Journal of Applied Fluid Mechanics, Vol. 9, No. 1, pp. 131-138, 2016.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.68.224.23967

MHD Flow of Micropolar Fluid due to a Curved Stretching Sheet with Thermal Radiation

M. Naveed[1†], Z. Abbas[1] and M. Sajid[2]

[1]*Department of Mathematics, The Islamia University of Bahawalpur, Bahawalpur 63100, Pakistan*
[2] *Theoretical Physics Division, PINSTECH, P.O. Nilore, Islamabad 44000, Pakistan*

†*Corresponding Author Email: rana.m.naveed@gmail.com*

(Received August 6, 2014; accepted October 1, 2014)

ABSTRACT

The two-dimensional boundary layer flow of an electrically conducting micropolar fluid and heat transfer subject to a transverse uniform magnetic field over a curved stretching sheet coiled in a circle of radius R has been studied. The effect of thermal radiation is also considered using linearized Rosseland approximation. For mathematical formulation of the flow equations, curvilinear coordinates system is used. The governing partial differential equations describing the flow phenomena and heat transfer characteristics are reduced to ordinary differential equations by means of suitable transformations. The system of differential equations is solved numerically by shooting method using Runge-Kutta algorithm combined with the Newtons-Raphson technique. Some physical features of the flow and heat transfer in terms of fluid velocity, angular velocity, temperature profile, the skin-friction coefficient, couple wall stress and the local Nusselt number for several values of fluid parameters are analyzed, discussed and presented in graphs and tables. Comparison of the present results with the published data for the flat surface i.e. $(k \to \infty)$ is found in good agreement.

Keywords: Micropolar fluid; MHD; Curved stretching surface; Thermal radiation; Numerical solution.

1. INTRODUCTION

The study of flow and heat transfer analysis over a stretching surface has great interests by many researchers in the last few years due to its wide range of applications in industries and engineering problems. Such applications are the extrusion of plastic sheets and rubber sheets, metal spinning and drawing plastic films, cooling of continues strips or filaments, crystal growing, glass blowing and paper production. In all the processes mentioned above the final product depends on the skin-friction and the rate of heat transfer at the surface. The study of the boundary layer flow over a flat surface moving with uniform speed was initiated by Sakiadis (1961). The pioneer work of Sakiadis has been extended by Crane (1970) for the stretching surface giving an exact closed form solution. Crane's work has been extended by many researchers in different directions for both Newtonian and non-Newtonian fluids. The study of the characteristics of heat transfer was carried out by Tsoue *et al*. (1967). The effects of combined heat and mass transfer on the fluid over a stretching surface with suction and injection were investigated by Gupta and Gupta (1977). A literature survey reveals that different aspects of the flow and heat transfer analysis with linear and power-law surface velocities have been investigated by many researchers. For details the readers are referred to Ishak *et al*. (2009), Bhattacharyya *et al*. (2013), Makinde (2010), Chamkha *et al*. (2006, 2010), Pal and Mondal (2014) and Anjalidevi and Kayalvizhi (2013).

The class of non-Newtonian fluids that deals with the micro-rotation of the suspended particles is the micropolar fluid. The micropolar theory was proposed by Eringen (1964, 1966), dealing with the effects of local rotational inertia and couple stresses, that cannot be explained by the classical Navier-Stokes equations. The mathematical modeling of micropolar fluid equations for the theory of lubrication and theory of porous media are given in the books by Eringen (2000) and Lukaszewiez (1999). The application of the micropolar fluids includes particle suspension, liquid crystals, animal blood, paints, lubrication and turbulent shear flows. Mixed convection flow of a micropolar fluid from an unsteady surface with viscous dissipation was investigated by El-Aziz

(2013).

The study of the MHD flow for an electrically conducting fluid with radiation effects on convective heat transfer are vital in process incorporating high temperature such as gas turbine, nuclear power plant, thermal energy storage, electrical power generation, solar power technology, space vehicle re-entry and other industrial area. The effects of thermal radiation and chemical reaction on MHD mixed convection heat and mass transfer in micropolar fluid was discussed by Srinivasacharya and Upendar (2013). Oahimire and Olajuwon (2014) have analyzed the effects of Hall current and thermal radiation on heat and mass transfer of a chemically reacting MHD flow of a micropolar fluid through porous medium. The effects of hydromagnetic convectional heat transfer in a micropolar fluid over a vertical plate are discussed by Ferdows *et al*. (2013). The combined effects of heat generation and radiation on MHD flow of a micropolar fluid past a moving surface is investigated by Reddy (2013). MHD flow and heat transfer of a micropolar fluid over a stretching surface with heat generation was carried out by Mostafa *et al*. (2012). The combined effects of heat and mass transfer in hydromagnetic micropolar fluid over a stretching sheet are analyzed by Kumar (2009). The effects of thermal radiation on unsteady MHD flow of a micropolar fluid with heat and mass transfer is discussed by Hayat and Qasim (2010). Rashidi *et al*. (2011) investigated the heat transfer of a micropolar fluid through a porous medium and obtained the analytic solution. Hayat *et al*. (2008) studied the mixed convection flow of a micropolar fluid over a non-linearly stretching sheet. The effects of rotating flow of a micropolar fluid induced by a stretching surface are discussed by Javed *et al*. (2010). The influence of suction on the micropolar fluid over a stretching/shrinking sheet through a porous medium is analyzed by Rosali *et al*. (2012). The effects of MHD stagnation point flow of a micropolar viscoelastic fluid towards a stretching/shrinking sheet with heat generation are investigated by Abbas *et al*. (2014).

