffusion, rotation and chemical reaction on double-diffusive magneto-convection and pattern selection have been explained. Baines and Gill (1969), Turner (1974), Huppert and Turner (1981), Chen and Johnson (1984) investigated the study of convection in a two and multi-component fluid layer where two scalar fields affect the density distribution. Ozoe and Maruo (1987) have investigated magnetic and gravitational natural convection of melted silicon-two dimensional numerical computations for the rate of heat transfer.

Siddheshwar and Pranesh (1999,2000) examined the effect of a transverse magnetic field on thermal/gravity convection in a weak electrically conducting fluid with internal angular momentum. Starchenko (2006) discussed double diffusion magneto-convection for Earth's type planets. Siddheshwar et al. (2012) performed a local non-linear stability analysis of Rayleigh-Bénard magneto-convection using Ginzburg-Landau equation. They showed that gravity or thermal modulation can be used to enhance or diminish the heat transport in stationary magneto-convection.

Many researchers, under different physical models have investigated thermal instability in a horizontal fluid layer with gravity modulation in the absence of magneto double diffusive convection. Some of them are Gershuni and Zhukhovitskii (1963) and Gresho and Sani (1970) were the first to study the effect of gravity modulation in a fluid layer. Biringen and Peltier (1990) investigated, numerically, the non-linear three dimensional Rayleigh-Bénard problem under gravity modulation, and confirmed the result of Gresho and Sani (1970). Wadih and Roux (1988) presented a study on instability of the convection in an infinitely long cylinder with gravity modulation oscillating along the vertical axis. Saunders et al. (1992) have discussed the effect of gravity modulation on thermosolutal convection in an infinite layer of fluid. Clever et al. (1993) performed a detailed non-linear analysis of Rayleigh-Bénard convection under gravity modulation and presented the stability limits to a much wider region of parameter space. Aniss et

al. (1995, 2000), Rogers et al. (2000, 2005), Bhadauria et al. (2005) showed that the gravitational modulation, which can be realized by vertically oscillating a horizontal fluid layer, acts on the entire volume of liquid and may have a stabilizing or destabilizing effect depending on the amplitude and frequency of the forcing. Boulal et al. (2007) focused attention on the influence of a quasi-periodic gravitational modulation on the convective instability threshold. They predicted that the threshold of convection corresponds precisely to quasi-periodic solutions. Bhadauria et al. (2012) studied thermally or gravity modulated non-linear stability problem in a rotating viscous fluid layer, using Ginzburg-Landau equation for stationary mode of convection. Bhadauria et al. (2013) studied internal heating effects weak non-linear Rayleigh-Bénard convection under gravity modulated, using Ginzburg-Landau equation for stationary mode of convection.

It was observed that most of the above studies considers only linear theory of Rayleigh-Bénard convection by assuming different physical models, which can't be used to quantify heat and mass transfer. It is important to consider external physical parameters for controlling convective instability in a horizontal fluid layer. The modulated gravitational field may change the structural characteristics of the convective flow. Especially convection in a horizontal fluid layer heated from below can be stabilized by vertical oscillation of the layer at suitable frequency and amplitude Gresho and Sani (1970) and imposing vertical magnetic field Rudraiah (1986). Adapting the method of Ginzburg-Landau model (Bhadauria and Kiran 2013a,b, 2014a,b) investigated internal heating effects in an electrically conducting fluid layer under vertical magnetic field and time periodic heating at the boundaries. Bhadauria and Kiran (2014c), investigated the effect of magnetic field modulation on electrically conducting fluid layer under gravity modulation. Bhadauria and Kiran (2014d,e) studied gravity modulation in a nanofluid layer under gravity modulation in the presence of internal heat source. Recently Bhadauria and Kiran (2014f) presented time periodic heating of the boundaries as external controlling mechanism to the convection in an electrically conducting two component fluid layer. It was found that temperature modulation as well as magnetic field can be used effectively to alternate the convection or heat and mass transfer in the system while suitably tuning the frequency and amplitude of modulation. So in our problem we considered such an external controlling convective mechanisms magnetic field and gravity modulation. It is found that magnetic field and gravity modulation together can be used strongly to control convective instability on magneto double diffusive convection.

2. GOVERNING EQUATION

We consider an electrically conducting fluid layer of depth d , confined between two infinitely parallel,

horizontal planes at $z=0$ and $z=d$. Cartesian coordinates have been taken with the origin at the bottom of the fluid layer, and the z -axis vertically upwards given in Fig.1. Under the Boussinesq approximation, the dimensional governing equations for the study of double-diffusive magneto-convection in an electrically conducting fluid layer are given by (Bhadauria et al. 2012, Bhadauria and Kiran 2014f):

