

# Effect of Sparse Distribution Pores in Thermohaline Convection in a Micropolar Ferromagnetic Fluid

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## ABSTRACT

Thermoconvective instability in multi-component fluids has wide range of applications in heat and mass transfer. This paper deals with the theoretical investigation on a horizontal fluid layer of micropolar ferromagnetic fluid heated from below and salted from above saturating a porous medium subjected to a transverse uniform magnetic field using Brinkman model. The salt is a ferromagnetic salt which modifies the magnetic field established. The effect of salinity has been included in the magnetization and density of the ferromagnetic fluid. A theory of linear stability analysis and normal mode technique have been carried out to analyze the onset of convection for a fluid layer contained between two free boundaries for which exact solution is obtained and the stationary and oscillatory instabilities have been carried out for various physical quantities. The results are depicted graphically and the stabilizing and destabilizing behaviors are studied.

**Keywords:** Thermohaline convection; Porous medium; Micropolar ferromagnetic fluid; Brinkman model.

## NOMENCLATURE

$a$	particle radius ( $m$ )	$S_a$	average salinity of the lower and upper surfaces of fluid layer ( $kg$ )
$B$	magnetic induction ( $T$ )	$T$	temperature (K)
$C_{v,H}$	effective heat capacity at constant volume and magnetic field ( $kJ/m^3K$ )	$\alpha_t$	coefficient of thermal expansion ( $K^{-1}$ )
$C_s$	specific heat of solid (porous matrix) material	$\alpha_s$	analogous solvent coefficient of expansion ( $K^{-1}$ )
$d$	thickness of the fluid layer ( $m$ )	$\beta_t$	uniform temperature gradient ( $Km^{-1}$ )
$D/Dt$	convective derivative ( $D/Dt = \partial/\partial t + \vec{q} \cdot \nabla$ ) ( $s^{-1}$ )	$\beta_s$	uniform concentration gradient ( $kgm^{-1}$ )
$g$	gravitational acceleration ( $0, 0, -g$ ) ( $ms^{-2}$ )	$\eta$	shear kinematic viscosity coefficient
$H$	magnetic field ( $amp/m$ )	$\zeta$	coupling viscosity coefficient or vortex viscosity
$k$	permeability of the porous medium ( $m^2$ )	$\lambda'$	bulk spin viscosity coefficient
$K_I$	thermal diffusivity ( $W/mK$ )	$\eta'$	shear spin viscosity coefficient
$K_s$	concentration diffusivity ( $W/mkg$ )	$\delta$	micropolar heat conduction coefficient
$k_0$	Resultant wave number ( $k_0 = \sqrt{k_x^2 + k_y^2}$ ) $m^{-1}$	$I$	moment of inertia
$k_x, k_y$	Wave number in the $x$ and $y$ direction ( $m^{-1}$ )	$\mu_0$	magnetic permeability of vacuum
$K$	pyromagnetic coefficient ( $\equiv -(\partial M/\partial T)_{H_0, T_0}$ ) ( $amp/mK$ )	$\mu$	dynamic viscosity ( $kgm^{-1}s^{-2}$ )
$K_1$	thermal conductivity ( $W/mK$ )	$\nu$	kinematic viscosity of a fluid ( $m^2/s$ )
$K_2$	salinity magnetic coefficient ( $\equiv (\partial M/\partial S)_{H_0, T_0}$ ) ( $amp/m kg$ )	$\theta$	perturbation in temperature (K)
$M$	magnetization ( $Ampm^{-1}$ )	$\rho_0$	mean density of the clean fluid ( $kgm^{-3}$ )
$M_0$	mean value of the magnetization at $H = H_0$ and $T = T_0$ .	$\rho$	density of the fluid ( $kgm^{-3}$ )
$P$	hydrodynamic pressure ( $N/m^2$ )	$\sigma$	growth rate ( $s^{-1}$ )
$q$	velocity of the ferrofluid ( $u, v, w$ ) ( $ms^{-1}$ )	$\phi$	magnetic scalar potential ( $Amp$ )
$S$	solute concentration ( $kg$ )	$\varphi$	viscous dissipation factor containing second order terms in velocity
$T_a$	average temperature of the lower and upper surfaces of fluid layer (K)	$\chi$	magnetic susceptibility ( $\equiv (\partial M/\partial H)_{H_0, T_0}$ )

$t$  time (s)

$\nabla$  vector differential operator  
 $\omega$  microrotation

## 1. INTRODUCTION

One of the most important features of colloidal suspension of magnetic nanoparticles, known as ferrofluids, is the relative change of their viscosity with changing magnetic field. Magnetic fluid has been used in a wide variety of applications for many years by NASA in 1960s for controlling liquids in space, damping system, ball bearings, avionics and lubrications. Such types of fluids have several applications like mechanical engineering, analytical instrumentation, heat transfer, electronic devices, aerospace, etc and are widely used in rotating X-ray tubes and sealing of computer hard disk drives. These are used as lubricants in bearing and dumpers. In biomedicine field, there is an idea to use ferrofluids for cancer treatment by heating the tumor soaked in ferrofluids by means of an alternating magnetic field.

Micropolar fluids are fluids with internal structures in which coupling between the spin of each particle and the macroscopic velocity field. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium, and they are important to engineers and scientists working with hydrodynamic fluid problems and phenomena. A general theory of micropolar fluids has been presented by Eringen (1964, 1980). Eringen (1966) introduced the theory of Micropolar fluid in order to describe some physical systems which do not satisfy the Navier–Stokes equations. The equations governing the micropolar fluid involve a spin vector and microinertia tensor in addition to the velocity vector. This theory is used to explain the flow of colloidal fluids, animal bloods and so on. Kazakia and Ariman (1971) and Eringen (1972) have developed the generalization of the theory of micropolar fluid including thermal effects.

The study of fluid flow through a porous medium is of considerable interest due to its natural occurrence and importance in many problems of engineering and technology like porous bearing, porous layer insulation consisting of solid and pores, porous rollers, etc. Additionally, these fluid flows are applicable to bio-mathematics particularly in the study of blood flow in lungs, arteries, cartilage, etc. The stability flow of a fluid through a porous medium taking into account the Darcy resistance was considered by Lapwood (1948) and Wooding (1960).

The study about a fluid heated from below in a porous medium is motivated both theoretically and also in engineering applications. An authoritative introduction to excellent reviews of convection of ferrofluid has been discussed in monograph by Rosensweig (1985) and the study of the effect of magnetization yields interesting information. Usually, this magnetization is a function of magnetic field, temperature and density of the fluid.

This in presence of a gradient of magnetic field gives convection in ferromagnetic fluids which is known as ferroconvection and is similar to Bénard convection in ordinary fluids (Chandrasekhar (1961)). Convective instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of uniform vertical magnetic field has been considered by Finlayson (1970). Further, Vaidyanathan *et al.* (1991) gave the convective instability of ferrofluid through a porous medium of large permeability and mentioned that stationary convection can occur and oscillatory convection cannot occur by use of Brinkman number. This work has been extended to an anisotropic porous medium by Sekar *et al.* (1996) and Vaidyanathan *et al.* (2002) modified the above work with use of Darcy model.

The interesting situation arises from both a geophysical and a mathematical point of view when the layer is simultaneously heated from below and salted from above. The buoyancy force can arise not only from density difference due to variations in temperature but also those due to variations in solute concentration. Double diffusive convection in fluid in a porous media is also of interest in geophysical system, electrochemistry, chemical technology, ground water hydrology, biomechanics, soil science and astrophysics. The thermohaline convection (double diffusive convection) in a layer of fluid heated from below and subjected to a stable salinity gradient has been investigated by Veronis (1965). The survey of double diffusive convection in a porous medium given in third edition of Nield and Bejan (2006) and the double diffusive instability that occurs when a solution of a slowly diffusing protein in layered over a denser solution of more rapidly diffusing sucrose has been explained by Brakke (1955). Vaidyanathan *et al.* (1995, 1997) illustrated ferro thermohaline convection in the presence and absence of a porous medium of sparse distribution of a two component ferroconvective system.