In all above studies, the flow is considered over a flat surface, in which Cartesian coordinates are used to model the flow phenomena. However, Sajid *et al*. (2010) modeled the flow problem using curvilinear coordinates systems by introducing a curvature in the surface. In another paper, Sajid *et al*. (2011) discussed the flow of a micropolar fluid over a curved stretching surface. The effect of partial slip embedded in a porous medium on a curved stretching sheet is analyzed by Sajid and Iqbal (2011). Recently, Abbas *et al*. (2013) discussed the effects of heat transfer over a curved surface in the presence of magnetic field. The aim of present study is to discuss the effects of radiation in the presence of uniform magnetic field for a micropolar fluid over a curved stretching surface. The present work is novel because it extends the analysis of stretching flow with thermal radiation effects in the general case for a curved stretching sheet. Moreover, the results of the previous studies can be obtained from the present solutions as special cases. Numerical solution for the fluid velocity and temperature distributions are obtained using Runge-Kutta algorithm. Numerical results are presented through graphs and tables.

2. MATHEMATICAL FORMULATION

Consider the steady, two-dimensional and incompressible flow of a micropolar fluid past a curved stretching sheet coiled in a circle of radius R. Two equal and opposite forces are applied along the s-direction so that the sheet is stretched keeping the origin fixed and r-direction is perpendicular to it. The stretching velocity of the surface is $u = as$, where $(a > 0)$ is the stretching constant. The fluid is electrically conducting and a constant magnetic field $B_0$is assumed to be applied in the r-direction. The magnetic Reynolds number is taken to be very small so that the induced magnetic field can be neglected. The temperature of the surface is maintained at T_w, where $T_w > T_\infty$ with T_∞ being the uniform temperature of the ambient fluid. Under these assumptions along with boundary layer approximation and neglecting viscous dissipation, the governing equations for micropolar fluid in the presence of Lorentz force are

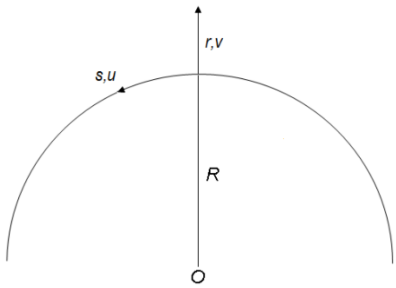

Fig. 1. Geometry of the curved surface.

$$\frac{\partial}{\partial r}\{(r+R)v\} + R\frac{\partial u}{\partial s} = 0, \tag{1}$$

$$\frac{u^2}{r+R} = \frac{1}{\rho}\frac{\partial p}{\partial r}, \tag{2}$$

$$v\frac{\partial u}{\partial r} + \frac{Ru}{r+R}\frac{\partial u}{\partial s} + \frac{uv}{r+R} = -\frac{1}{\rho}\frac{R}{r+R}\frac{\partial p}{\partial s} + \left(\nu + \frac{\kappa}{\rho}\right)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r+R}\frac{\partial u}{\partial r} - \frac{u}{(r+R)^2}\right) - \frac{\kappa}{\rho}\frac{\partial N}{\partial r} - \frac{\sigma B_0^2}{\rho}u, \tag{3}$$

$$v\frac{\partial N}{\partial r} + \frac{Ru}{r+R}\frac{\partial N}{\partial s} = \frac{\gamma}{\rho j}\left(\frac{\partial^2 N}{\partial r^2} + \frac{1}{r+R}\frac{\partial N}{\partial r}\right) - \frac{\kappa}{\rho j}\left(2N + \frac{\partial u}{\partial r} + \frac{u}{r+R}\right), \tag{4}$$

$$\rho c_p\left[v\frac{\partial T}{\partial r} + \frac{uR}{r+R}\frac{\partial T}{\partial s}\right] = k_1\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r+R}\frac{\partial T}{\partial r}\right] - \frac{1}{(r+R)}\frac{\partial}{\partial r}(r+R)q_r, \tag{5}$$

where u and v are the velocity components in s and r-directions, respectively, ρ is the fluid density, p is the pressure, ν is the kinematics viscosity of fluid, σ is the electrical conductivity, $N$is the micro-rotation in the rs-plane, j is the micro-inertial per unit mass, γ is the spin gradient viscosity, κ is the vortex viscosity, c_p is the specific heat at constant pressure, q_r is the radiative heat flux, $k_1$is the thermal conductivity and T is the temperature. It may be noted that for a curved stretching surface pressure is no more constant inside the boundary layer (Sajid *et al*. (2010)).