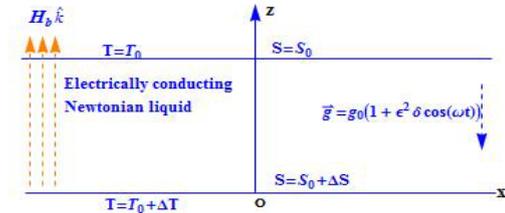


Fig. 1. Physical configuration of problem

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

$$\nabla \cdot \vec{H} = 0, \tag{2}$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} = -\frac{1}{\rho_0} \nabla p + \frac{\rho}{\rho_0} \vec{g} - \frac{\mu}{\rho_0} \nabla^2 \vec{q} + \frac{\mu_m}{\rho_0} \vec{H} \cdot \nabla \vec{H} \tag{3}$$

$$\gamma \frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = \kappa_T \nabla^2 T, \tag{4}$$

$$\frac{\partial S}{\partial t} + (\vec{q} \cdot \nabla) S = \kappa_S \nabla^2 S, \tag{5}$$

$$\frac{\partial \vec{H}}{\partial t} + (\vec{q} \cdot \nabla) \vec{H} - (\vec{H} \cdot \nabla) \vec{q} = \nu_m \nabla^2 \vec{H}, \tag{6}$$

$$\rho = \rho_0 [1 - \alpha(T - T_0) + \beta(S - S_0)], \tag{7}$$

$$\vec{g} = g_0 [1 + \varepsilon^2 \delta \cos(\omega t)] \mathbf{k}. \tag{8}$$

The physical quantities which are mentioned in above equations have their own meanings given in Nomenclature. The externally imposed solutal and thermal boundary conditions are given by

$$T = T_0 \quad z = d$$

$$= T_0 + \nabla T \quad z = 0 \tag{9}$$

$$S = S_0 \quad z = d$$

$$= S_0 + \nabla S \quad z = 0 \tag{10}$$

The basic state is assumed to be quiescent and the quantities in this state are given by:

$$\vec{q}_b = 0, T = T_b(z), S = S_b(z), \tag{11}$$

$$p = p_b(z, t), \rho = \rho_b(z, t);$$

Using Eq. (2.11) in Eqs. (2.1,2.8) and applying the Eqs. (2.9,2.10) we obtain the expressions for basic state temperature and solute as:

$$T_b = T_0 + \Delta T \left(1 - \frac{z}{d}\right), \tag{12}$$

$$S_b = S_0 + \Delta S \left(1 - \frac{z}{d}\right). \tag{13}$$

We assume and impose finite amplitude perturbations on the basic state in the form:

$$\vec{q} = \vec{q}_b + \vec{q}', T = T_b + T', S = S_b + S', p = p_b + p',$$

$$\rho = \rho_b + \rho', \quad \bar{H} = H_b + \bar{H}'. \quad (14)$$

Now substituting Eq. (2.14) into Eqs. (1-8), using stream function ψ as $u' = \frac{\partial \psi}{\partial z}, w' = -\frac{\partial \psi}{\partial x}$; and

$$\text{magnetic potential } \Phi \text{ as } \bar{H}'_x = \frac{\partial \Phi}{\partial z},$$

$$\bar{H}'_z = -\frac{\partial \Phi}{\partial x}; \text{ finally introducing the non}$$

dimensional parameters as $(x', z') = (x^*, z^*),$

$$T' = T^* \Delta T, \quad \psi = \kappa_T \psi^*, \quad S' = S^* \Delta S, \quad t' = \frac{d^2}{\kappa_T} t^*$$

$$\bar{q}' = \frac{\kappa_T}{d^2} \bar{q}^*, \quad \Phi' = d H_b \Phi^*; \text{ then we obtain the}$$

following non dimensionalized equations: (for simplicity we drop asterisk)

$$\begin{aligned} -\nabla^4 \psi + g_m Ra_T \frac{\partial T}{\partial x} - g_m Ras \frac{\partial S}{\partial x} - QPm \frac{\partial}{\partial z} \nabla^2 \Phi \\ = \frac{-1}{Pr} \frac{\partial \nabla^2 \psi}{\partial t} + \frac{1}{Pr} \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, z)} \end{aligned} \quad (15)$$

$$-QPm \frac{\partial(\Phi, \nabla^2 \Phi)}{\partial(x, z)},$$

$$\frac{\partial \psi}{\partial x} - \nabla^2 T = -\frac{\partial \psi}{\partial t} + \frac{\partial(\psi, T)}{\partial(x, z)}, \quad (16)$$

$$\frac{\partial \psi}{\partial x} - \frac{1}{Le} \nabla^2 S = -\frac{\partial \psi}{\partial t} + \frac{\partial(\psi, S)}{\partial(x, z)}, \quad (17)$$

$$-\frac{\partial \psi}{\partial z} - Pm \nabla^2 \Phi = -\frac{\partial \Phi}{\partial t} + \frac{\partial(\psi, \Phi)}{\partial(x, z)}. \quad (18)$$

Where $g_m = 1 + \varepsilon^2 \delta \cos(\omega t)$; the non dimensionalized numbers in the above equations are given in Nomenclature. We assume small variations of time and re-scaling it as $\tau = \varepsilon^2 t$; to study the stationary convection of the system. The considered boundary conditions to solve the above system of Eqs. (2.15-2.18) is:

$$\psi = \nabla^2 \psi = 0; \quad \Phi = D\Phi = 0 \text{ at } z=0 \text{ and } z=1 \quad (19)$$

where $D = d / dz$.

