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# Mixed Convection Flow of Couple Stress Fluid in a Vertical Channel with Radiation and Soret Effects

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## ABSTRACT

The radiation and thermal diffusion effects on mixed convection flow of couple stress fluid through a channel are investigated. The governing non-linear partial differential equations are transformed into a system of ordinary differential equations using similarity transformations. The resulting equations are then solved using the Spectral Quasi-linearization Method (QLM). The results, which are discussed with the aid of the dimensionless parameters entering the problem, are seen to depend sensitively on the parameters.

**Keywords**: Couple stress fluid; Mixed convection; Soret effect; Radiation effect; SQLM.

## NOMENCLATURE

| | |
|---|---|
| A | constant pressure gradient |
| Br | Brinkman number |
| C | concentration |
| $C_p$ | specific heat at constant pressure |
| $C_S$ | concentration susceptibility |
| $C_T$ | temperature ratio |
| D | solutal diffusivity |
| f | reduced stream function |
| g | acceleration due to gravity |
| $Gr_C$ | mass Grashof number |
| $Gr_T$ | Temperature Grashof number |
| $K_f$ | coefficient of thermal conductivity |
| $K_T$ | thermal diffusion ratio |
| Nu | Nusselt number |
| p | pressure |
| Pr | Prandtl number |
| $q^r$ | radiation heat flux |
| R | suction induction parameter |
| Re | Reynolds number |
| S | couple stress parameter |
| Sc | Schmidt number |
| Sh | Sherwood number |
| $S_r$ | Soret number |

| | |
|---|---|
| $T$ | temperature |
| $T_m$ | mean fluid temperature |
| u, v | velocity components in the x and y directions respectively |
| x, y | cartesian coordinates along the plate and normal to it |
| $\alpha$ | thermal diffusivity |
| $\beta_T$, $\beta_C$ | coefficients of thermal and solutal expansion |
| $\chi$ | mean absorption coefficient |
| $\eta$ | similarity variable |
| $\eta_1$ | coupling material constant |
| $\sigma$ | stefan-Boltzman constant |
| $\theta$ | dimensionless temperature |
| $\phi$ | dimensionless concentration |
| $\mu$ | dynamic viscosity |
| $\nu$ | kinematic viscosity |
| $\rho$ | density of the fluid |

## 1. INTRODUCTION

In space technology applications and at higher operating temperatures, radiation effects can be quite significant. Since radiation is quite complicated, many aspects of its effect on free convection or combined convection have not been studied in recent years. It is therefore of great significance and interest to the researchers to investigate combined convective and radiative flow and heat transfer aspects. Radiative convective flows are frequently encountered in many scientific and environmental processes such as astrophysical flows, water evaporation from open reservoirs, heating and cooling of chambers and solar power technology. Heat transfer by simultaneous radiation and convection has applications in numerous technological problems including combustion, furnace design, the design of high temperature gas cooled in nuclear reactors, nuclear reactor safety, fluidized bed heat exchanger, fire spreads, solar fans, solar collectors, natural convection in cavities, turbid water bodies, photo chemical reactors and many others. Where as mixed convection in vertical parallel-plate channel is relevant to a wide range of applications such as the cooling of electronic equipments in which circuit cards containing heat generating electronic devices are arrayed to form vertical channels, the design of solar panels, energy efficient buildings, heat removal in nuclear technology, and a host of others. The combined radiation and mixed convection from a vertical wall with suction/injection in a non- Darcy porous medium was studied by Murthy *et al*. (2004). Grosan and Pop (2004) considered the effect of thermal radiation on fully developed mixed convection flow in a vertical channel. Raptis (2009) studied the influence of radiation on free convection flow through a porous medium. Mahfooz and Hossain (2012) presented a numerical study of conduction radiation effect on transient natural convection with thermophoresis. Most recently, radiation effects on mixed convection about a cone embedded in a porous medium filled with a nanofluid have been presented numerically by Chamkha *et al*. (2013). The diffusion of mass due to temperature gradient is called Soret or thermo-diffusion effect and this effect might become significant when large density differences exist in the flow regime. For example, Soret effect can be significant when species are introduced at a surface in fluid domain, with a density lower than the surrounding fluid. The Soret parameter has been utilized for isotope separation and in a mixture between gases with very light molecular weight ($N_2$, He) and of medium molecular weight ($N_2$, air). Dursunkaya and Worek (1992) studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams (1995) presented the same effects on mixed convective and mass transfer steady laminar boundary layer flow over a vertical flat plate with temperature dependent viscosity. Cheng (2009) studied the Soret and Dufour effects on natural convection heat and mass transfer from a vertical cone in a porous medium. Maleque (2010) examined the Dufour and Soret effects on unsteady MHD convective heat and mass transfer flow due to a rotating disk. Pal and Mondal (2011) analyzed the MHD non-Darcian mixed convection over a non-linear stretching sheet with Soret - Dufour effects with chemical reaction. Recently, Srinivasacharya and Kaladhar (2013) have investigated the Soret and Dufour effects on free convection flow of a couple stress fluid in a vertical channel with chemical reaction. Most recently, they (Srinivasacharya and Kaladhar (2014)) presented the chemical reaction, Soret and Dufour effects on mixed convection flow of couple stress fluid between vertical parallel plates.