According to the study of Nazar *et al*. (2004) and Rees and Pop (1998), the definition of γ is

$$\gamma = \left(\mu + \frac{\kappa}{2}\right) j, \tag{6}$$

where μ is the fluid viscosity and $j = \nu/as$ is the reference length. As represented by Ahmadi (1976), relation (6) is mentioned to allow Eqs. (1)-(4) to present the correct behavior in the limiting case when microstructure effects become negligible and micro-rotation N reduces to the angular velocity.

The appropriate boundary conditions for the flow problem are

$$u = as,\ v = 0,\ N = -m_0 \frac{\partial u}{\partial r},\ T = T_w\ at\ r = 0,$$
$$u \to 0,\ \frac{\partial u}{\partial r} \to 0,\ N \to 0,\ T \to T_\infty\ as\ r \to \infty. \tag{7}$$

where $m_0\ (0 \le m_0 \le 1)\ is$ a constant and a has the dimension of (time)$^{-1}$. The case $m_0 = 0$, which leads $N = 0$ at the wall, represents concentrated particle in which the microelements close to the wall sheet are unable to rotate (Jena and Mathur (1981)). This case is also known as the strong concentration of microelements (Guram and Smith (1980)). The case $m_0 = 1/2$ shows the vanishing of anti-symmetric part of the stress tensor and denotes weak concentration (Ahmadi, G. (1976)) of microelements. It can, however, be easily proved that for $m_0 = 1/2$ the governing equations can be converted to the classical problem of steady boundary layer flow of a viscous (Newtonian) incompressible fluid near the plane wall. The case $m_0 = 1$, as pointed by Peddieson (1972) is used for the modeling of turbulent boundary layer flows. The true boundary condition to be used to the spin is still an open question (see Nazar *et al*. (2004)). However, the most common boundary condition discussed in the literature is the vanishing of the spin on the boundary, so-called strong interaction. The opposite extreme, the weak interaction, is the vanishing of the momentum stress on the boundary microelements (Guram and Smith (1980)). A third or compromise is the vanishing of a linear combination of spin, shearing stress and couple stress, involving some friction coefficients a particular case of which was the condition used by Peddieson (1972). In the present analysis, we shall consider here only the case of $m_0 = 0$.

Under the Rosseland approximation for radiation (Rosseland (1931)) applies to optically thick media, the radiative heat flux is given by

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial r}, \tag{8}$$

where σ^* is the Stefan-Boltzman constant and k^* is the mean absorption coefficient. Under the assumption of temperature differences within the flow are sufficiently small, we may expand the term T^4 as a linear function of the temperature in a Taylor series about T_∞ and neglecting the higher terms, one can get

$$T^4 = 4T_\infty^3 T - 3T_\infty^4. \tag{9}$$

Now with Eqs. (8) and (9), Eq. (5) can be written as

$$\left[v\frac{\partial T}{\partial r} + \frac{uR}{r+R}\frac{\partial T}{\partial s}\right] = \frac{1}{\rho c_p}\left(k_1 + \frac{16\sigma^* T_\infty^3}{3k^*}\right)\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r+R}\frac{\partial T}{\partial r}\right]. \tag{10}$$

Following Magyari and Pantokratoras (2011), we take $N_r = 16\sigma^* T_\infty^3/3k_1k^*$ as a radiation parameter, so Eq. (10) becomes

$$\left[v\frac{\partial T}{\partial r} + \frac{uR}{r+R}\frac{\partial T}{\partial s}\right] = \left(\frac{1+N_r}{P_r}\right)\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r+R}\frac{\partial T}{\partial r}\right], \tag{11}$$

where $P_r = \mu c_p/k_1$ is the Prandtl number.
To simplify the flow equations, we define the following new variables

$$u = asf'(\eta)\ \ ,\ v = \frac{-R}{r+R}\sqrt{a\nu} f(\eta),\ N = as\sqrt{\frac{a}{\nu}}g,$$
$$\eta = \sqrt{\frac{a}{\nu}}r,\ \ p = \rho a^2 s^2 P(\eta),\ \ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \tag{12}$$

With the help of above equation, the continuity equation given by Eq. (1) is automatically satisfied and Eqs. (2)-(5) yield

$$\frac{\partial P}{\partial \eta} = \frac{f'^2}{\eta + k}, \tag{13}$$

$$\frac{2k}{\eta+k}P = (1+K)\left[f''' + \frac{f''}{\eta+k} - \frac{f'}{(\eta+k)^2}\right] - \frac{k}{\eta+k}f'^2 + \frac{k}{\eta+k}ff'' + \frac{k}{(\eta+k)^2}ff' - Kg' - M^2 f', \tag{14}$$

$$\left(1+\frac{K}{2}\right)\left[g'' + \frac{g'}{\eta+k}\right] + \frac{k}{\eta+k}fg' - \frac{k}{\eta+k}f'g - K\left(2g + f'' + \frac{f'}{\eta+k}\right) = 0, \tag{15}$$