3. FINITE AMPLITUDE EQUATION AND HEAT TRANSPORT FOR THE STATIONARY INSTABILITY

We now introduce the following asymptotic expansions in Eqs. (2.15-2.18):

$$Ra_T = R0c + \varepsilon^2 R_2 + \dots; \quad (20)$$

$$\psi = \varepsilon \psi_1 + \varepsilon^2 \psi_2 + \dots; \quad (21)$$

$$T = \varepsilon T_1 + \varepsilon^2 T_2 + \dots; \quad (22)$$

$$S = \varepsilon S_1 + \varepsilon^2 S_2 + \dots; \quad (23)$$

$$\Phi = \varepsilon \Phi_1 + \varepsilon^2 \Phi_2 + \dots; \quad (24)$$

where $R0c$ is the critical value of the Rayleigh number at which the onset of convection takes place in the absence of gravity modulation. Now we solve the above system Eqs. (2.15-2.18):

for different orders of ε :

At the lowest order we obtain the solutions according to the boundary conditions given in Eq. (2.19) as:

$$\psi_1 = A \sin k_c x \sin \pi z, \quad (25)$$

$$T_1 = -\frac{k_c}{\delta^2} A \cos k_c x \sin \pi z, \quad (26)$$

$$S_1 = -\frac{k_c}{\delta^2} Le A \cos k_c x \sin \pi z, \quad (27)$$

$$\Phi_1 = \frac{\pi}{Pm \delta^2} A \sin k_c x \cos \pi z, \quad (28)$$

where $\delta^2 = k_c^2 + \pi^2$. The critical value of the Rayleigh number for the onset of magneto double diffusive convection in the absence of gravity modulation is:

$$R0c = \frac{\delta^2(\delta^4 + Q\pi^2) + RasLe k_c^2}{k_c^2}. \quad (29)$$

In the absence of magnetic field and $Ras=0$, we obtain the classical results obtained by Chandrasekhar (1961).

At the second order we obtain the solutions as:

$$\psi_2 = 0, \quad (30)$$

$$T_2 = -\frac{k_c^2}{8\pi\delta^2} A^2 \sin 2\pi z, \quad (31)$$

$$S_2 = -\frac{k_c^2 Le^2}{8\pi\delta^2} A^2 \sin 2\pi z, \quad (32)$$

$$\Phi_2 = -\frac{\pi^2}{8k_c Pm^2 \delta^2} A^2 \sin 2k_c x. \quad (33)$$

The horizontally averaged Nusselt Nu and Sherwood Sh numbers, for the stationary mode of double-diffusive magneto-convection (the mode considered in this problem) are given by:

$$Nu = 1 + \frac{k_c^2}{4\delta^2} A^2, \quad (34)$$

$$Sh = 1 + \frac{k_c^2 Le^2}{4\delta^2} A^2. \quad (35)$$

The above results are same as obtained by (Siddheswar et al. 2012, Bhadauria et al. 2013a, b). For existing the solution of third order system under solvability condition (Siddheswar et al. 2012, Bhadauria and Kiran. 2013a,b; 2014a-c), we obtain the following an amplitude equation (Ginzburg-Landau equation)

$$B_1 \frac{\partial A}{\partial \tau} - B_2 A + B_3 A^3 = 0, \quad (36)$$

where

$$B_1 = \frac{\delta^2}{Pr} + \frac{R0c k_c^2}{\delta^4} - \frac{Q\pi^2}{Pm \delta^2} - \frac{Ras k_c^2 Le^2}{\delta^4}; \quad (37)$$

$$B_2 = \frac{R0c k_c^2}{8\delta^2}$$

$$\left(\frac{R_2}{R0c} + \delta \cos(\omega \tau) - \frac{Le Ras}{R0c} \delta \cos(\omega \tau) \right); \quad (38)$$

$$B_3 = \frac{R0ck_c^4}{8\delta^4} + \frac{Q\pi^4k_c^2}{2Pm^2\delta^2} - \frac{Rask_c^4Le^3}{8\delta^4}. \quad (39)$$

4. ANALYTICAL SOLUTION FOR UN-MODULATED CASE

In the case of unmodulated fluid layer, the above amplitude equation can be written as:

$$B_1 \frac{\partial A_u}{\partial \tau} - B_2 A_u + B_3 A_u^3 = 0, \quad (40)$$

where B_1, B_3 take the form as in modulated system,

but $B_2 = \frac{R_2 k_c^2}{8\delta^2}$; the solution of Eq. (4.1), is given

by:

$$A_u = \frac{1}{\sqrt{B_3 / 2B_2 + ce^{-2A_2/A_1}}}; \quad (41)$$

where c is a constant, it can be calculated for given suitable initial condition. The horizontal averaged Nusselt and Sherwood numbers in this case is obtained from Eq. (3.15,3.16), by using the value of A_u in the place of A : The amplitude equation given in Eq. (3.17) is Bernoulli equation and obtaining its analytical solution is difficult, due to its non-autonomous nature. So that it has been solved numerically using the in-built function `NDSolve` Mathematica8, subjected to the initial condition $A(0) = a0$; where $a0$ is the chosen initial amplitude of convection. In our calculations we may use $R_2 = R0c$; to keep the parameters to the minimum.