Micropolar ferromagnetic fluid saturating a porous medium subjected to a transverse uniform magnetic field has been analysed by Sunil and Pavan Kumar Bharti (2006). The thermosolutal convection of micropolar fluids in Hydromagnetics in a porous medium has been studied by V. Sharma and S. Sharma (2000) and they found that Rayleigh number increases with magnetic field and solute parameter. Sunil *et al.* (2004, 2005) have studied thermosolutal convection in a ferromagnetic fluid in the presence and absence of porous medium and double diffusive convection in a micropolar ferromagnetic fluid in a porous and non-porous medium have been analyzed by Sunil *et al.* (2007, 2007a). Here, they investigated that the stabilizing effect of stable solute gradient. Ryskin *et al.* (2003) attempted to study Soret-driven convection in ferrofluids using nonlinear analysis. Vaidyanathan *et al.* (2005) and Sekar *et al.* (2013) attempted to

study the Soret effect due to thermoconvective instability in a ferrofluid by use of Brinkman and Darcy models and Sekar *et al.* (2006) further studied the analysis to the condition of a porous medium of ferroconvective instability of multi-component fluid heated from below and salted from above using Brinkman model. Very more recently, the presence and absence of magnetic field dependent viscosity and rotation on Soret-driven ferrothermohaline convection in an anisotropic porous medium have studied by Sekar and Raju (2013) and Sekar *et al.* (2013a). The effect of rotation on Soret driven ferro thermoconvective instability in the presence of an anisotropic porous medium with uniform magnetic field has analyzed by Sekar *et al.* (2013b) and temperature dependent viscosity is studied on ferrothermohaline convection with Soret effect in a porous medium by Sekar and Raju (2014). Sekar and Raju (2014a) analyzed the effect of magnetic field dependent viscosity on Soret driven ferrothermohaline convection in an anisotropic porous medium.

Keeping in mind the importance of ferromagnetic fluids in different field of applications and in view of the above analyses, we intend to extend our investigation to the problem of thermohaline convection in Eringen’s micropolar fluid saturating a Brinkman porous medium with uniform magnetic field. The linear stability analysis is carried out to study the onset of ferroconvection both by stationary as well as oscillatory instabilities. It is attempted to study the effect of salinity gradient on micropolar ferromagnetic fluid heated from below and salted from above in the presence of a porous medium of large permeability. The understanding of these micropolar ferromagnetic fluid stability problems plays an important role in microgravity environmental applications.

## 2. MATHEMATICAL FORMULATION OF PROBLEM

An infinite horizontal layer of thickness ‘*d*’ of an electrically non-conducting incompressible thin micropolar ferromagnetic fluid heated from below and salted from above saturating a porous podium is considered. The temperature and salinity at the bottom and top surfaces  $z = \mp d/2$  are  $T_0 \pm (\Delta T)/2$  and  $S_0 \mp (\Delta S)/2$ , respectively and a uniform temperature gradient  $\beta_T (= |dT/dz|)$  and a uniform salinity gradient  $\beta_S (= |dS/dz|)$  are maintained (see Fig.1). The temperature gradient thus maintained is qualified as adverse since, on the account of thermal expansion, the fluid at the bottom will be lighter than the fluid at the top and this is a top-heavy arrangement, which is potentially unstable. On the other hand, the heavier salt at the upper part of the layer has exactly the opposite effect and this acts to prevent motion through convection overturning. Thus, these two physical effects are competing against each other. Here both the boundaries are taken to be free and perfect conductors of heat and salt. The gravity field  $\mathbf{g} = (0, 0, -g)$  and uniform vertical magnetic field intensity

$\mathbf{H} = (0, 0, H_0)$  pervade the system. The fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity  $\varepsilon$  and medium permeability  $k$ . The angular velocity field of rotation of particles is introduced. Correspondingly, only one (vector) equation is added-it represents the conservation of the angular momentum.

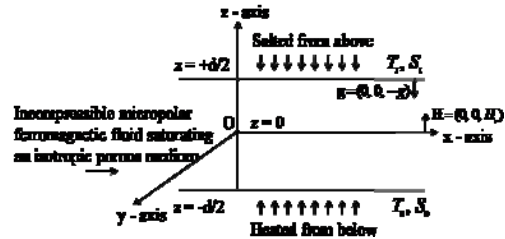


Fig. 1. Geometrical configuration.

### 2.1 Basic Equations

The continuity equation for an incompressible fluid is

$$\nabla \cdot \mathbf{q} = 0 \tag{1}$$

The momentum and internal angular momentum equations for a Brinkman model are

$$\left. \begin{aligned} \rho_0 \left( \frac{\partial}{\partial t} + \frac{1}{\varepsilon} (\mathbf{q} \cdot \nabla) \right) \mathbf{q} &= -\nabla p + \rho \mathbf{g} \\ &+ \nabla \cdot (\mathbf{H}\mathbf{B}) - \frac{1}{k} (\zeta + \eta) \mathbf{q} + 2\zeta (\nabla \times \boldsymbol{\omega}) + \frac{1}{\varepsilon} (\zeta + \eta) \nabla^2 \mathbf{q} \end{aligned} \right\} \tag{2}$$

$$\left. \begin{aligned} \rho_0 I \left( \frac{\partial}{\partial t} + \frac{1}{\varepsilon} (\mathbf{q} \cdot \nabla) \right) \boldsymbol{\omega} &= 2\zeta \left( \frac{1}{\varepsilon} \nabla \times \mathbf{q} - 2\boldsymbol{\omega} \right) + \mu_0 (\mathbf{M} \times \mathbf{H}) \\ &+ (\lambda + \eta') \nabla (\nabla \cdot \boldsymbol{\omega}) + \eta' \nabla^2 \boldsymbol{\omega} \end{aligned} \right\} \tag{3}$$

The temperature equation for an incompressible micropolar ferromagnetic fluid is

$$\left. \begin{aligned} \left[ \rho_0 C_{v,H} - \mu_0 \mathbf{H} \cdot \left( \frac{\partial \mathbf{M}}{\partial T} \right)_{v,H} \right] \frac{DT}{Dt} \\ + (1 - \varepsilon) \rho_s C_s \left( \frac{\partial T}{\partial t} \right) + \mu_0 T \left( \frac{\partial \mathbf{M}}{\partial T} \right)_{v,H} \cdot \frac{D\mathbf{H}}{Dt} \\ = K_1 \nabla^2 T + \delta (\nabla \times \boldsymbol{\omega}) \cdot \nabla T + \phi \end{aligned} \right\} \tag{4}$$

The mass flux equation is given by

$$\rho_0 (\partial / \partial t + \mathbf{q} \cdot \nabla) S = K_s \nabla^2 S \tag{5}$$

The partial derivatives of  $\mathbf{M}$  are material properties which can be evaluated once the magnetic equation of state, such as (9), is known. There are many situations of practical occurrence in which the basic equations can be simplified considerably. These situations occur when variability in the density and in the various coefficients is due to variations in the temperature not exceeding  $10^0$  (say), the variation of the small amount can be ignored. But there is an important exception that the variability of  $\rho$  in the gravitational body force term in the equation of the motion cannot be ignored, so we may treat  $\rho$  as a constant in all terms in the equation of motion

except the one in the external force. Thus, in writing Eq. (2), we use the Boussinesq approximation by allowing the density to change only in the gravitation body force term.

A porous medium of large permeability allows us to use the Brinkman model. For a medium of very small stable particle suspension, the permeability tends to be more justifying the use of Brinkman model. This is because the viscous drag force is necessarily important.

Also, the Darcy's law governs the flow of ferromagnetic fluid through an isotropic and homogeneous porous medium. However, to be mathematically compatible and physically consistent with Navier – Stokes equations, Brinkman (1947) heuristically proposed the introduction of the term  $(\eta/\varepsilon)\nabla^2\mathbf{q}$  (now known as the Brinkman term) in addition to the Darcian term  $(-\eta/k)\mathbf{q}$ . So that, we considered a term  $(\eta/\varepsilon)\nabla^2\mathbf{q}$  in Eq. (2) and also the additional term pertinent to a ferromagnetic fluid is the magnetic stress, which was derived by Landau and Lifshitz (1960) and Cowley and Rosensweig (1967).

When the permeability of the porous medium is large, then the internal force becomes relatively significance as compared with the viscous drag when flow is considered. Therefore, Brinkman model is proposed heuristically to govern the flow of this micropolar ferromagnetic fluid saturating a porous medium. Furthermore, Eq. (5) is considered for the system is getting salt from the above.