$$(1+N_r)\left(\theta'' + \frac{\theta'}{\eta+k}\right) + P_r\frac{k}{\eta+k}f\theta' = 0, \tag{16}$$

where $K = \kappa/\nu$ is the material parameter, $k = R\sqrt{a/\nu}$ is the dimensionless radius of curvature and $M^2 = \sigma B_0^2/\rho a$ is the Hartmann number or magnetic parameter. The results in the case when there is no thermal radiation can be retrieved by taking $N_r = 0$. The corresponding boundary conditions becomes

$$f(0) = 0,\quad f'(0) = 1,\quad g(0) = 0,\quad \theta(0) = 1,$$
$$f'(\infty) = 0,\ \ f''(\infty) = 0,\quad g(\infty) = 0,\ \theta(\infty) = 0. \tag{17}$$

On elimination of pressure between Eqs. (13) and (14), one obtain

$$f^{iv} + \frac{2f'''}{\eta+k} - \frac{f''}{(\eta+k)^2} + \frac{f'}{(\eta+k)^3} - \frac{k}{\eta+k}(f'f'' - ff''') - \frac{k}{(\eta+k)^2}\left(f'^2 - ff''\right) - \frac{k}{(\eta+k)^3}ff' - M^2\left(f'' + \frac{f'}{\eta+k}\right) = 0. \tag{18}$$

Once we obtain the fluid velocity $f(\eta)$, the pressure can be determined from Eq. (14) in the following form

$$P = \frac{\eta+k}{2k}\left[(1+K)\{f''' + \frac{f''}{\eta+k} - \frac{f'}{(\eta+k)^2}\} - \frac{k}{\eta+k}f'^2 + \frac{k}{\eta+k}ff'' + \frac{k}{(\eta+k)^2}ff' - Kg' - M^2 f'\right]. \tag{19}$$

The physical quantities of interest are the skin-friction coefficient, couple stress coefficient and the local Nusselt number along the s-direction, which are defined as

$$C_f = \frac{\tau_{rs}}{\rho u_w^2},\ C_m = \frac{M_w}{\mu j u_w},\ Nu_s = \frac{s q_w}{k_1(T_w - T_\infty)}, \tag{20}$$

in which τ_{rs} is the wall shear stress, M_w is the wall couple stress and q_w is the heat flux at the wall along the s-direction, which are given by

$$\tau_{rs} = (\mu+\kappa)\left(\frac{\partial u}{\partial r}-\frac{u}{r+R}\right)+\kappa N\Big|_{r=0}, \quad M_w = \gamma \frac{\partial N}{\partial r}\Big|_{r=0},$$

$$q_w = -k_1\left(1+\frac{16\sigma^* T_\infty^3}{3k_1 k^*}\right)\frac{\partial T}{\partial r}\Big|_{r=0}, \qquad (21)$$

Using Eqs. (12) and (21), Eq. (20) becomes

$$Re_s^{\frac{1}{2}} C_f = (1+K)\left\{f''(0)-\frac{f'(0)}{k}\right\},$$

$$Re_s C_m = \left(1+\frac{K}{2}\right)g'(0),$$

$$Re_s^{-1/2} Nu_s = -(1+N_r)\theta'(0),$$

where $R_{es} = as^2/\nu$ is the local Reynolds number.

3. RESULTS AND DISCUSSION

We compute the velocity profile and characteristics of the heat transfer in the flow by solving Eqs. (15), (16) and (18) subject to boundary conditions (17) numerically using fourth order Runge-Kutta algorithm along with Newton-Raphson method. The graphical results for magnetic parameter M, material parameter K, Prandtl number P_r and radiation parameter N_r are presented through s. 2-11. The effect of dimensionless radius of curvature and fluid parameters on pressure distribution has already been discussed in Abbas *et al*. (2013). Furthermore, the magnitude of the skin friction coefficient $Re_s^{1/2} C_f$, local Nusselt number $Re_s^{-1/2} Nu_s$ and couple stress coefficient $Re_s C_m$ for different parameters are presented both graphically and in tabular form.

Fig. 2 is made to show the variation of the material parameter Kon the horizontal component of velocity $f'(\eta)$ by keeping $k = 7$ and $M = 0.8$ fixed. It can be seen from this Fig. that the velocity of the fluid increases with increase in K. It is due to the fact that microconcentration of the fluid increased by increase in material parameter which alter the flow field and hence enhance the boundary layer thickness.

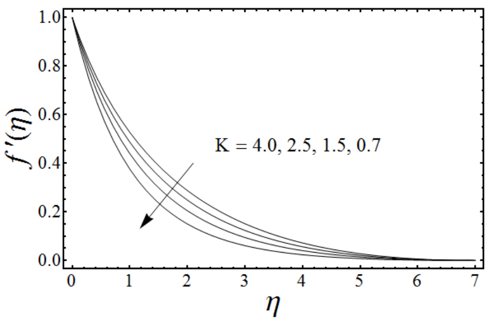

Fig. 2. Influence of the material parameter K on the velocity profile $f'(\eta)$ with $M = 0.8$ and $k = 7$.