5. RESULTS AND DISCUSSION

External regulation of convection of Rayleigh-Benard convection is important to study the thermal instability in a fluid layer. In this present paper we have consider two such candidates, namely vertical magnetic field and gravity modulation for either enhancing or inhibiting convective heat or mass transports as is required by a real applications (Thermal and Engineering sciences). This paper deals with double-diffusive magnetoconvection under gravity modulation by using a non-autonomous Ginzburg-Landau equation. It is necessary to consider a weak nonlinear theory to study heat and mass transfer, which is not possible by the linear theory. We consider the direct mode ($\kappa_S / \kappa_T < 1$, otherwise Hopf mode) in which the salt and heat make opposing contributions $\kappa_T \neq \kappa_S$. We also consider the effect of gravity modulation to be of order $O(\epsilon^2)$ this leads to small amplitude of modulation. Such an assumption will help us in obtaining the amplitude equation of convection in a rather simple and elegant manner and is much easier to obtain than in the case of the Lorenz model.

Before discussing the results obtained in the present analysis, we would like to make some comments on

the various aspects of the problem, such as:

1. The need for nonlinear stability analysis:
2. The relation of the problem to a real application and:
3. The selection of all dimensionless parameters utilized in computations, and:
4. Consideration of numerical values for different parameters.

The parameters that arise in our problem are $Q, Pr, Le, Pm, Ras, \theta, \delta, \omega$, these parameters influence the convective heat and mass transfer. The first five parameters are related to the fluid layer, and the last three are concerning to the external mechanism of controlling convection. The effect of electrical conductivity and magnetic field comes through Pm, Q . There is the property of the fluid coming into picture through Prandtl number Pr . Also the fluid layer is not considered to be highly viscous, therefore only moderate values of Prandtl number Pr is taken for calculations. The effect of gravity modulation is represented by an amplitude δ of modulation, which takes the values between 0 to 0.3, since we are studying the effect of small amplitude modulation on the heat and mass transport. Further; as the effect of low frequencies on the onset of convection as well as on the heat and mass transport is maximum therefore, the modulation of gravity is assumed to be of low frequency. It is important at this stage to consider the effect of $Q, Pr, Le, Pm, Ras, \theta, \delta, \omega$ on the current problem. Heat and Mass transfer quantified by the Nusselt and Sherwood numbers which are given in Eqs. (3.15-3.16).

The Figs. 2, 3 show that, the individual effect of each non-dimensional parameter on Heat and Mass transfer. Here we present our results on Heat, Mass transfer with respect to Nusselt and Sherwood numbers.

1. The Figs. 2-3a shows that, the effect of Chandrasekhar number Q which is ratio of Lorentz force to viscous force is to delay the onset of convection, hence heat and mass transfer. So increase in Chandrasekhar number Q decreases in Nu and Sh , which means that an externally imposed magnetic field stabilizes the system these are the results where the vertical magnetic field (Kaaddeche et al. 2003) has stabilizing effect. Since the applied magnetic field is working in vertical direction, therefore the direction of the Lorentz force will be horizontally therefore, the applied magnetic field will not allow moving the fluid vertically as free as without magnetic field. Hence the magnetic field will have stabilizing effect on the system. Increasing magnetic field will increase Q , and the system will be more stabilizing.

2. The effect of Prandtl number Pr is to advances the convection and hence heat and mass transfer for lower values of Pr , given in Figs. 2-3b.