Understanding and modeling the flows of non-Newtonian fluids are of both fundamental and practical significance in the industrial and engineering applications. With the growing importance of non- Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. The theory of polar fluids and related theories are models for fluids whose microstructure is mechanically significant. The micro-continuum theory of couple stress fluid proposed by Stokes (1966), defines the rotational field in terms of the velocity field for setting up the constitutive relationship between the stress and strain rate. Also, the study of a couple stress fluids is very useful in understanding various physical problems because it possesses the mechanism to describe rheological complex fluids such as liquid crystals and human blood. The rheological characteristics of such fluids are important in the flows of nuclear fuel slurries, lubrication with heavy oils and greases, paper coating, plasma and mercury, fossil fuels, polymers etc. In view of applications, Sunil and Devi (2012) described the global stability for thermal convection in a couple stress fluid saturating in a porous medium with temperature-pressure dependent viscosity using Galerkin method. Srinivasacharya and Kaladhar (2011, 2012) discussed the convection flow of couple stress fluid with Hall and Ion-slip effects in different geometries. Recently, Muthuraj *et al*. (2013) have studied the heat and mass transfer effects on MHD flow of a couple-stress fluid in a horizontal wavy channel with viscous dissipation and porous medium. Most recently Hayat *et al*. (2013) analyzed the stagnation point flow of couple stress fluid with melting heat transfer and the analytical study of Hall and Ion-slip effects on mixed convection flow of couple stress fluid between parallel disks have been presented by Srinivasacharya and Kaladhar (2013).

In this paper, the mixed convection flow of a couple stress fluid is investigated through a vertical channel in presence of thermal radiation and Soret effect. The Spectral quasilinearization method is employed to solve the nonlinear problem. The quasilinearization method was proposed by Bellman and Kalaba (1965) as a generalization of the Newton-Raphson method. Mandelzweig and his co-workers Krivec and Mandelzweig (2001); Mandelzweig and Tabakin (2001); Mandelzweig (2005) have extended the application of the quasilinearization method to a wide variety of nonlinear BVPs and established that the method converges quadratically. Most recently, the accuracy and validity of the Spectral quasilinearization schemes is presented by Motsa and Sibanda (2013). The behavior of emerging flow parameters on the velocity, temperature and concentration is discussed.

## 2. MATHEMATICAL FORMULATION

The Consider a steady fully developed laminar mixed convection flow of a couple stress fluid between two permeable vertical plates distance $2d$ apart. Choose the coordinate system such that $x$ -axis be taken along vertically upward direction through the central line of the channel, $y$ is perpendicular to the plates and the two plates are infinitely extended in the direction of $x$. The plate $y = -d$ has given the uniform temperature $T_1$ and concentration $C_1$, while the plate $y = d$ is subjected to a uniform temperature $T_2$ and concentration $C_2$. Since the boundaries in the $x$ direction are of infinite dimensions, without loss of generality, we assume that the physical quantities depend on $y$ only. The fluid properties are assumed to be constant except for density variations in the buoyancy force term. In addition, the thermo diffusion with thermal radiation effects considered. The flow is a mixed convection flow taking place under thermal buoyancy and uniform pressure gradient in the flow direction. The flow configuration and the coordinates system are shown in Figure 1. The fluid velocity $u$ is assumed to be parallel to the $x$-axis, so that only the $x$-component $u$ of the velocity vector does not vanish but the transpiration cross-flow velocity $v_0$ remains constant, where $v_0 < 0$ is the velocity of suction and $v_0 > 0$ is the velocity of injection.