Fig. 3 elucidate the change in the horizontal component of velocity $f'(\eta)$ for various values of the magnetic parameter M by keeping $K = 1$ and $k = 7$ fixed. It is observed from this Fig. that both the velocity and momentum boundary layer thickness decreased by increasing the value of M. It is because that the influence of the magnetic field act as a resistance to the fluid velocity, which reduce the velocity of the fluid.

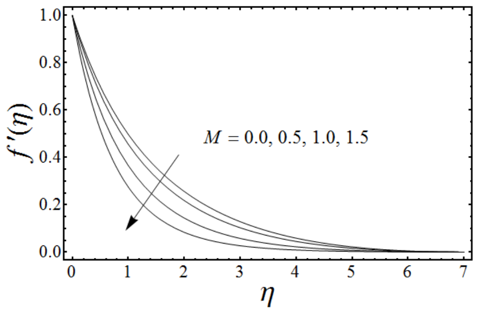

Fig. 3. Influence of the magnetic parameter M on the velocity profile $f'(\eta)$ with $K = 1$ and $k = 7$.

The effect of material parameter on the microrotation profile is displayed in Fig. 4. It is noticed from this Fig. that the microrotation of the fluid increases with an increase in the K.

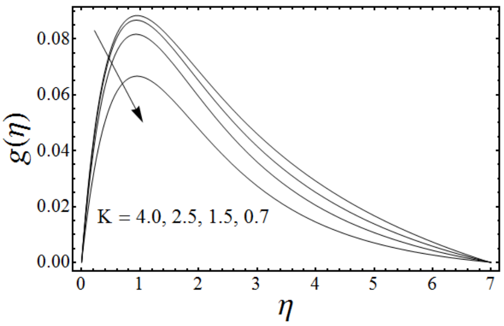

Fig. 4. Influence of the material parameter K on the Micro-Rotation $g(\eta)$ with $M = 0.5$ and $k = 7$.

Fig. 5 depicts the effects of magnetic parameter M on the microrotation. It is noticed from this Fig. that initially the microrotation of the fluid decreases by increasing the value of M but after $(\eta = 2)$ it has reverse behavior, i.e. the microrotation of the fluid increases as M increased.

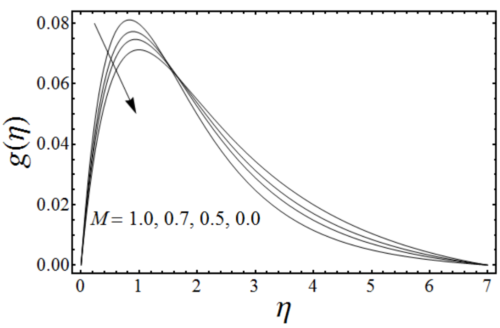

Fig.5. Influence of the magnetic parameter M on the Micro-Rotation $g(\eta)$ with $K = 1$ and $k = 7$.

Fig. 6 demonstrates the effects of Prandtl number P_r on the temperature distribution by keeping $K = 1$, $k = 7$, $N_r = 2$ and $M = 0.5$ fixed. It is found that temperature of the fluid decreases as P_r increases.

The thermal boundary layer also decreases in this case. The reason for this behavior is that, by increasing P_r rate of thermal diffusion decreases.

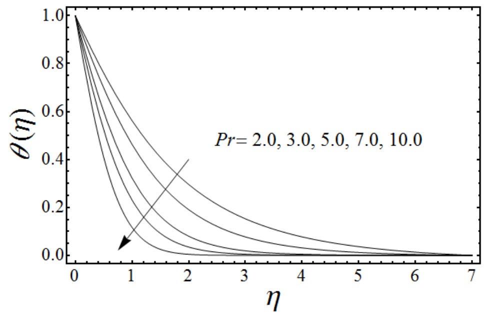

Fig. 6. Influence of the Prandtl number P_r on the temperature profile $\theta(\eta)$ with $K = 1$, $N_r = 2$, $M = 0.5$ and $k = 7$.

Fig. 7 illustrates the variation of the radiation parameter N_r on the temperature distribution. It is observed from this Fig. that the temperature and thermal boundary layer thickness is increased by increase in radiation parameter N_r.

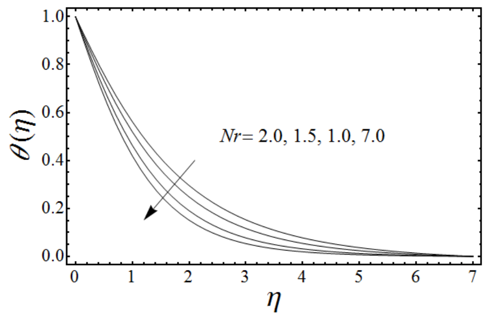

Fig. 7. Influence of the radiation parameter N_r on the temperature profile $\theta(\eta)$ with $K = 1$, $P_r = 2$, $M = 0.5$ and $k = 7$.