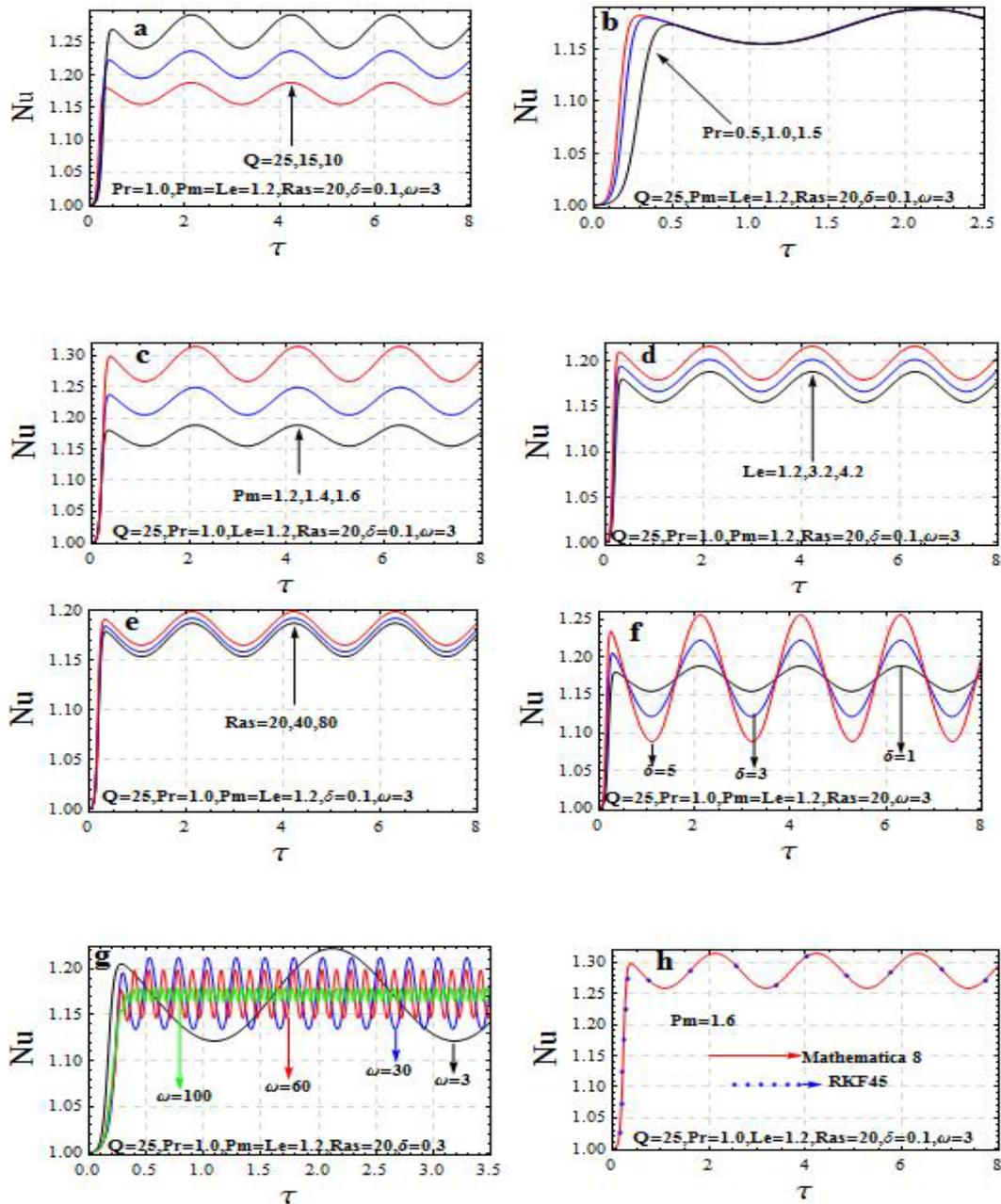


Fig. 2. Nu versus τ (a) Q (b) Pr (c) Pm (d) Le (e) Ras (f) δ (g) ω (h) Comparison.

3. The effect of Magnetic Prandtl Pm and Lewis Le numbers is to advances the convection and hence heat and mass transfer. Hence both Pm , Le numbers has destabilizing effect of the system given in Figs. 2c-d and 3c-d.

4. The effect of solutal Rayleigh number Ras is to increase Nu and Sh so that heat and mass transfer. Hence it has destabilizing effect given by the Figs. 2-3e. Though the presence of a stabilizing gradient of solute will prevent the onset of convection, the strong finite-amplitude motions, which exist for large Rayleigh numbers, tend to mix the solute and redistribute it so that the interior layers of the fluid are more neutrally stratified. As a consequence, the inhibiting effect of the solute

gradient is greatly reduced and hence fluid will convect more and more heat and mass when Ras is increased (Stommel et al. 1956 and Bhadauria and Kiran 2014f).

5. The increment in amplitude of g-jitter δ leads to increment in Nu , Sh hence heat and mass transfer given in Figs. 2-3f, and also the increment in ω shortens the wavelength and decreases in magnitude of Nu , Sh and hence heat and mass transfer given in Figs. 2-3g, (Bhadauria and Kiran 2014f).