Maxwell's equations, simplified for a non-conducting fluid with no displacement currents, become

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = 0 \tag{6a, b}$$

Further  $\mathbf{B}$  and  $\mathbf{H}$  are related by

$$\mathbf{B} = \mu_0(\mathbf{M} + \mathbf{H}) \tag{7}$$

Using Maxwell's equation for non-conducting fluids (Sekar *et al.* (2013a, 2013b)), one can assume that the magnetization is aligned with the magnetic field and depends on the magnitude of the magnetic field, temperature and salinity, so that

$$\mathbf{M} = \frac{\mathbf{H}}{H} M(H, T, S). \tag{8}$$

The magnetic equation of state is linearized about the magnetic field  $H_0$ , the average temperature  $T_a$  and the average salinity  $S_a$  to become

$$M = M_0 + \chi(H - H_0) - K(T - T_0) + K_2(S - S_0) \tag{9}$$

The density equation of state for a two-component fluid is

$$\rho = \rho_0[1 - \alpha_T(T - T_a) + \alpha_S(S - S_a)] \tag{10}$$

### 2.2 Basic State

The basic state is assumed to be quiescent state. The basic state quantities are obtained by substituting velocity of quiescent state in Eqs. (2)-(5). The basic state quantities obtained are

$$\begin{aligned} \mathbf{q} &= \mathbf{q}_b = (0, 0, 0), \quad \boldsymbol{\omega} = \boldsymbol{\omega}_b = (0, 0, 0), \\ T_b &= T_0 - \beta_T z, \quad S_b = S_0 - \beta_S z, \\ \rho(z) &= \rho_0[1 + \alpha_T \beta_T z - \alpha_S \beta_S z], \quad p = p_b(z), \\ \mathbf{H}_b(z) &= \left[ H_0 - \frac{K \beta_T z}{1 + \chi} + \frac{K_2 \beta_S z}{1 + \chi} \right] \mathbf{k}, \\ \mathbf{M}_b(z) &= \left[ M_0 + \frac{K \beta_T z}{1 + \chi} - \frac{K_2 \beta_S z}{1 + \chi} \right] \mathbf{k}, \quad M_0 + H_0 = H_0^{ext} \end{aligned} \tag{11}$$

### 2.3 Perturbed State and Equations

A small thermal perturbation has been imparted on all the dynamical variables. Let the components of perturbed magnetization and magnetic field can taken as  $[M_1', M_2', M_0(z) + M_3']$  and  $[H_1', H_2', H_0(z) + H_3']$ , respectively. The perturbed physical quantities are

$$\left. \begin{aligned} \mathbf{q} &= \mathbf{q}_b + \mathbf{q}', \quad \boldsymbol{\omega} = \boldsymbol{\omega}_b + \boldsymbol{\omega}', \quad \rho = \rho_b + \rho', \\ p &= p_b(z) + p', \quad T = T_b(z) + \theta, \quad S = S_b(z) + S', \\ \mathbf{H} &= \mathbf{H}_b(z) + \mathbf{H}', \quad \mathbf{M} = \mathbf{M}_b(z) + \mathbf{M}' \end{aligned} \right\} \tag{12}$$

where  $\mathbf{q}' = (u, v, w)$ ,  $\boldsymbol{\omega}' = (\omega_1, \omega_2, \omega_3)$ ,  $\mathbf{H}'$ ,  $\mathbf{M}'$ ,  $p'$ ,  $\theta$  and  $S'$  are perturbations in velocity  $\mathbf{q}$ , spin (microrotation)  $\boldsymbol{\omega}$ , magnetic field intensity  $\mathbf{H}$ , magnetization  $\mathbf{M}$ , pressure  $p$ , temperature  $T$  and salinity  $S$ . The change in density  $\rho'$ , caused mainly by the perturbations  $\theta$  and  $S'$  in temperature and salinity, respectively, is given by

$$\rho' = \rho_0(-\alpha_T \theta + \alpha_S S') \tag{13}$$

Therefore using linear theory and assuming  $K \beta_T d \ll (1 + \chi)H_0$  and  $K_2 \beta_S d \ll (1 + \chi)H_0$ , one

$$\left. \begin{aligned} H_i' + M_i' &= [1 + (M_0/H_0)] H_i' \quad (i = 1, 2) \\ H_3' + M_3' &= (1 + \chi)H_3' - (K_2 S + K \theta) \end{aligned} \right\} \text{ gets} \tag{14}$$

If  $\mathbf{B}$  denotes the components  $(B_1, B_2, B_3)$ , then sing Eqs. (7) and (13) become

$$\left. \begin{aligned} B_i' &= \mu_0 [1 + (M_0/H_0)] H_i', \quad (i = 1, 2) \\ B_3' &= \mu_0(M_0 + H_0) + \mu_0(1 + \chi)H_3' - (\mu_0 K_2 S + \mu_0 K \theta) \end{aligned} \right\} \tag{15}$$

Using Eq. (6b), one get  $\mathbf{H}' = \nabla \phi'$ , where  $\phi'$  is a magnetic scalar potential and  $\mathbf{H}'$  has the components  $(H_1', H_2', H_3')$ . Further investigation has been carried out using the analyses similar to Sekar *et al.* (2013a, 2013b) and Vaidyanathan *et al.* (2005). The linearized perturbation equation of the magnetized ferrofluid become

$$\left. \begin{aligned} \frac{\rho_0}{\varepsilon} \frac{\partial u}{\partial t} &= -\frac{\partial p}{\partial x} + \mu_0(M_0 + H_0) \frac{\partial H_1'}{\partial z} \\ &\quad - \frac{1}{k}(\zeta + \eta)u + 2\zeta \Omega_1' + \frac{1}{\varepsilon}(\zeta + \eta)\nabla^2 u \end{aligned} \right\} \tag{16}$$

$$\left. \begin{aligned} \frac{\rho_0}{\varepsilon} \frac{\partial v}{\partial t} &= -\frac{\partial p}{\partial y} + \mu_0(M_0 + H_0) \frac{\partial H_2'}{\partial z} \\ &\quad - \frac{1}{k}(\zeta + \eta)v + 2\zeta \Omega_2' + \frac{1}{\varepsilon}(\zeta + \eta)\nabla^2 v \end{aligned} \right\} \quad (17)$$

$$\left. \begin{aligned} \frac{\rho_0}{\varepsilon} \frac{\partial w}{\partial t} &= -\frac{\partial p}{\partial z} + \mu_0(M_0 + H_0) \frac{\partial H_3'}{\partial z} - \mu_0 K \beta_t H_3' \\ &\quad + \frac{\mu_0 K^2 \beta_t \theta}{1 + \chi} - \frac{\mu_0 K K_2 \beta_t S'}{1 + \chi} + \mu_0 K_2 \beta_s H_3' \\ &\quad - \frac{\mu_0 K K_2 \beta_s \theta}{1 + \chi} + \frac{\mu_0 K_2^2 \beta_s S'}{1 + \chi} + \rho_0 g \alpha_t \theta - \rho_0 g \alpha_t S' \\ &\quad - \frac{1}{k}(\zeta + \eta)w + 2\zeta \Omega_3' + \frac{1}{\varepsilon}(\zeta + \eta)\nabla^2 w \end{aligned} \right\} \quad (18)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (19)$$

$$\rho_0 I \left( \frac{\partial \Omega_3'}{\partial t} \right) = -2\zeta \left( \frac{1}{\varepsilon} \nabla^2 w + 2\Omega_3' \right) + \eta' \nabla^2 \Omega_3' \quad (20)$$

$$\left. \begin{aligned} \rho C_1 \frac{\partial \theta}{\partial t} - \mu_0 K T_0 \varepsilon \frac{\partial}{\partial t} \left( \frac{\partial \phi}{\partial z} \right) + K_1 \nabla^2 \theta - \delta \beta_t \Omega_3' \\ = \left[ \rho C_2 \beta_t - \left( \frac{\mu_0 K^2 T_0^2 \beta_t}{1 + \chi} \right) + \left( \frac{\mu_0 K K_2 T_0 \beta_s}{1 + \chi} \right) \right] w \end{aligned} \right\} \quad (21)$$

$$\frac{\partial S}{\partial t} + \beta_s w = K_s \nabla^2 S' \quad (22)$$

where

$$\nabla^2 = \nabla_1^2 + (\partial^2 / \partial z^2), \quad \nabla_1^2 = (\partial^2 / \partial x^2) + (\partial^2 / \partial y^2),$$

$$\rho C_1 = \varepsilon \rho_0 C_{v,H} + (1 - \varepsilon) \rho_s C_s + \varepsilon \mu_0 K_2 H_0 \quad \text{and}$$

$$\rho C_2 = \rho_0 C_{v,H} + \mu_0 K_2 H_0.$$

### 3. NORMAL MODE ANALYSIS METHOD

Analyzing the disturbance into normal modes, we assume that the perturbation quantities are of the form

$$f(x, y, z, t) = f(z, t) \exp[ik_x x + ik_y y] \quad (23)$$

where  $f(z, t)$  represents  $w(z, t)$ ,  $\theta(z, t)$ ,  $\phi(z, t)$  and  $S(z, t)$ .