Fig. 8 shows the behavior of the temperature profile with material parameter by keeping other parameters fixed. It is clear from this Fig. that temperature of the fluid is decreased with an increase in the value of K.

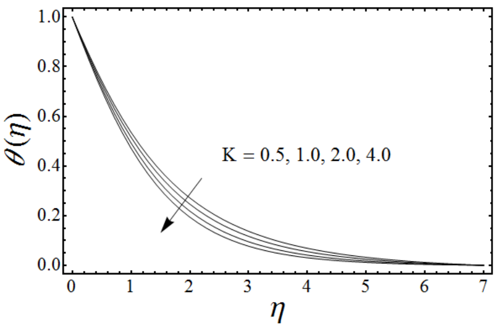

Fig. 8. Influence of the material parameter K on the temperature profile $\theta(\eta)$ with $M = 0.8$, $P_r = 1.5$, $N_r = 0.7$ and $k = 7$.

The effect of magnetic parameter M on temperature profile is shown in Fig. 9. It is found that the temperature of the fluid increases by increasing M.

Fig. 10 gives the change in the magnitude of the local Nusselt number $Re_s^{-1/2}Nu_s$ verses Prandtl number P_r for different values of the radiation parameter N_r. From this Fig. it is evident that the absolute values of $Re_s^{-1/2}Nu_s$ are increased by increasing the Prandtl number P_r and radiation parameter N_r.

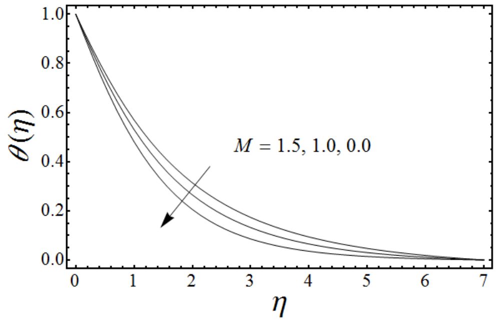

Fig. 9. Influence of the magnetic parameter M on the temperature profile $\theta(\eta)$ with $K = 1$, $P_r = 1.5$, $N_r = 0.7$ and $k = 7$.

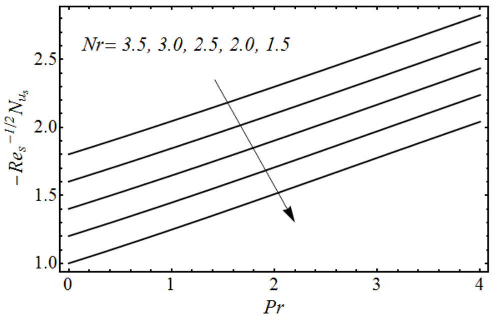

Fig. 10. Influence of radiation parameter N_r on the local Nusselt number $Re_s^{-1/2}Nu_s$ with P_r and $k = 7$, $M = 0.2$ and $K = 0.2$.

Fig. 11 indicates the absolute value of the local Nusselt number $Re_s^{-1/2}Nu_s$ verses radiation parameter Nr for various values of the magnetic parameter M. It is observed from this Fig. that the absolute value of $Re_s^{-1/2}Nu_s$ increases by increasing the value of Nr, however it is decreased by increasing the value of M.

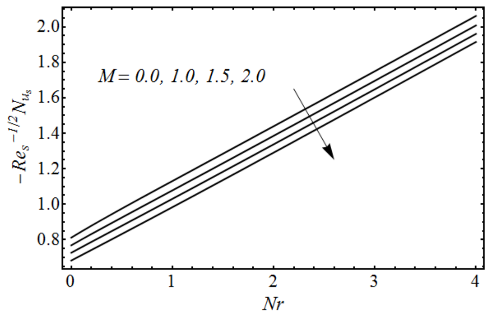

Fig. 11. Influence of magnetic parameter M on the local Nusselt number $Re_s^{-1/2}Nu_s$ with N_r and $k = 7$, $P_r = 1.5$ and $K = 1$.

Table. 1 is made to check the validity and accuracy of the present results with the results published by Hayat and Qasim (2010) (in the case of flat stretching sheet) by taking the dimensionless radius of curvature $k \to \infty$ i.e. $k = 1000$, and are found in good agreement. Hayat and Qasim (2010) obtained the analytic solutions using homotopy analysis method while we have obtained numerical solutions by shooting method using Runge-Kutta algorithm and both gives the same results in the special case when the surface is flat.

Table 1 Comparison of the values of skin friction coefficient $Re_s^{1/2}C_f$ for different values of M and K keeping $k = 1000$ fixed

M	K	Hayat and Qasim (2010)	Present results
0.0	1	1.3678	1.3678
0.5		1.5305	1.5305
1.0		1.9422	1.9422
1.5		2.4873	2.4873
0.5	0	1.1180	1.1180
	1	1.5305	1.5305
	2	1.8152	1.8152
	4	2.2456	2.2456

Table. 2 indicate the numerical values of the couple stress coefficient $Re_s C_m$ for different values of K and M by keeping $k = 7$ fixed. It is noticed from this table that magnitude of couple stress coefficient increases by increasing the value of K and M.