6. From Figs. 4a-b we observe that Q has strongly stabilizing effect (Siddheshwar et al.2012,

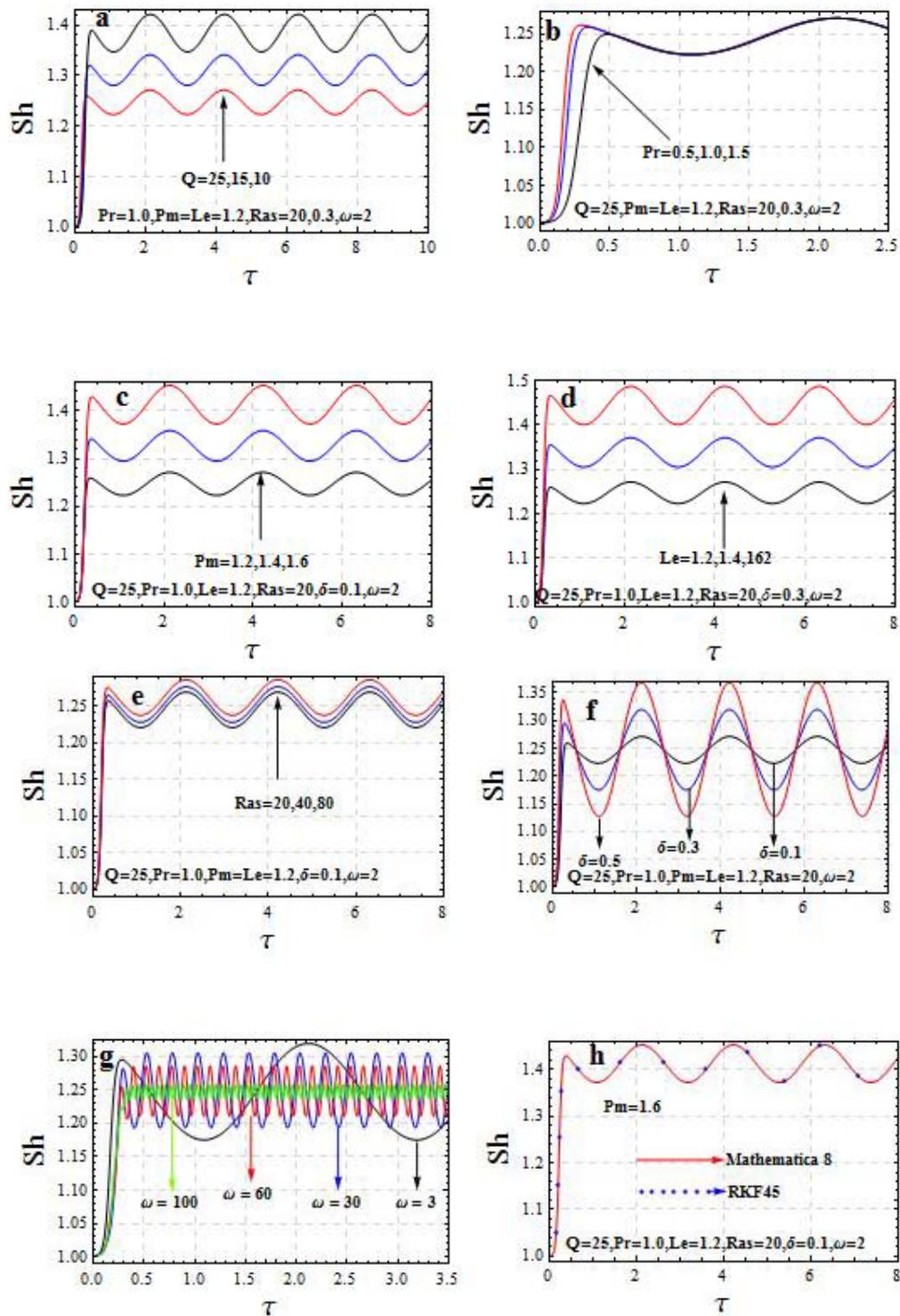


Fig. 3. Nu versus τ (a) Q (b) Pr (c) Pm (d) Le (e) Ras (f) δ (g) ω (h) Comparison.

Bhadauria and Kiran 2014b,f).

$$[Nu / Sh]_{Q=0} > [Nu / Sh]_{Q \neq 0}$$

7. In Fig . 4a-b the presence of magnetic field ($Q \neq 0$) strongly stabilizing with low values of Nu and Sh leads less heat and mass transfer, but opposite effect when ($Q = 0$).

8. Fig . 4c-d, deals with the plots with or without

g-jitter, in the case of unmodulated system (without g-jitter) we found an amplitude of convection analytically in Eq. (4.2), and obtained Nusselt and Sherwood numbers, depicted a plot Nu, Sh versus τ It is clear from the Figs. 4c-d that, the average of Nu and Sh has oscillatory behavior in modulated system but in unmodulated system for small values of