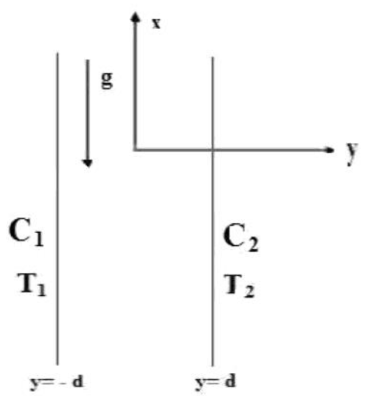


**Fig. 1. Physical model and coordinate system.**

With the above assumptions and Boussinesq approximations with energy and concentration, the equations governing the steady flow of an incompressible couple stress fluid are

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho v_0 \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} - \eta_1 \frac{\partial^4 u}{\partial y^4} - \frac{\partial p}{\partial x} + \rho g \beta_T (T - T_1) + \rho g \beta_C (C - C_1) \tag{2}$$

$$v_0 \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q^r}{\partial y} + 2\frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\eta_1}{\rho C_p}\left(\frac{\partial^2 u}{\partial y^2}\right)^2 \tag{3}$$

$$v_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{D K_T}{T_m} \frac{\partial^2 T}{\partial y^2} \tag{4}$$

where $u$ is the velocity component along $x$ direction, $\rho$ is the density, $g$ is the acceleration due to gravity, $p$ is the pressure, $\mu$ is the coefficient of viscosity, $\beta_T$ is the coefficient of thermal expansion, $\beta_C$ is the coefficient of solutal expansion, $\alpha$ is the thermal diffusivity, $D$ is the mass diffusivity, $C_P$ is the specific heat capacity, $C_S$ is the concentration susceptibility, $T_m$ is the mean fluid temperature, $K_T$ is the thermal diffusion ratio, $K_f$ is the coefficient of thermal conductivity, $\eta_1$ is the additional viscosity coefficient which specifies the character of couple-stresses in the fluid and $q^r$ is the radiation heat flux, We assume that $q^r$ under the Rosseland approximation has the following form:

$$q^r = -\frac{4\sigma}{3\chi} \frac{\partial T^4}{\partial y} \tag{5}$$

where $\sigma$ is the Stefan-Boltzman constant, $\chi$ is the mean absorption coefficient

The boundary conditions are

$u=0$ at $y=\pm d$ (6a)

$u_{yy}=0$ at $y=\pm d$ (6b)

$T=T_1$, $C=C_1$ at $y=-d$ (6c)

$T=T_2$, $C=C_2$ at $y=d$ (6d)

The boundary condition (6a) corresponds to the classical no-slip condition from viscous fluid dynamics. The boundary condition (6b) implies that the couple stresses are zero at the plate surfaces.

Introducing the following similarity transformations

$$y = \eta d, u = u_0 f, T - T_1 = (T_2 - T_1)\theta$$
$$C - C_1 == (C_2 - C_1)\varphi, p = \frac{\mu u_0}{d^2} P \tag{7}$$

in equations (2) - (4), we get the following nonlinear system of differential equations

$$S^2 f^{(iv)} - f'' + R f' - \frac{Gr_T}{\mathrm{Re}}\theta - \frac{Gr_C}{\mathrm{Re}}\varphi + A = 0 \tag{8}$$

$$\theta'' - R \Pr \theta' + \frac{4}{3} R_d \left[ \left(C_T + \theta\right)' \right]' + 2Br(f')^2 + S^2 Br(f'')^2 = 0 \tag{9}$$

$$\varphi'' - R\, Sc\, \varphi' + S_r Sc\, \theta'' = 0 \tag{10}$$

where primes denote differentiation with respect to η alone, $\mathrm{Re} = \frac{u_0 d}{\nu}$ is the Reynolds number, $R = \frac{v_0 d}{\nu}$ is the suction/induction parameter, $C_T = \frac{T_1}{T_2 - T_1}$ is the temperature ratio, $A = \frac{dP}{dx}$ is the constant pressure gradient, $Gr_T = \frac{g\beta_T (T_2 - T_1) d^3}{\nu^2}$ is the temperature Grashof number, $Gr_C = \frac{g\beta_T (C_2 - C_1) d^3}{\nu^2}$ is the mass Grashof number, $\Pr = \frac{\mu C_P}{K_T}$ is the Prandtl number, $Sc = \frac{\nu}{D}$ is the Schmidth number,

$R_d = \frac{4\sigma(T_2 - T_1)^3}{K_T \chi}$ is the radiation parameter, $Br = \frac{\mu \nu^2}{K_T d^2 (T_2 - T_1)}$ is the Brinkman number, $S_r = \frac{DK_T(T_2 - T_1)}{\nu T_m (C_2 - C_1)}$ is the Soret number, $S = \frac{1}{d}\sqrt{\frac{\eta_1}{\mu}}$ is the couple stress parameter.