The vertical component of the momentum equation can be written as

$$\left. \begin{aligned} \left( \frac{\rho_0}{\varepsilon} \frac{\partial}{\partial t} + \frac{1}{k}(\zeta + \eta) \right) \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) w \\ = \mu_0 K \beta_t k_0^2 \frac{\partial \phi}{\partial z} - \left( \frac{\mu_0 K^2 \beta_t}{1 + \chi} \right) k_0^2 \theta + \left( \frac{\mu_0 K K_2 \beta_t}{1 + \chi} \right) k_0^2 S' \\ - \mu_0 K_2 \beta_s k_0^2 \frac{\partial \phi}{\partial z} + \left( \frac{\mu_0 K K_2 \beta_s}{1 + \chi} \right) k_0^2 \theta - \left( \frac{\mu_0 K_2^2 \beta_s}{1 + \chi} \right) k_0^2 S' \\ - \rho_0 g \alpha_t k_0^2 \theta + \rho_0 g \alpha_s k_0^2 S + 2\zeta \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) \Omega_3' \\ + \frac{1}{\varepsilon}(\zeta + \eta) \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right)^2 w \end{aligned} \right\} \quad (24)$$

From Eq. (3) after doing mathematical manipulation, we get

$$\left. \begin{aligned} \rho_0 I \frac{\partial \Omega_3'}{\partial t} &= -2\zeta \left[ \frac{1}{\varepsilon} \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) w + 2\Omega_3' \right] \\ &\quad + \eta' \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) \Omega_3' \end{aligned} \right\} \quad (25)$$

The modified Fourier heat conduction equation is

$$\left. \begin{aligned} \rho C_1 \frac{\partial \theta}{\partial t} - \mu_0 K T_0 \varepsilon \frac{\partial}{\partial t} \left( \frac{\partial \phi}{\partial z} \right) \\ = K_1 \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) \theta + \left[ \rho C_2 \beta_t - \left( \frac{\mu_0 K^2 T_0^2 \beta_t}{1 + \chi} \right) + \left( \frac{\mu_0 K K_2 T_0 \beta_s}{1 + \chi} \right) \right] w - \delta \beta_t \Omega_3' \end{aligned} \right\} \quad (26)$$

The Salinity equation is

$$\left( \frac{\partial S}{\partial t} \right) + \beta_s w = K_s \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) S' \quad (27)$$

Using the analysis similar to Sekar *et al.* [29], one gets

$$(1 + \chi) \frac{\partial^2 \phi}{\partial z^2} - \left( 1 + \frac{M_0}{H_0} \right) k_0^2 \phi - K \frac{\partial \theta}{\partial z} + K_2 \frac{\partial S}{\partial z} = 0 \quad (28)$$

Following the analyses Sekar *et al.* (2013a, 2013b) and Vaidyanathan *et al.* (2005), the equations in non-dimensional form can be written using

$$\left. \begin{aligned} w^* &= \frac{wd}{v}, \quad t^* = \frac{vt}{d^2}, \quad T^* = \left( \frac{K_1 a R^{1/2}}{\rho_0 C_{v,H} \beta_t v d} \right) \theta, \quad v = \frac{\mu}{\rho_0}, \\ \phi^* &= \left( \frac{(1 + \chi) K_1 a R^{1/2}}{\rho_0 C_{v,H} K \beta_t v d^2} \right) \phi, \quad z^* = \frac{z}{d}, \quad \Omega_3^* = \frac{\Omega_3}{v} d^3 \\ a &= k_0 d, \quad D = \frac{\partial}{\partial z^*}, \quad S^* = \left( \frac{K_s a R_S^{1/2}}{\rho_0 C_{v,H} \beta_s v d} \right) S, \quad k^* = \frac{k}{d^2}, \end{aligned} \right\} \quad (29)$$

Then the Eqs. (24)–(28) become

$$\left. \begin{aligned} \left( \frac{1}{\varepsilon} \frac{\partial}{\partial t^*} + \frac{1}{k^*} (1 + N_1) \right) (D^2 - a^2) w^* \\ = a R^{1/2} M_1 D \phi^* - (1 + M_1) a R^{1/2} T^* \\ + a R^{1/2} M_1 M_5 D \phi^* + a R_S^{1/2} (1 + M_4) S^* \\ - a R^{1/2} M_1 M_5 T^* + a R_S^{1/2} \frac{M_4}{M_5} S^* \\ + 2N_1 (D^2 - a^2) \Omega_3^* + \frac{1}{\varepsilon} (\zeta + \eta) (D^2 - a^2)^2 w^* \\ I' \frac{\partial \Omega_3^*}{\partial t^*} = -2N_1 \left[ \frac{1}{\varepsilon} (D^2 - a^2) w^* + 2\Omega_3^* \right] \\ + N_3 (D^2 - a^2) \Omega_3^* \end{aligned} \right\} \quad (30)$$

$$\left. \begin{aligned} \left[ P_r \frac{\partial T^*}{\partial t^*} - \varepsilon P_r M_2 \frac{\partial}{\partial t^*} (D \phi^*) \right] \\ = (D^2 - a^2) T^* + a R^{1/2} (1 - M_2 - M_2 M_5) w^* - a R^{1/2} N_5 \Omega_3^* \end{aligned} \right\} \quad (31)$$

$$P_r \frac{\partial S^*}{\partial t^*} = \tau (D^2 - a^2) S^* - a R_S^{1/2} \left( \frac{M_5}{M_6} \right) w^* \quad (32)$$

$$D^2 \phi^* - M_3 a^2 \phi^* - D T^* + \frac{M_5}{\tau} \left( \frac{R}{R_S} \right)^{1/2} D S^* = 0 \quad (33)$$

where the non-dimensional parameters used are

$$\left. \begin{aligned} M_1 &= \frac{\mu_0 K^2 \beta_t}{(1+\chi)\rho_0 g \alpha_s}, M_2 = \frac{\mu_0 K^2 T}{(1+\chi)\rho_0 C_{v,H}}, \\ M_3 &= \frac{1+M_0/H_0}{(1+\chi)}, M_6 = \frac{K_s}{K_1}, \tau = \rho_0 C_{v,H} \left( \frac{K_s}{K_1} \right), \\ M_4 &= \frac{\mu_0 K^2 \beta_s}{(1+\chi)\rho_0 g \alpha_s}, M_5 = \frac{K_2 \beta_s}{K \beta_t}, R = \frac{\rho_0 C_{v,H} \beta_t \alpha_s g d^4}{\nu K_1}, \\ N_1 &= \frac{\zeta}{\eta}, N_3 = \frac{\eta'}{\eta d^2}, N_5 = \frac{\delta}{\rho C_2 d^2}, P_r = \frac{\nu}{K_1} \rho C_2, \\ P_r' &= \frac{\nu}{K_1} \rho C_1, R_s = \frac{\rho_0 C_{v,H} \beta_s \alpha_s g d^4}{\nu K_s}, I' = \frac{I}{d^2}, \end{aligned} \right\} \quad (35)$$

**4. LINEAR STABILITY THEORY**

In this section we predict the thresholds of both stationary and oscillatory convections using linear theory. The boundary conditions on velocity, temperature, salinity and angular momentum are

$$w^* = D^2 w^* = T^* = D\phi^* = S^* = \Omega_3^* = 0 \quad \text{at} \quad z^* = \pm 1/2. \quad (36)$$

The exact solutions satisfying above Eq. (36) are

$$\left. \begin{aligned} w^* &= A e^{\sigma z^*} \cos \pi z^*, \quad T^* = B e^{\sigma z^*} \cos \pi z^*, \\ S^* &= C e^{\sigma z^*} \cos \pi z^*, \quad D\phi^* = E e^{\sigma z^*} \cos \pi z^*, \\ \phi^* &= \frac{E}{\pi} \sin \pi z^*, \quad \Omega_3^* = F e^{\sigma z^*} \cos \pi z^* \end{aligned} \right\} \quad (37)$$

where A, B, C, E, and F are constants. Using Eq. (36) in Eqs. (30)–(34), we get

$$\left. \begin{aligned} &\left[ \left( \frac{\sigma}{\varepsilon} + \frac{1+N_1}{k} \right) (\pi^2 + a^2) + \frac{1}{\varepsilon} (1+N_1) (\pi^2 + a^2)^2 \right] A \\ &- aR^{1/2} [1+M_1(1+M_5)] B + aR_s^{1/2} (1+M_4+M_4M_5^{-1}) C \\ &+ aR^{1/2} M_1 (1+M_5) E - 2N_1 (\pi^2 + a^2) F = 0 \end{aligned} \right\} \quad (38)$$

$$-\frac{2N_1}{\varepsilon} (\pi^2 + a^2) A + [4N_1 + (\pi^2 + a^2) N_3' + I' \sigma] F = 0, \quad (39)$$

$$\left. \begin{aligned} aR^{1/2} (1-M_2 - M_2 M_5) A - (\pi^2 + a^2 + P_r \sigma) B \\ + P_r \varepsilon \sigma M_2 E - aR^{1/2} N_5' F = 0, \end{aligned} \right\} \quad (40)$$

$$aR_s^{1/2} M_6 A + [\tau (\pi^2 + a^2) + \sigma P_r] C = 0, \quad (41)$$

$$-R_s^{1/2} \pi^2 B + R^{1/2} \pi^2 (M_5 / M_6^{-1}) C + R_s^{1/2} (\pi^2 + a^2 M_3) E = 0, \quad (42)$$