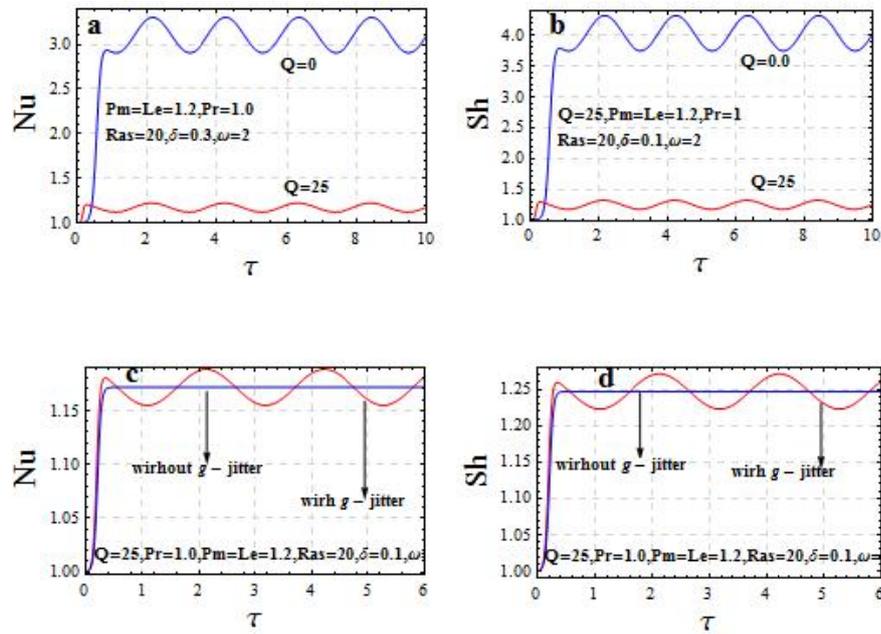


Fig. 4. Comparison with or without Q and modulated or unmodulated system

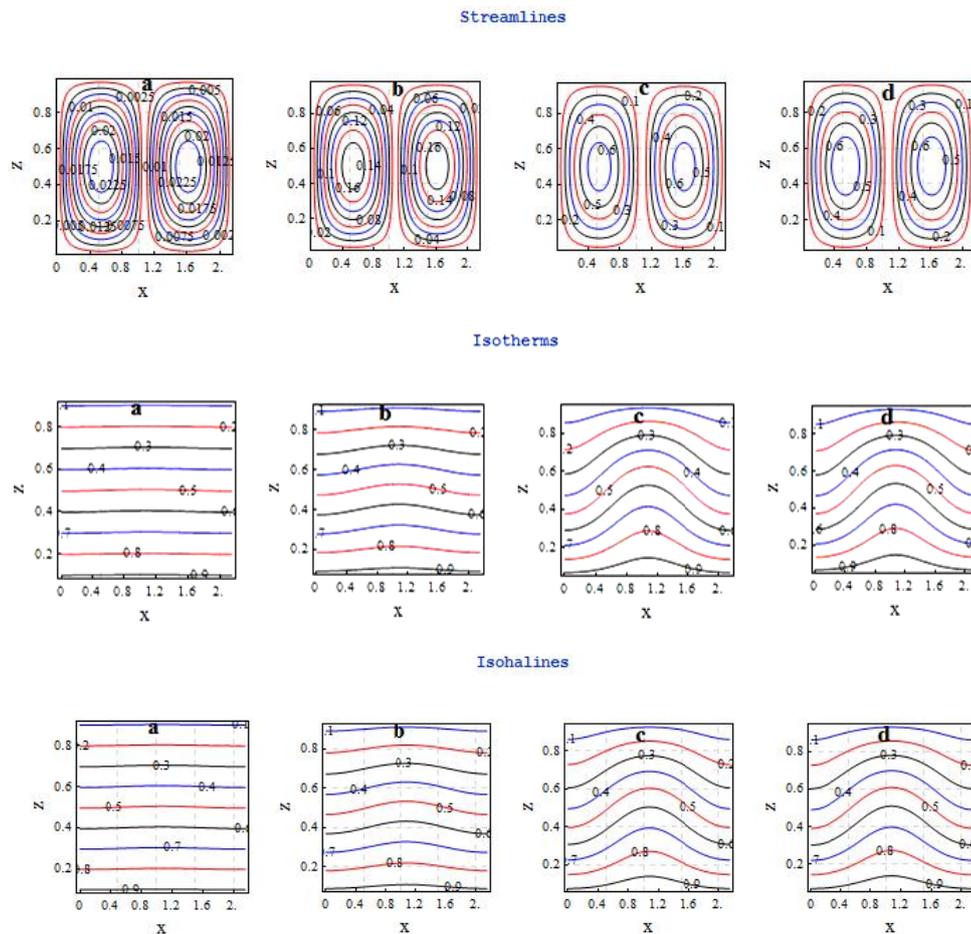


Fig. 5. For $\epsilon = 0.5: Q=25: Pr = 1: Ras = 20: Pm = Le = 1.2: \delta = 0.1: \omega = 3.0$

time it varies and becomes steady further values of time.9. Variation of stream lines, isotherms, at different instant of time is shown graphically in

Figs. 5, 6, with or without Q . From the Fig. 5 it is clear that the magnitudes of streamlines increases as time increases slowly when $Q \neq 0$