Boundary conditions (6) in terms of $f$, $\theta$, $\phi$ become

$$f = 0,\ f'' = 0,\ \theta = 0,\ \varphi = 0 \quad at\ \eta = -1$$
$$f = 0,\ f'' = 0,\ \theta = 1,\ \varphi = 1 \quad at\ \eta = 1 \tag{11}$$

The physical quantities of interest in this problem are the skin friction coefficient, the Nusselt number and the Sherwood number. The shearing stress, heat, mass fluxes at the vertical plate surfaces can be obtained from

$$\left.\begin{aligned} &\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=\pm d}, q_m = \left[-D\frac{\partial C}{\partial y}\right]_{y=\pm d} \\ &q_w = \left[-K_f \frac{\partial T}{\partial y} + q^r\right]_{y=\pm d} \end{aligned}\right\} \tag{12}$$

The non-dimensional shear stress $C_f = \frac{\tau_w}{\rho u_0^2}$, the Nusselt number $Nu = \frac{q_w d}{K_f (T_2 - T_1)}$ and the Sherwood number $Sh = \frac{q_m d}{D(C_2 - C_1)}$ are given by

$$\left.\begin{aligned} &\mathrm{Re} C_{f_1} = 2f'(-1), \mathrm{Re} C_{f_2} = 2f'(1) \\ &Nu_{1,2} = -\left[1 + \frac{4}{3}(C_T + \theta)^3\right]\left.\theta'\right|_{\eta=-1,1} \\ &Sh_{1,2} = -\left.\varphi'\right|_{\eta=-1,1} \end{aligned}\right\} \tag{13}$$

Effect of the various parameters involved in the investigation on these coefficients is discussed in the following section.

## 3. THE SPECTRAL QUASI-LINEARISATION (QLM)SOLUTION OF THE PROBLEM

In this section, we introduce the quasi-linearisation (QLM) method for solving the governing system of equations (8) - (10) subject to the boundary conditions (11). The quasilinea- risation technique is essentially a generalized Newton- Raphson Method that was originally used by Bellman and Kalaba (1965) for solving functional equations applying the QLM on (8) - (10) gives the following iterative sequence of linear differential equations;

$$\begin{aligned} &S^2 f_{r+1}^{iv} - f_{r+1}'' + R f_{r+1}' - \frac{Gr_T}{\mathrm{Re}}\theta_{r+1} \\ &- \frac{Gr_C}{\mathrm{Re}}\varphi_{r+1} = -A \end{aligned} \tag{14}$$

$$\begin{aligned} &a_{1,r} f_{r+1}'' + a_{2,r} f_{r+1}' + a_{3,r}\theta_{r+1}'' \\ &\quad + a_{4,r}\theta_{r+1}' + a_{5,r}\theta_{r+1} = a_{6,r} \end{aligned} \tag{15}$$

$$\varphi_{r+1}'' - R\,Sc\,\varphi_{r+1}' + S_r Sc\,\theta_{r+1}'' = 0 \tag{16}$$

where the coefficients $a_{s,r}$ ($s$ = 1, 2. . . 6) are known functions (from previous calculations) and are defined as

$$a_{1,r} = 2S^2 Br f_r'', a_{2,r} = 4Br f_r'$$

$$a_{3,r} = 1 + \frac{4}{3} Rd\left(C_T + \theta_r\right)^3$$

$$a_{4,r} = -R\Pr + 8Rd\left(C_T + \theta_r\right)^2 \theta_r'$$

$$a_{5,r} = 8Rd\left(C_T + \theta_r\right)\left(\theta_r'\right)^2 + 4Rd\left(C_T + \theta_r\right)^2 \theta_r''$$

$$\begin{aligned} a_{6,r} = &-4Rd\left(C_T + \theta_r\right)^2\left(\theta_r'\right)^2 + S^2 Br\left(f_r''\right)^2 \\ &- 8Rd\left(C_T + \theta_r\right)\left(\theta_r'\right)^2 \theta_r \\ &- \left(C_T + \theta_r\right)^2 \theta_r''\theta_r + 12Br\left(f_r'\right)^2 \end{aligned}$$