The determinant of the co-efficient of A, B, C, E and F in Eqs. (38)–(42) must vanish for the existence of non-trivial Eigen functions. The techniques and analyses of Finlayson (1970) and Vaidyanathan *et al.* (1991), Eqs. (38)–(42) have been adopted to obtain

$$U \sigma^4 + V \sigma^3 + W \sigma^2 + X \sigma + Y = 0 \quad (43)$$

where

$$U = -P_r I' h_1 h_8,$$

$$\begin{aligned} V &= -P_r h_2 I' (h_1^2 h_8 + P_r (h_1 h_7 + h_1^2)) \\ &\quad - P_r h_1 h_2 h_8 (P_r h_3 + h_1 I'), \end{aligned}$$

$$\begin{aligned} W &= a^2 P_r I'^2 h_4 R + 4h_2 P_r^2 N_1^2 h_1^2 h_8 \\ &\quad + h_1 h_2 (P_r h_3 + \tau h_1 I') (h_8 + P_r (h_1 + h_7)) \\ &\quad + P_r h_2 I' (a^2 (1+h_4) R - (h_1^2 (h_1 + h_7))) - h_1 h_2 h_8 P_r^2 I' \\ &\quad - a^2 M_6 I' h_4 h_5 \pi^2 R + a^2 M_6 I' h_2 h_6 P_r R_s, \end{aligned}$$

$$\begin{aligned} X &= -a^2 \pi^2 h_1 \tau h_4 R - a^2 \pi^2 P_r h_4 (h_3 - 2N_1 h_1 h_8 N_5') \\ &\quad + 2N_1 P_r h_1 h_2 h_3 (a^2 (1+h_4) N_5' + 2N_1 h_1^2) \\ &\quad + a^2 h_2 (P_r h_3 + \tau h_1 I') (1+h_4) R - (h_1^2 (h_1 + h_7)) \\ &\quad + h_1 h_2 h_3 \tau (h_1^2 h_8 + P_r h_1 + (h_1 + h_7)) \\ &\quad + a^2 M_6 h_2 R_s (4N_1 + h_1 h_2 P_r + I' h_1 h_6) \\ &\quad - a^2 \pi^2 M_6 I' h_1 h_4 h_5 R + a^2 h_3 h_4 M_5 R, \\ &\quad + 4\tau h_1^2 h_2 h_8 P_r N_1^2 \end{aligned}$$

$$Y = -a^2 \pi^2 R \tau h_1 h_4 (h_3 - 2N_1 N_5' h_1 h_8)$$

$$\begin{aligned} &- 2a^2 \tau h_1^2 h_2 h_8 N_1 (1+h_4) N_5' R \\ &- 4\tau h_1^4 h_2 h_8 N_1^2 + a^2 \tau h_1 h_2 h_8 (1+h_4) R \\ &- a^2 M_6 h_1 h_3 h_4 R + a^2 M_6 h_1 h_2 h_4 h_6 R_s, \end{aligned}$$

$$h_1 = \pi^2 + a^2, \quad h_2 = \pi^2 + a^2 M_3,$$

$$h_3 = 4N_1 + h_1 N_3', \quad h_4 = M_1 (1+M_5), \quad h_5 = M_5 / M_6^{-1}$$

$$h_6 = 1+M_4 + M_4^{-1} M_5, \quad h_7 = (1+N_1) / k \quad \text{and} \quad h_8 = 1 / \varepsilon.$$

**4.1 Stationary State**

For the validity of principle of exchange of stabilities (i.e. steady case), we have  $\sigma = 0$  at the margin of stability. Then Eq. (43) helps one to obtain Eigen value  $R_{sc}$  for which solution exists. Therefore the critical magnetic Rayleigh number  $R_{sc}$  have been calculated

$$R_{sc} = Nr / Dr \quad (44)$$

where

$$Nr = (\pi^2 + a^2)^2 \left( \left( 4N_1 + (\pi^2 + a^2) N_3' \right) (1+N_1) \right. \\ \left. - \left( \frac{1}{k} + \frac{1}{\varepsilon} (\pi^2 + a^2) \right) - \frac{4N_1^2}{\varepsilon} (\pi^2 + a^2) \right)$$

$$- a^2 (1+M_4 + M_4 M_5^{-1}) (4N_1 + (\pi^2 + a^2) N_3') M_6 R_s \tau^{-1}$$

$$Dr = a^2 (1+M_1 (1+M_5)) \left( 4N_1 + (\pi^2 + a^2) \left( N_3' - \frac{2N_1 N_5'}{\varepsilon} \right) \right)$$

$$- \frac{a^2 \pi^2 M_1 (1+M_5)}{(\pi^2 + a^2 M_3)} \left( \left( 4N_1 + (\pi^2 + a^2) N_3' \right) (1+M_5 \tau^{-1}) \right)^F \\ \left( - \frac{2N_1 N_5'}{\varepsilon} (\pi^2 + a^2) \right)$$

or  $M_1$  very large, one gets the results for the magnetic mechanism and the critical thermal magnetic Rayleigh number ( $N_{sc} = M_1 R_{sc}$ ) for stationary mode is calculated using

$$N_{sc} = Nr / Dr_1 \quad (45)$$

where

$$Dr_1 = a^2 (1+M_5) \left( 4N_1 + (\pi^2 + a^2) \left( N_3' - \frac{2N_1 N_5'}{\varepsilon} \right) \right)$$

$$- \frac{a^2 (1+M_5) \pi^2}{(\pi^2 + a^2 M_3)} \left( \left( 4N_1 + (\pi^2 + a^2) N_3' \right) (1+M_5 \tau^{-1}) \right) \\ \left( - \frac{2N_1 N_5'}{\varepsilon} (\pi^2 + a^2) \right)$$

Here  $a$  is a critical wave number denoted as  $a_c$ . The classical results in respect of ferromagnetic fluids can be obtained as the limiting case of present study.

Setting  $\varepsilon = 1$ ,  $N_1 = 0$ ,  $N_3' = 1$  and  $N_5' = 0$  in Eq. (45), we get

$$N_{sc} = \frac{(\pi^2 + a^2)^2 \left( \frac{1}{k} + (\pi^2 + a^2) \right) - a^2(1 + M_4 + M_4 M_5^{-1}) M_6 \tau^{-1} R_S}{a^2(1 + M_5) \left( 1 - \pi^2 (1 + M_5 \tau^{-1}) / (\pi^2 + a^2 M_3) \right)} \quad (46)$$

which is the expression for the critical thermal Rayleigh number of ferrothermohaline convection in a porous medium for multi-component fluid (Vaidyanathan et al. (1995)).

Further in the case of single component system,  $M_4, M_6, \tau^{-1}, R_S = 0$  Eq. (46) gives

$$N_{sc} = \frac{(\pi^2 + a^2)^2 (\pi^2 + a^2 M_3) \left( \frac{1}{k} + (\pi^2 + a^2) \right)}{a^2 \pi^2} \quad (47)$$

This has exactly given by Vaidyanathan *et al.* (1991). Further when  $1/k = 0$ , i.e., in the absence of porous medium, Eq. (47) gives the Finlayson (1970).