Table 2 Numerical values of the couple stress coefficient for $Re_s C_m$ for different values of K and M keeping $k = 7$ fixed

K	M	$\mathrm{Re}_s\, C_m$
1	1	0.4297
2		0.7201
3		0.9790
4		1.2258
1	0	0.3688
	0.5	0.3860
	1	0.4297
	1.5	0.4843
	2	0.5379

4. CONCLUDING REMARKS

In the present analysis, the boundary layer flow of a viscous fluid over a steady curved stretching sheet with effects of radiation is discussed. The resultant non-linear equations are solved numerically using a Runge-Kutta method combined with Newton-Raphson technique. The physical interpretation of the fluid velocity, temperature distribution, skin friction coefficient, couple stress coefficient and local Nusselt number for various values of involved parameters are given graphically and in tabular form. The following conclusions have been made from this study:

- By increasing the value of K the velocity and microrotation of the fluid increases while temperature distribution decreases.
- Both the momentum boundary layer thickness and the flow velocity are decreased by increasing the value of M but the microrotation of the fluid and temperature of the fluid increased by increasing M.
- An increase in a radiation parameter Nr temperature and the thermal boundary layer increases.
- Temperature and the thermal boundary layer thickness decreases by an increase in the Prandtl and radiation parameter.
- The absolute value of the local Nusselt number increases by increasing both the Prandtl number Pr as well as radiation parameter Nr but it decreases by increasing M.
- The magnitude of couple stress coefficient increases with an increase in material parameter K and magnetic parameter M.

The present work can be extended for other non-Newtonian fluids that exhibit different nonlinear characteristics like normal stress effects, shear thinning, shear thickening, stress relaxation and retardation.

REFERENCES

Abbas, Z., M. Naveed and M. Sajid (2013). Heat transfer analysis for stretching flow over curved surface with magnetic field. *Journal of Engineering Thermo Physics.* 22(4),337-345.

Abbas, Z., Sheikh, M. and M. Sajid (2014). Hydromagnetic stagnation point flow of a micropolar viscoelastic fluid towards a stretching/shrinking sheet in the presence of heat generation. *Canadian Journal of Physics*.

Abd El-Aziz, M. (2013). Mixed convection flow of a micropolar fluid from an unsteady stretching surface with viscous dissipation. *Journal of the Egyptian Mathematical Society 21, 385-394.*

Ahmadi, G. (1976). Self-similar solution of incompressible micropolar boundary layer flow over a semi-infinite flat plate, *International Journal of Engineering Science 14, 639-646.*

Anjalidevi, S. P. and M. Kayalvizhi (2013). Nonlinear hydromagnetic flow with radiation and heat source over a stretching surface with prescribed heat and mass transfer embedded in a porous medium, Journal of Applied Fluid Mechanics 6(2),157-165.

Bhattacharyya, K., S. Mukhopadhyay and G. C. Layek (2013). Unsteady MHD boundary layer flow with diffusion and first-order chemical reaction over a permeable stretching sheet wth suction or blowing. *Chemical Engineering Communication 200, 379-397.*

Crane, L. J. (1970). Flow past a stretching plat. *Journal of Applied Mathematics and Mechanics 21, 645-647*.

Chamkha, A. J., I. Pop and H. S. Takhar (2006). Marangoni mixed convection boundary layer flow. *Meccanica 41, 219-232*.

Chamkha, A. J., A. M. Aly and M. A. Mansour (2010). Similarity solution for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction/injection and chemical reaction effects. *Chemical Engineering Communications 197, 846-858*.

Eringen, A. C. (1964). Simple microfluids. *International Journal of Engineering Science 2, 205-207.*

Eringen, A. C. (1966). Theory of micropolar fluids. *Journal of Mathematics and Mechanics 16, 1-18.*

Eringen, A. C. (2000). *Microcontinuum Field Theories II: Fluent Media*, Springer, New York.

Ferdows, M., P. Nag, A. Postelnicu and K. Vajrevelu (2013). Hydro-magnetic convection heat transfer in a micropolar fluid over a vertical plate. *Journal of Applied Fluid Mechanics 6,(2), 285-299.*

Gnaneswara Reddy, M. (2013). Heat generation and radiation effects on steady MHD free convection flow of a micropolar fluid past a moving surface. *Journal of Computational and Applied Research in Mechanical Engineering 2(2),1-10.*

Gupta, P. S. and A. S. Gupta (1977). Heat and mass transfer on a stretching sheet with suction and blowing. The *Canadian Journal of Chemical Engineering 55, 744-746.*

Guram, G. S. and A. C. Smith (1980). Stagnation flows of micropolar fluids with strong and weak interactions. *Computers and Mathematics* 6, 213-233.