It must be pointed out that the above system (14) – (16) constitute a linear system of coupled differential equations with variable coefficients and can be solved iteratively using any numerical method for $r$ = 1, 2, 3, . . . In this work, as will be discussed below, the Chebyshev pseudo-spectral method was used to solve the QLM scheme (14) – (16). Starting from a given set of initial approximations $f_0$, $\theta_0$, $\phi_0$, the iteration schemes (14 - 16) can be solved iteratively for $f_{r+1}(\eta)$, $\theta_{r+1}(\eta)$, $\phi_{r+1}(\eta)$ when $r$ = 0, 1, 2, .... To solve equation (14) – (16) we discretize the equation using the Chebyshev spectral collocation method. The basic idea behind the spectral collocation method is the introduction of a differentiation matrix ***D*** which is used to approximate the derivatives of the unknown functions $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ at the collocation points $\eta_j = Cos\frac{j\pi}{N_x}$ ($j$ = 0, 1, 2... $N_x$) as the matrix vector product

$$\left.\frac{df}{d\eta}\right|_{\eta=\eta_j} = \sum_{k=0}^{N_x} D_{jk} f(\eta_k) = DF, \quad j = 0,1,..,N_x \tag{17}$$

where $N_x$ + 1 is the number of collocation points, and $F = \left[f(\eta_0), f(\eta_1), ..., f(\eta_x)\right]^T$ is the vector function at the collocation points. Similar vector functions corresponding to $\phi$ and $\theta$ are denoted by $\Phi$ and $\Theta$, respectively. Higher order derivatives are obtained as powers of **D**, that is

$$f^{(p)} = D^p F, \varphi^{(p)} = D^p \Phi, \theta^{(p)} = D^p \Theta \tag{18}$$

where $p$ is the order of the derivative. The matrix $D$ is of size ($N_x$ + 1) × ($N_x$ + 1) and its entries are defined in (Canuto *et al*. 1988; Trefethen 2000). Thus, applying the spectral method as described above on Eqs. (14)- (16) gives

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix}\begin{bmatrix} F_{r+1} \\ \Theta_{r+1} \\ \Phi_{r+1} \end{bmatrix} = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} \tag{19}$$

where

$$A_{1,1}=S^2D^4-D^2+\mathrm{Re}\,D, A_{1,2}=-I,$$

$$A_{1,3}=-NI, A_{2,1}=a_{1,r}D^2+a_{2,r}D,$$

$$A_{2,2}=a_{3,r}D^3+a_{4,r}D+a_{5,r}, A_{2,3}=O,$$

$$A_{3,1}=O, A_{3,2}=SrScD^2, A_{3,3}=D^2-\mathrm{Re}\,ScD,$$

$$K_1=O_1, K_2=a_{6,r}, K_3=O_1$$

the matrices $a_{s,\,r}$ denoted that the vector $a_{s,r}$ ($s$ = 1, 2) is placed on the main diagonal of a matrix of size $(N_x+1)\times(N_x+1)$, $I$ is a $(N_x+1)\times(N_x+1)$ identity matrix, **O** is a $(N_x+1)\times(N_x+1)$ matrix of zeros, and $\mathbf{O}_1$ is a $(N_x+1)\times 1$ vector. The approximate solutions for $f$, θ and ϕ are obtained by solving the matrix system (19).

## 4. RESULTS AND DISCUSSION

The solutions for $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ have been computed and shown graphically in Figs. 2 to 10. The effects of radiation parameter ($Rd$), Soret parameter ($Sr$), couple stress fluid parameter ($S$) and the temperature ration ($C_T$) have been discussed. To study the effect of $Rd$, $Sr$, $C_T$ and $S$, computations were carried out by taking $Pr$ = 0.7, $Gr_T = Gr_C$ =10, $Re$ = 2, $R$=2, $Br$ = 0.1, $Sc$ = 0.7, $C_T$ = 0.1 and $A$=1.