### 4.2 Oscillatory State

The conditions for the onset of oscillatory stabilities are obtained as follows. We put  $\sigma = i\sigma_1$  ( $\sigma_1 > 0$ ) in Eq. (43) and rearranging the terms to get the oscillatory Rayleigh number  $R_{oc}$  at the margin of stability, in the form

$$R_{oc} = \left( \frac{X_1 U \sigma_1^6 + (X_2 U + X_3 V) \sigma_1^4}{+(X_1 X_4 + X_3 X_5) \sigma_1^2 + X_2 X_4} \right) / Dr$$

where

$$U = -P_r I' h_1 h_8,$$

$$V = -h_2 P_r I' (h_1^2 h_8 + P_r h_1 (h_1 + h_7)) - h_1 h_2 P_r h_8 (P_r h_3 + h_1 I')$$

$$W = X_1 + 4h_2 h_1^2 h_8 P_r N_1^2 - h_2 (P_r h_3 + \tau h_1 I') (h_1 h_8 + P_r (h_1 h_7 + h_1^2)) - P_r^2 h_2 I' h_1 h_8 + a^2 M_6 h_1 h_2 h_4 h_6 R_S$$

$$X_1 = a^2 P_r I'^2 h_4 + h_2 P_r I' a^2 (1 + h_4) - a^2 \pi^2 M_6 I' h_4 P_r h_5$$

$$X_2 = \tau a^2 \pi^2 h_1 h_4 (h_3 - 2N_1 N_5 h_1 h_8) + h_2 \tau h_1^2 h_8 2N_1 N_5 a^2 (1 + h_4) - h_2 \tau h_8 a^2 (1 + h_4) + a^2 M_6 h_1 h_3 h_4$$

$$X_3 = \tau a^2 \pi^2 h_1 I' h_4 - a^2 h_2 (1 + h_4) (P_r h_3 + \tau h_1 I') + a^2 M_6 I' h_4 \pi^2 h_1 h_5 - a^2 h_3 h_4 P_r M_5$$

$$X_4 = -4h_2 P_r^2 N_1^2 h_1^2 h_8 + P_r h_2 I' (h_1^2 (h_1 + h_7)) - h_2 (P_r h_3 + \tau h_1 I') (h_1 h_8 + P_r (h_1 h_7 + h_1^2)) + h_2 P_r^2 I' h_1 h_8 + 4h_2 \tau h_1^4 N_1^2 h_8 - a^2 M_6 h_1 h_2 h_4 h_6 R_S + h_2 \tau h_1^3 h_3 (h_1 + h_7)$$

$$X_5 = a^2 \pi^2 h_4 P_r (h_3 - 2N_1 N_5 h_1 h_8) - 2P_r N_1 N_3 h_1 h_2 (a^2 (1 + h_4) N_5 + 2N_1 h_1^2) - 4P_r \tau h_2 h_1^3 N_1^2 h_8 + h_1^2 (h_1 + h_7) - \tau h_1 h_2 h_3 (h_1^2 h_8 + P_r h_1 (h_1 + h_7)) - a^2 M_6 h_7 R_S (4N_1 + h_1 h_7 P_r + I' h_1 h_6)$$

$$\sigma_1^2 = (-B_2 \pm \sqrt{B_2^2 - 4B_1 B_3}) / 2B_1,$$

$$B_1 = VX_1 - X_3 U,$$

$$B_2 = X_1 X_5 + UX_2,$$

$$B_3 = X_2 X_5 - X_3 X_4$$

$$\text{and } Dr = (X_1 \sigma_1^2 + X_2)^2 - \sigma_1^2 X_3^2.$$

In the next section we perform the results and discussion and many physical parameters have been studied.

### 5. RESULTS AND DISCUSSION

Brinkman model is made on the effect of linearity on thermoconvective instability in a micropolar ferromagnetic fluid with uniform angular velocity heated from below and salted from above has been analyzed. A linear stability analysis is carried out as perturbations are small and normal mode technique is applied. The conditions for both stationary and oscillatory modes have been calculated and a discussion of the results depicted by figures.

Before we investigate the effect of different parameters, we first make some physical comments on various parameters like buoyancy magnetization parameter  $M_1$  is taken to be 1000 (Sekar *et al.* 2013a) and the value of  $M_2$  is assumed to be zero (Finlayson 1970) for these types of fluids. The non-buoyancy magnetization parameter  $M_3$  is allowed to vary from 5 to 25, because this parameter cannot take a value less than one (Vaidyanathan *et al.* 2005). The range of permeability of the porous medium  $k$  is varied from 0.1 to 0.9 (Sekar *et al.* 2013a). The ratio of mass transport to the heat transport  $\tau$  is taken as 0.05 (0.02) 0.11 and the Prandtl number  $P_r$  is assumed to be 0.01 (Sekar *et al.* 2013a). The salinity Rayleigh number  $R_s$  is taken values from -500 to 500 and magnetization parameters  $M_4$  and  $M_6$  are assumed to be 0.1 and  $M_5 = 0.5$ . Further, the coupling parameter  $N_1$  (coupling between vorticity and spin effects), spin diffusion parameter  $N_3'$  and micropolar heat conduction parameter  $N_5'$  (coupling between spin and heat flux) are arising some comments due to the suspended particles. Assuming the Clausius-Duhem inequality, Eringen (1964) presented certain thermodynamic restrictions which lead to non-negativeness of  $N_1, N_3'$  and  $N_5'$ . It is obvious the couple stress comes into play at small values of  $N_3'$ . This supports the condition that (Sunil *et al.* 2007) and that  $N_3'$  is small positive real number (Sunil *et al.* 2007a) and the micropolar heat conduction parameter  $N_5'$  has to be finite because the increasing of concentration has to practically stop somewhere and hence it has to be a positive

finite number. This typical order of magnitudes on  $N_1$ ,  $N_3'$  and  $N_5'$  mentioned above applies to fluid system encountered in material processing under microgravity in space.

Fig. 2 (a) represents the plot of critical thermal magnetic Rayleigh number  $N_{sc}$  versus the medium permeability  $k$  for various values of non-buoyancy magnetization parameter  $M_3$  in the presence and absence of a coupling parameter  $N_1$ . This shows that the non-buoyancy magnetization and medium permeability have a destabilizing behavior. In order to investigate our results, we must review the results and its physical explanation. When the fluid layer is assumed to be flowing through homogeneous and an isotropic porous medium, then the medium permeability has a destabilizing behavior. This is because, as medium permeability increases, the void space increases and as result of this, the flow quantities perpendicular to the planes will clearly be increased. Thus, increase in heat transfer is responsible for early onset of convection.

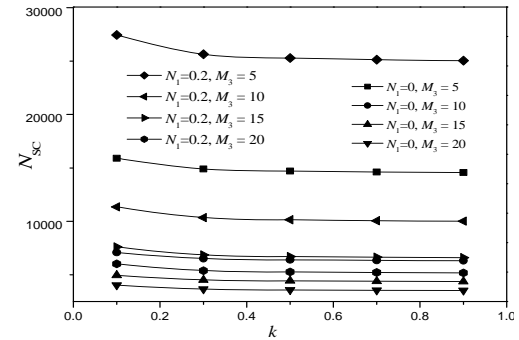
Fig. 2 (b) gives the critical thermal magnetic Rayleigh number  $N_{sc}$  variation with respect to the ratio of mass transport to heat transport  $\tau$ , indicate the system destabilizes as the non-buoyancy magnetization parameter  $M_3$  increases. This is indicated by a decrease in  $N_{sc}$ . This is because variation in magnetization releases extra energy which adds up to thermal energy to destabilize the system.

In Fig. 2 (c), the variation of critical thermal magnetic Rayleigh number  $N_{sc}$  versus the salinity Rayleigh number  $R_S$  for different values of  $k$  and presence and absence of coupling parameter  $N_1$  is analyzed. It is obvious from the Fig. 2 (c) that salinity Rayleigh number  $R_S$  has a destabilizing effect on the system. This is indicated by a decrease in  $N_{sc}$ . In the presence of coupling parameter  $N_1 (= 0.2)$ , system gets heavy energy but in the absence of coupling parameter  $N_1 (= 0)$ , system gets low energy. However, the critical magnetic thermal Rayleigh number  $N_{sc}$  converges to zero when the value of  $R_S$  is 500. In other words, the system has a null effect.

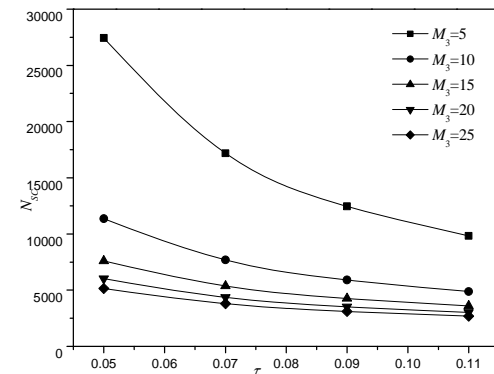
Figs. 3 (a) – (c) show the variations of critical magnetic thermal Rayleigh number  $N_{sc}$  with respect to the micropolar heat conduction parameter  $N_5'$  for different values of porous medium  $k$  with non-buoyancy magnetization parameter  $M_3$ , salinity Rayleigh number  $R_S$  and the ratio of mass transport to heat transport  $\tau$ , respectively.