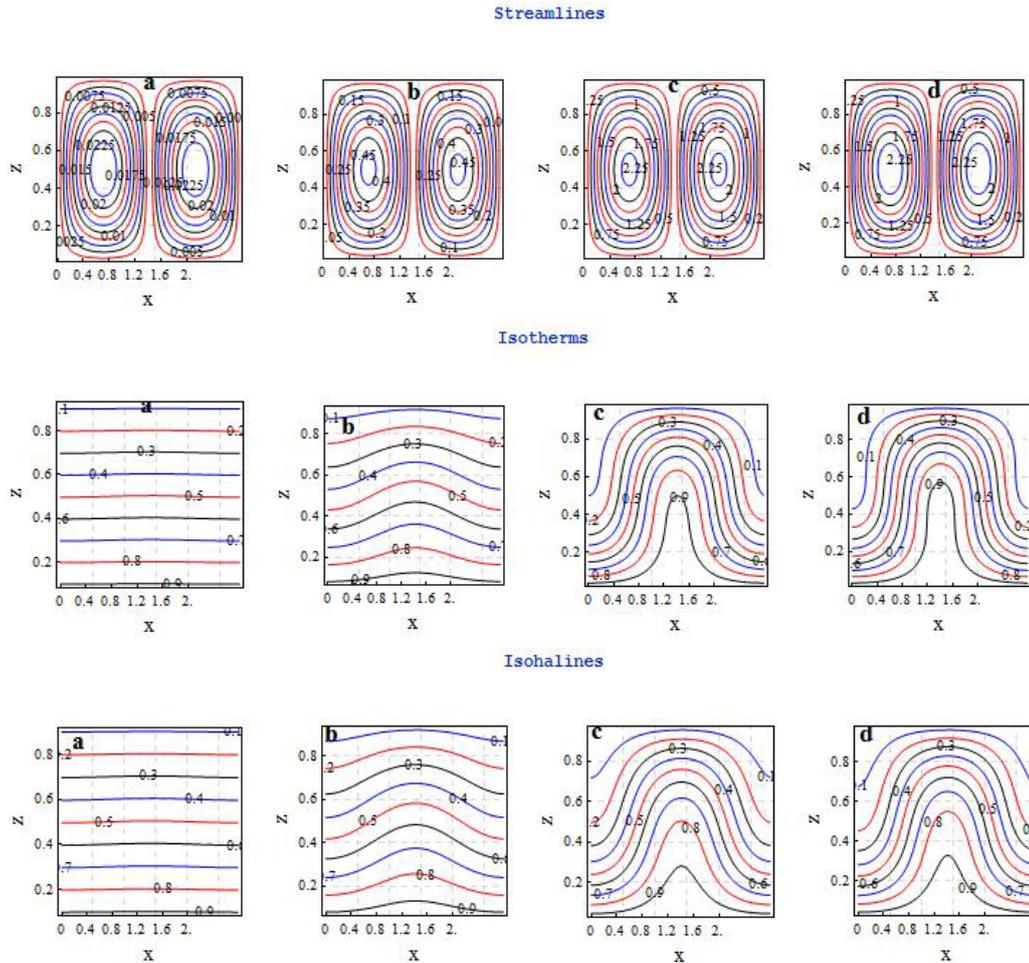


Fig. 6. For $\varepsilon = 0.5$: $Q=0$: $Pr = 1$: $Ras = 20$: $Pm = Le = 1.2$: $\delta = 0.1$: $\omega = 3.0$.

further increasing time achieves steady state. Similarly isotherms drawn at different instant of time and it is found that, from the graph initially isotherms are at parallel state showing that heat transport is only by conduction and as time increases isotherms starts oscillating slowly, showing that convective regime is in place and then forms contour showing that as time increases convection contributes in heattransport. The similar behavior is observed for streamlines isotherms when $Q = 0$ in Fig. 6, the magnitude of streamlines increases very fast and isotherms loses it's evenness fast, showing that convection advances and heat and mass transfer more.

10. To check the validity of our results, we have compared our results (preset) with the results obtained by RKF45 (step size $h=0.01$ and $A(0)=0.05$), it is observed that the good approximation in both results presented in the Figs. 2-3h.

11. The results of this work can be summarized as follows from the Figs. 2, 4.

1. $[Nu / Sh]_{Q=25} < [Nu / Sh]_{Q=15} < [Nu / Sh]_{Q=10}$
2. $[Nu / Sh]_{Pr=0.5} < [Nu / Sh]_{Pr=1.0} < [Nu / Sh]_{Q=1.5}$

3. $[Nu]_{Le=1.2} < [Nu]_{Le=3.2} < [Nu]_{Le=4.2}$
4. $[Sh]_{Le=1.2} < [Sh]_{Le=1.4} < [Sh]_{Le=1.6}$
5. $[Nu / Sh]_{Pm=1.2} < [Nu / Sh]_{Pm=1.4} < [Nu / Sh]_{Pm=1.6}$
6. $[Nu / Sh]_{Ras=20} < [Nu / Sh]_{Ras=40} < [Nu / Sh]_{Ras=80}$
7. $[Nu / Sh]_{\delta=0.1} < [Nu / Sh]_{\delta=0.3} < [Nu / Sh]_{\delta=0.5}$
8. $[Nu / Sh]_{\omega=100} < [Nu / Sh]_{\omega=60} < [Nu / Sh]_{\omega=30} < [Nu / Sh]_{\omega=3}$

CONCLUSIONS

The effect of gravity modulation on weak nonlinear double diffusive magneto convection has been analyzed by using Ginzburg-Landau equation. The effect of various parameters on the system is discussed in detail. The following conclusions are drawn.

1. The effect of Chandrasekhar number Q is to stabilize the system.
2. The effect of increasing Pr for lower values there is enhancement in the heat and mass transfer further increment in it no effect is found (Bhadauria et al. 2013, Bhadauria and Kiran 2014a,f).

3. The effect of increasing Le , Pm , Ras is found to increase in Nu and Sh thus increasing heat and mass transfer Bhadauria and Kiran (2014f).
4. The effect of increasing δ is to increase the value of Nu and Sh , hence heat and mass transfer.
5. The effect of increasing ω is to decrease the value of Nu and Sh , hence heat and mass transfer.
6. The natures of Nu and Sh remain oscillatory.
7. Initially when τ is small, the values of Nusselt and Sherwood numbers start with 1, corresponding to the conduction state. However as τ increases, Nu and Sh also increase, thus increasing the heat and mass transfer.
8. The modulated system has an oscillatory behavior so the heat and mass transfer but steady state in unmodulated system so heat and mass transfer.
9. The effect of magnetic field is to stabilize the system.

ACKNOWLEDGMENT

This work was done during the lien sanctioned to the author B.S Bhadauria by Banaras Hindu University, Varanasi to work as professor of Mathematics at Department of Applied Mathematics, School for Physical Sciences, Babasaheb Bhimrao Ambedkar Central University, Lucknow, India. The author B.S.B. gratefully acknowledges Banaras Hindu University, Varanasi for the same. Further, the author Palle Kiran gratefully acknowledges the financial assistance from Babasaheb Bhimrao Ambedkar University as a research fellowship.

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