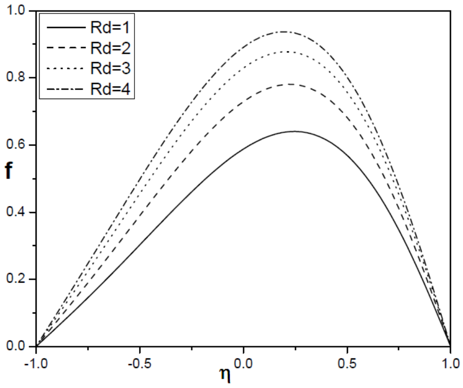


**Fig. 2. Radiation parameter (*Rd*) effect on *f* at *S*=0.5, *Sr*=0.5.**

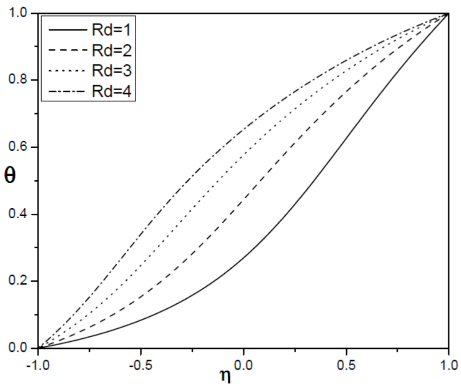


**Fig. 3. Radiation parameter (*Rd*) effect on θ at *S*=0.5, *Sr*=0.5**

Figures 2 to 4 represent the effect of radiation parameter Rd on $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$. It can be seen from these figures that the velocity $f(\eta)$ increase with an increase in the parameter $Rd$. This implies that the radiation have a retarding influence on the mixed convection flow. The dimensionless temperature increases as $Rd$ increases. The effect of radiation parameter $Ra$ is to increase the temperature significantly in the flow region. The increase in radiation parameter means the release of heat energy from the flow region and so the fluid temperature increases. The concentration $\phi(\eta)$ decreases with an increase in the parameter $Rd$.

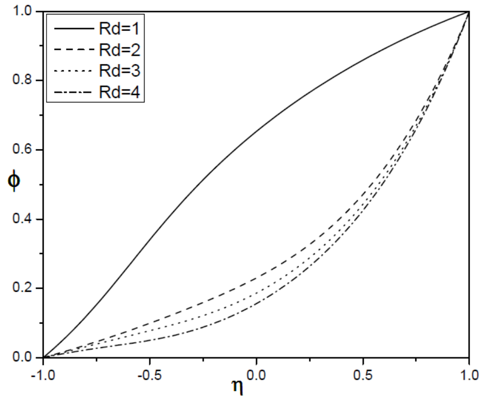


**Fig. 4. Radiation parameter (*Rd*) effect on ϕ at *S*=0.5, *Sr*=0.5**

Figure 5 displays the effect of the thermal diffusion parameter $Sr$ on $f(\eta)$. It can be observed that the velocity $f(\eta)$ decreases with an increase in the parameter Sr. Fig. 6 depicts the variation of temperature with $Sr$. The temperature $\theta(\eta)$ decreases with an increase in the parameter $Sr$. Figure 7 demonstrates the dimensionless concentration for different values of Soret number $Sr$. It is seen that the concentration of the fluid increases with the increase of Soret number. The present analysis shows that the flow field is appreciably influenced by the Soret number.

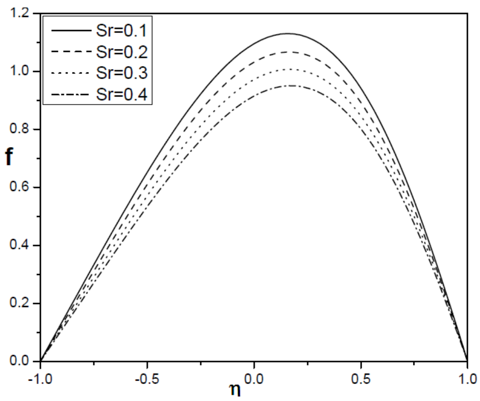


**Fig. 5. Effect of *Sr* on *f* at *S* = 0.5, *Rd* = 5.**

Figures 8 to 10 indicate the effect of the couple stress fluid parameter S on $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$. As the couple stress fluid parameter $S$ increases, the velocity $f(\eta)$ decreases. It is also clear that the temperature $\theta(\eta)$ decreases with an increase in $S$. It can be noted that the velocity in case of couple stress fluid is less than that of a Newtonian fluid case.