It is observed from the Fig. 3 (a) that when  $N_5'$  is increases, the heat induced into the fluid due to microelements is also increased, thus inducing the heat transfer from the bottom to the top. The decrease in heat transfer is responsible for delaying the onset of convection. Thus increasing of  $N_5'$  leads to increase in  $N_{sc}$ . Therefore,  $N_5'$  have a stabilizing flow. An increase in micropolar heat conduction parameter  $N_5'$  is found to cause large

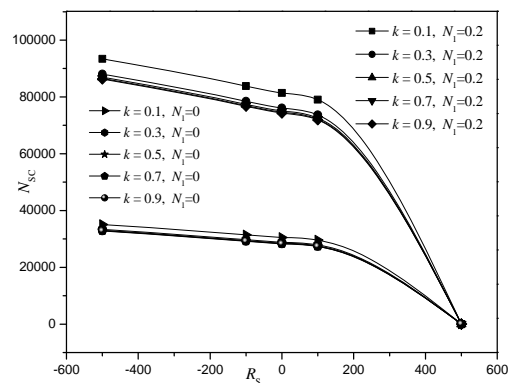
stabilization. This can be observed from Fig. 3 (b) in which the increase in  $N_5'$  and  $N_{sc}$ . As  $R_S$  increases from -500 to 500,  $N_{sc}$  values tend to increase leading to stabilization uniformly in Fig. 3 (c). This is because adding magnetic salt from above it makes the system heavier at the above, thereby delays the onset of convection.



**Fig. 2(a).** Marginal instability curve for variation of  $N_{sc}$  versus  $k$  for different  $M_3$  and  $N_1 (= 0$  and  $0.2)$ ,  $N_3' = 2$ ,  $N_5' = 0.5$ ,  $\varepsilon = 0.5$ ,  $\tau = 0.03$  and  $R_S = -500$ .

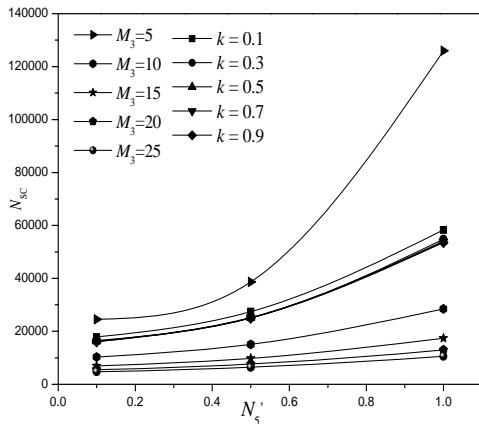


**Fig. 2(b).** Marginal instability curve for variation of  $N_{sc}$  versus  $\tau$  for different  $M_3$ ,  $N_1 = 0.2$ ,  $N_3' = 2$ ,  $N_5' = 0.5$ ,  $\varepsilon = 0.5$  and  $R_S = -500$ .

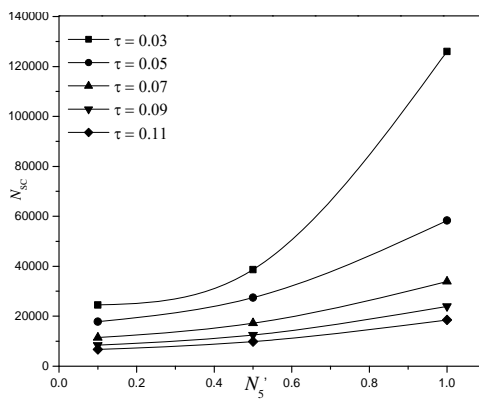


**Fig. 2(c).** Marginal instability curve for variation of  $N_{sc}$  versus  $R_S$  for different  $k$  and  $N_1 (= 0$  and  $0.2)$ ,  $N_3' = 2$ ,  $N_5' = 0.5$  and  $\varepsilon = 0.5$ .

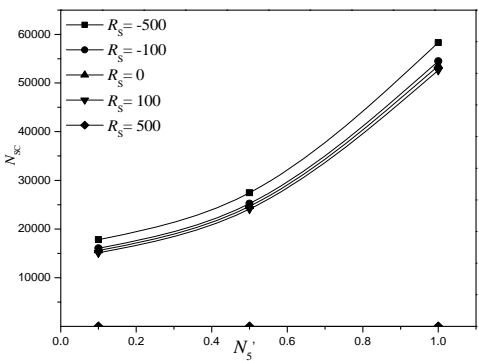




**Fig. 3(a).** Marginal instability curve for variation of  $N_{sc}$  versus  $N_5'$  for different  $M_3$  and  $k$ ,  $N_1=0.2$ ,  $N_3'=2$ ,  $\varepsilon=0.5$ ,  $\tau=0.03$  and  $R_S=-500$ .



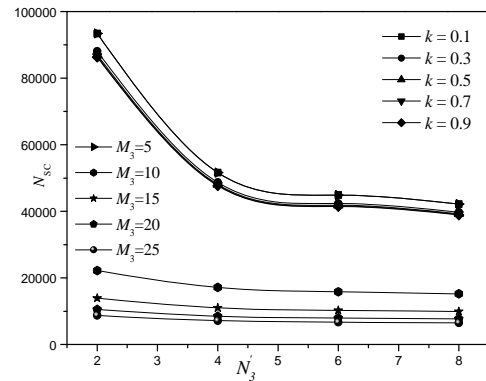
**Fig. 3(b).** Marginal instability curve for variation of  $N_{sc}$  versus  $N_5'$  for different  $\tau$ ,  $N_1=0.2$ ,  $N_3'=2$ ,  $\varepsilon=0.5$ ,  $M_3=5$ ,  $k=0.1$  and  $R_S=-500$ .



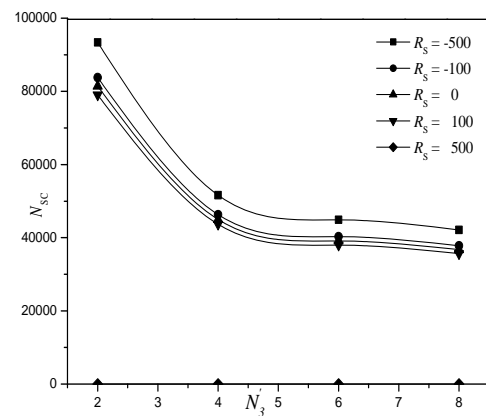
**Fig. 3(c).** Marginal instability curve for variation of  $N_{sc}$  versus  $N_5'$  for different  $R_S$ ,  $M_3=5$ ,  $N_1=0.2$ ,  $N_3'=2$ ,  $\varepsilon=0.5$ ,  $\tau=0.03$  and  $k=0.1$ .

Figs. 4 (a) – (c) give the variation of critical magnetic thermal Rayleigh number  $N_{sc}$  with respect to the spin diffusion parameter  $N_3'$  for various values of  $M_3$ ,  $k$ ,  $R_S$  and  $\tau$ . In these figures, it is clear that the spin diffusion parameter  $N_3'$  has a destabilizing effect on the system. It shows that the spin diffusion parameter  $N_3'$  increases with

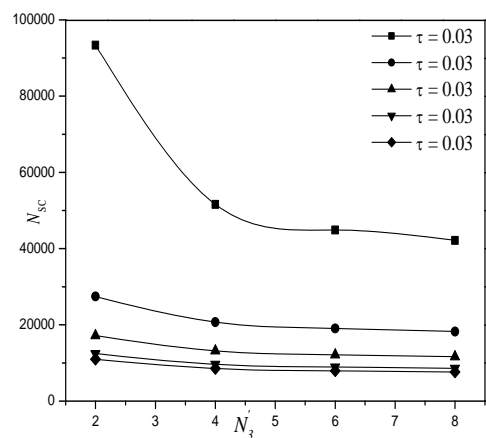
increasing of medium permeability  $k$ , non-buoyancy magnetization parameter  $M_3$ , salinity Rayleigh number  $R_S$  and the ratio of mass transport to heat transport  $\tau$ ,  $N_{sc}$  decreases. Moreover, we observe that as  $N_3'$  increases, the couple stress of the fluid increases, which causes the microrotation to decrease; rendering the system prone to instability.