Hayat, T., Z. Abbas and T. Javed (2008). Mixed convection flow of a micropolar fluid over a non-linearly stretching sheet. *Physics Letters A 372, 637-647.*

Hayat, T. and M. Qasim (2010). Effects of thermal radiation on unsteady magnetohydrodynamic flow of a micropolar fluid with heat and mass transfer. *Naturforsch 65(a), 950-960*.

Ishak, A., R. Nazar and I. Pop (2009). Heat transfer over an unsteady stretching permeable surface with prescribed wall temperature, Non-linear Analysis. *Real World Applications 10, 2909-2913.*

Javed, T., I. Ahmad, Z. Abbas and T. Hayat (2010). Rotating flow of a micropolar fluid induced by a stretching surface. *Z. Naturforsch 65(a), 829-836.*

Jena, S. K. and M. N. Mathur (1981). Similarity solutions for laminar free convection flow of a thermo-micropolar fluid past a non-isothermal flat plate. *International Journal of Engineering Science 19, 1431-1439*.

Kumar, L. (2009). Finite element analysis of combined heat and mass transfer in hydromagneticmicropolar flow along a stretching sheet. *Computational Materials* Science 46, 841-848.

Lukaszewicz, G. (1999*). Micropolar Fluids: Theory and Applications*, Birkhauser, Boston.

Mahmood, M. A. A. and S. E. Waheed (2012). MHD flow and heat transfer of a micropolar fluid over a stretching surface with heat generation (absorption) and slip velocity. *Journal of the Egyptian Mathematical Society 20, 20-27*.

Makinde, O. D. (2010). Similarity solution of hydromagnetic heat and mass transfer over a vertical plate with a convective surface boundary condition. International Journal of the Physical Sciences 5(6),700-710.

Nazar, R., N. Amin, D. Filip and I. Pop (2004). Stagnation point flow of a micropolar fluid towards a stretching sheet, *International Journal of Non-linear Mechanics 39, 1227-1235.*

Oahimire, J. I. and B. I. Olajuwon (2014). Effects of Hall current and thermal radiation on heat and mass transfer of a chemically reacting MHD flow of a micropolar fluid through a porous medium. *Journal of King Saud University Engineering Sciences 26, 112-121.*

Pal, D. and H. Mondal (2014). Soret-Dufour effects on hydromagnetic non-Darcy convective radiative heat and mass transfer over a stretching sheet in porous medium with viscous dissipation and Ohmic heating. *Journal of Applied Fluid Mechanics 7(3),513-523.*

Peddieson, J. (1972). An application of the micropolar fluid model to the calculation of turbulent shear flow. *International Journal of Engineering Science 10, 23-32*.

Rashidi, M. M., S. A. Mohimanian and S. Abbasbandy (2011). Analytic approximation solutions for heat transfer of a micropolar fluid through a porous medium with radiation. *Communications in Non-linear Science and Numerical Simulation 16, 1874-1879.*

Rees, D. A. S. and I. Pop (1998). Free convection boundary layer flow of a micropolar fluid from a vertical flat plate. *IMA Journal of Applied Mathematics 61, 179-197.*

Rosali, H., A. Ishak and I. Pop (2012). Micropolar fluid flow towards a stretching/shrinking sheet in a porous medium with suction, International Communications in Heat and Mass Transfer 39, 826-829.

Rosseland, S. (1931). Astrophysik und Atom-Theoretische Grundlagen, Springer-Verlag, Berlin 41-44.

Rosseland, S., E. Magyari and A. Pantokratoras (2011). Note on the effect of thermal radiation in the linearized Rosseland approximation on the heat transfer characteristic of various boundary layer flows. *International Communications in Heat and Mass Transfer 38, 554-556.*

Sakiadis, B. C. (1961). Boundary layer behavior on continuous solid surface: I. The boundary layer on a continuous flat surface. *AICHE 7(1),26-28.*

Sakiadis, B. C. (1961). Boundary layer behavior on continuous solid surface II. The boundary layer on a continuous flat surface. *AICHE 7(2), 221-225.*

Sajid, M., N. Ali, T. Javed and Z. Abbas (2010). Stretching a curved surface in a viscous fluid, Chinese Physics Letters 27, 024703.

Sajid, M., N. Ali, T. Javed and Z. Abbas (2011). Flow of a micropolar fluid over a curved stretching surface. *Journal of Engineering Physics and Thermo physics 84, 798-804.*

Sajid, M. and S. A. Iqbal (2011). Stretching a curved surface through porous medium in a micropolar fluid with partial slip. *International Journal of Mechanics Systems Engineering 1(1),1-7.*

Srinivasacharya, D. and M. Upendar (2013). Thermal radiation and chemical reaction effects on MHD mixed convection heat and mass transfer in a micropolar fluid. *Mechanica 19(2),518-525.*

Tsou, K., E. M. Sparrow and R. J. Goldstein (1967). Flow and heat transfer in the boundary layer on a continuous moving surface. *International Journal of Heat and Mass Transfer 10, 219-235*.