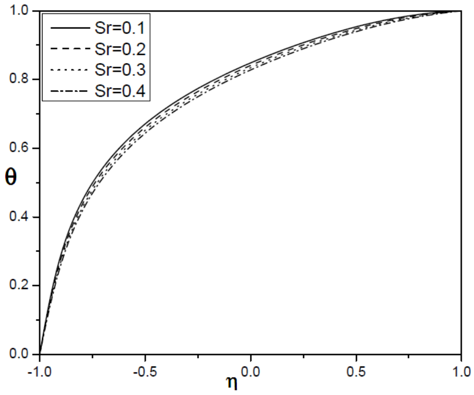


**Fig. 6. Effect of *Sr* on θ at *S* = 0.5, *Rd* = 5.**

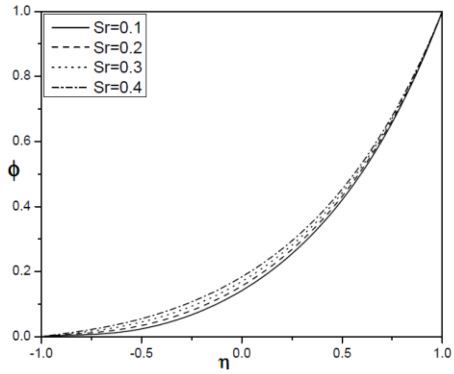


**Fig. 7. Effect of *Sr* on ϕ at *S* = 0.5, *Rd*=5.**

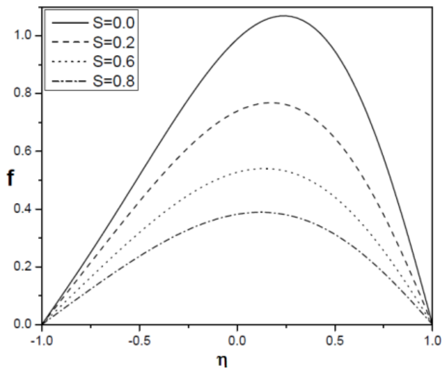


**Fig. 8. *S* Effect on *f* at *Rd* = 1, *Sr* = 1.**

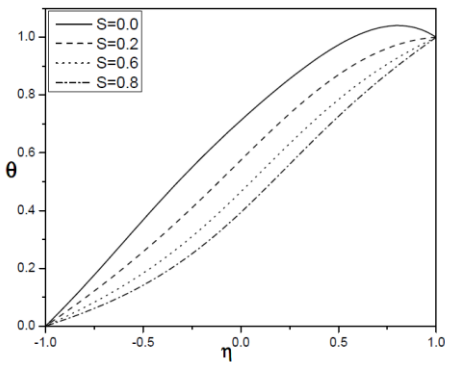


**Fig. 9. *S* Effect on θ at *Rd* = 1, *Sr* = 1.**

Thus, the presence of couple stresses in the fluid decreases the velocity and temperature. It can be seen from Fig. 10 that the concentration of the fluid increases with the increase of couple stress fluid parameter *S*.

Figure 11 displays the effect of the temperature ration $C_T$ on $f(\eta)$. It can be observed that the velocity $f(\eta)$ increases with an increase in the parameter $C_T$. Fig. 12 depicts the variation of temperature with $C_T$. The temperature $\theta(\eta)$ increases with an increase in the parameter $C_T$. Figure 13 demonstrates the dimensionless concentra- tion for different values of temperature ratio $C_T$. It is seen that the concentration of the fluid decreases with the increase of $C_T$. The present analysis shows that the flow field is appreciably influenced by the temperature ratio $C_T$.

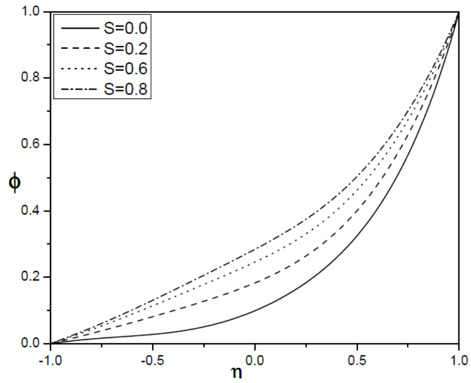


**Fig. 10. *S* Effect on ϕ at *Rd* = 1, *Sr* = 1.**

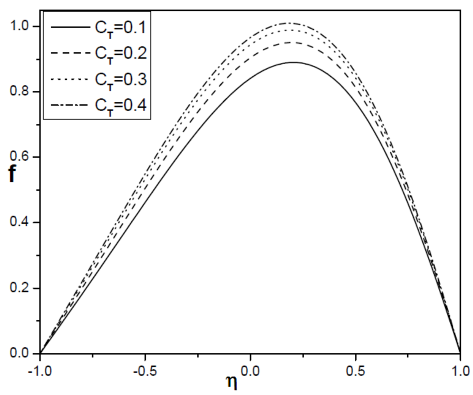


**Fig. 11. $C_T$ Effect on *f* at *Rd* = 2, *Sr* = 0.2, *S*=0.5.**

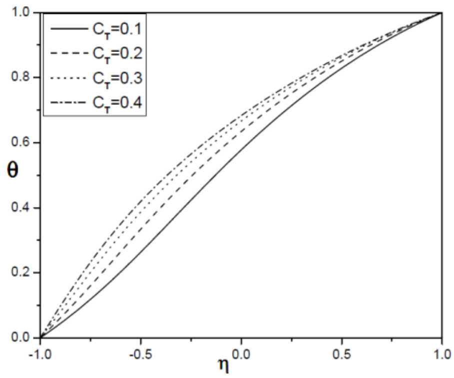


**Fig. 12. $C_T$ Effect on θ at *Rd* = *2*, *Sr* = *0.2*, *S=0.5*.**

**table 1 Effects of skin friction coefficient, heat and mass transfer rates for various values of *Sr*, *Rd* and *S* when *CT* = 0.1, *Re* = 2, *Pr* = 0.7, *Br* =0.5, *Gr* = 1 and *Sc* = 0.7**