**Fig. 4(a).** Marginal instability curve for variation of  $N_{sc}$  versus  $N_3'$  for various  $M_3$  and  $k$ ,  $N_1=0.2$ ,  $N_5'=0.2$ ,  $\varepsilon=0.5$ ,  $R_S=-500$  and  $\tau=0.03$ .



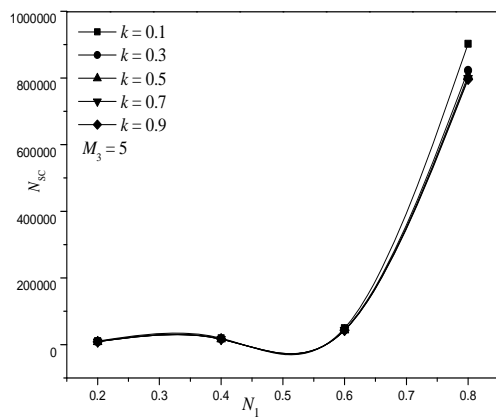
**Fig. 4(b).** Marginal instability curve for variation of  $N_{sc}$  versus  $N_3'$  for various  $R_S$ ,  $N_1=0.2$ ,  $N_5'=0.2$ ,  $\varepsilon=0.5$ ,  $\tau=0.03$ ,  $M_3=5$  and  $k=0.1$ .



**Fig. 4(c).** Marginal instability curve for variation of  $N_{sc}$  versus  $N_3'$  for various  $\tau$ ,  $N_1=0.2$ ,  $N_5'=0.2$ ,  $R_S=-500$ ,  $M_3=5$  and  $k=0.1$ .

Figs. 5 (a) – (c) represent the plots of critical magnetic thermal Rayleigh number  $N_{sc}$  versus coupling parameter  $N_1$  for various values of medium permeability  $k$ , salinity Rayleigh number  $R_s$  and the ratio of mass transport to heat transport  $\tau$  and increasing value of non-buoyancy magnetization parameter  $M_3$  from 5 to 25. These figures indicate that the coupling parameter  $N_1$  has a stabilizing behavior.

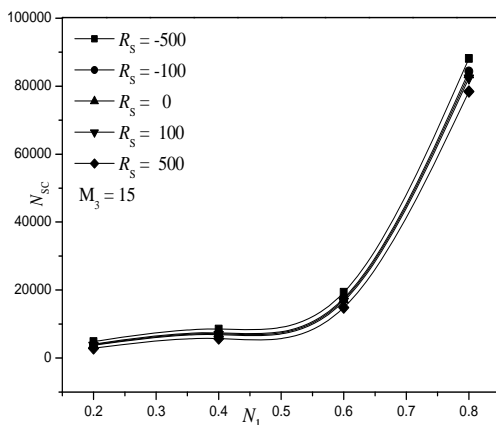
It is observed from Fig. 5 (a) that  $N_{sc}$  increases with increasing value of  $N_1$  for the fixed value of non-buoyancy magnetization parameter  $M_3 = 5$ . When  $N_1$  is increases, the concentration of microelements also increases, and as a result of this a greater part of the energy of the system is consumed by these elements in developing twist velocities in the fluid and onset of convection is delayed.



**Fig. 5 (a). Marginal instability curve for variation of  $N_{sc}$  versus  $N_1$  for various  $k$ ,**  
 $N_3' = 2, N_5' = 0.2,$

$\varepsilon = 0.5, \tau = 0.03, R_s = -500$  and  $M_3 = 5$ .

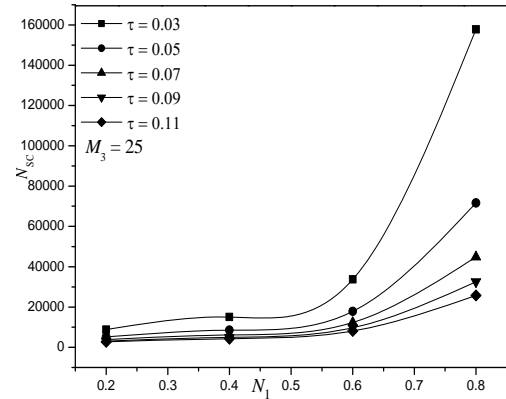
Moreover, Fig. 5 (b) analyzed for the non-buoyancy magnetization parameter  $M_3 = 15$  and the convective system have a same stabilization effect.



**Fig. 5(b). Marginal instability curve for variation of  $N_{sc}$  versus  $N_1$  for various  $R_s$ ,**  
 $N_3' = 2, N_5' = 0.2, \tau = 0.03, \varepsilon = 0.5, k = 0.1$   
**and  $M_3 = 15$ .**

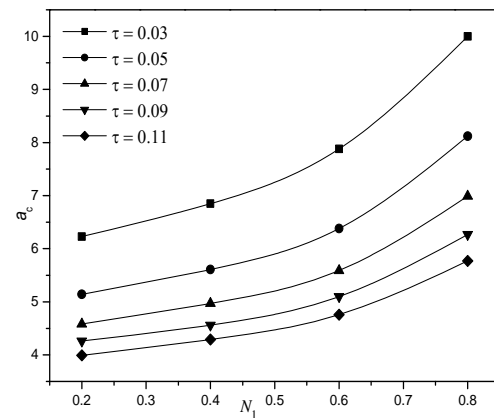
Likewise, Fig. 5 (c) investigated for  $M_3 = 25$  and the

system have a same stabilizing effect. However, nature of an increasing of non-buoyancy magnetization parameter  $M_3$  is destabilizing effect has been investigated by many authors, Vaidyanathan *et al.* (1995, 1997), Sekar *et al.* (2013a, 2013b). But an introducing of coupling parameter  $N_1$  on the convective system, the system has a stabilizing behavior.



**Fig. 5(c). Marginal instability curve for variation of  $N_{sc}$  versus  $N_1$  for various  $\tau$ ,**  
 $N_3' = 2, N_5' = 0.2,$   
 $k = 0.1, R_s = -500$  and  $M_3 = 25$ .

From Fig. 6, the cell shape and critical wave number  $a_c$  variation with respect to coupling parameter  $N_1$  for various  $\tau$ , indicate that the system stabilizes as coupling parameter  $N_1$  increases. This is indicated by an increase in  $a_c$ . This trend is seen for various values of the ratio of mass transport to heat transport  $\tau$ .



**Fig. 6. Variation of  $a_c$  versus  $N_1$  for various  $\tau$ ,**  
 $N_3' = 2, N_5' = 0.2, M_3 = 25, k = 0.1$  and  $R_s = -500$ .

## 6. CONCLUSION

The linear stability analysis of thermohaline convection in a micropolar ferromagnetic fluid layer heated from below and salted from above saturating a porous medium subject to transversed uniform magnetic field has been considered. In this investigation, the simplest boundary condition is chosen, namely free-free. Also, the case of two free

boundaries is mathematically important because one can derive exact solution, whose properties guide our analysis. Here we have investigated the different parameters like permeability of the porous medium  $k$ , buoyancy magnetization parameter  $M_1$ , non-buoyancy magnetization parameter  $M_3$ , thermal Rayleigh number  $R_C$ , Salinity Rayleigh number  $R_S$ , coupling parameter (coupling between vorticity and spin effects)  $N_1$ , spin diffusion  $N_3'$ , micropolar heat conduction parameter  $N_5'$ , ratio of mass transport to heat transport  $\tau$  and magnetic numbers  $M_4$ ,  $M_5$  and  $M_6$  on the onset of convection.

The critical magnetic thermal Rayleigh number for the onset of instability is depicted graphically for sufficient large values of buoyancy magnetization parameter  $M_1$ . Further the principle of exchange of instability is applied to find out mode of attaining instability.

We see in conclusion that convection can encourage in a micropolar ferromagnetic fluid by means of spatial variation in magnetization, which is induced when the magnetization of the fluid depends on temperature and salinity, and a uniform temperature and salinity gradients are established across the layer. This problem represents thermal-salinity-microrotational-mechanical interaction in a porous medium arising through the stress tensor, salinity and microrotation.

The destabilizing effect of non-buoyancy magnetization, medium permeability and spin diffusion parameter and stabilizing effect of coupling parameter and micropolar heat conduction parameter are discussed in different physical situations. We conclude that the magnetization parameters, micropolar parameters and salinity gradient have a profound influence on the onset of convection in a porous medium and coupling parameter and micropolar heat conduction parameter dominant the system. Because, the non-buoyancy magnetization parameter has a destabilizing effect in some of analyses Sekar *et al.* (2006, 2013, 2013a, 2013b) and Vaidyanathan *et al.* (2005), but an introducing of coupling and micropolar heat conduction effects on the convective system, the system leads stabilizing behavior.

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