| *Sr* | *Rd* | *S* | *f*'(-1) | *f*'(1) | θ'(-1) | θ'(1) | ϕ'(-1) | ϕ'(1) |
|---|---|---|---|---|---|---|---|---|
| 0.2 | 5 | 0.5 | 1.1851 | -2.2831 | -1.0773 | -1.6564 | 0.0434 | 1.5697 |
| 0.4 | 5 | 0.5 | 1.1240 | -2.1993 | -1.0557 | -1.6996 | 0.0009 | 1.6464 |
| 0.6 | 5 | 0.5 | 1.0651 | -2.1184 | -1.0358 | -1.7397 | -0.0372 | 1.7210 |
| 1 | 1 | 0.5 | 0.7820 | -1.7974 | -0.2601 | -1.1823 | 0.2070 | 1.4906 |
| 1 | 2 | 0.5 | 0.8757 | -1.9080 | -0.4122 | -1.2642 | 0.1745 | 1.6677 |
| 1 | 3 | 0.5 | 0.9218 | -1.9497 | -0.5919 | -1.4148 | 0.1062 | 1.7624 |
| 1 | 5 | 0.2 | 0.9443 | -2.3764 | -1.0179 | -1.7108 | -0.1118 | 1.8747 |
| 1 | 5 | 0.4 | 0.9045 | -2.3764 | -0.9791 | -1.8941 | -0.0910 | 1.8556 |
| 1 | 5 | 0.6 | 0.7434 | -1.1481 | -0.9381 | -2.0097 | -0.0682 | 1.8417 |

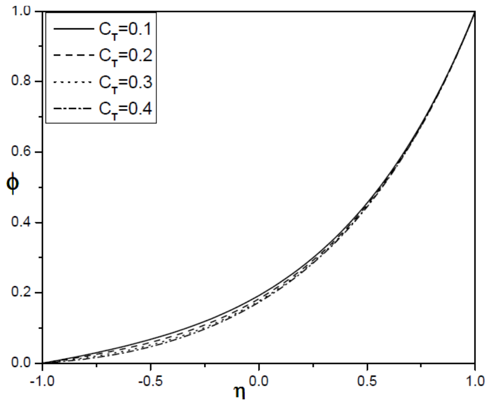


**Fig. 13. $C_T$ Effect on ϕ at *Rd* = 2, *Sr* = 0.2, *S*=0.5**

Variation of couple stress parameter (*S*), Radiation parameter (*Rd*) together with the Soret number (*Sr*) is presented in Table 1 with fixed values of other parameters. Skin friction coefficient, heat transfer rates increase but mass transfer rate decreases with the increasing value of the Soret parameter (*Sr*). Further, it can be noted that the skin friction coefficient and mass transfer rates decrease with an increase in *Rd* where as heat transfer rate increases with an increase in the parameter *Rd*. Finally, for fixed values of *Rd*, *Sr*, the effect of couple stress parameter on the skin-friction coefficient, the rate of heat and mass transfers are shown in this table. The behavior of these parameters is self evident from the Table 1 and hence is not discussed for brevity.

## 5. CONCLUSIONS

In this paper, the Radiation and Soret effects on couple stress fluid flow between vertical parallel plates has been studied. The governing equations are expressed in the non-dimensional form and are solved by using QLM. The effects of emerging parameters on velocity, temperature, concentration profiles are presented, also the radial friction factor, heat and mass transfer rates are presented in table form. The main findings are summarized as:

- The concentration, skin friction and the mass transfer rate of the fluid decreases and velocity, temperature, heat transfer rate increases as radiation parameter increases.
- The velocity, temperature, friction and the heat transfer rates are decreases, while mass transfer rate increases with the increase in the soret parameter.
- It is noticed that the presence of couple stresses in the fluid decreases the velocity, temperature, friction and the heat transfer rate and increases the concentration and mass transfer rate.

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