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The Transient MHD Flow Generated by a Periodic Wall Motion in a Porous Space

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Abstract

The problem of transient flow of incompressible third grade fluid on the two-dimensional magnetohydrodynamic (MHD) flow in a porous space is analyzed. The flow is generated due to the motion of the plate in its plane with a periodic velocity. Under the flow assumptions, the governing nonlinear partial differential equation is transformed into steady-state and transient nonlinear equations. The reduced equation for the transient flow is solved analytically using symmetry approach while the nonlinear steady-state equation is solved using a modified version of He's homotopy perturbation method. The effect of several operating parameters on the flow hydromagnetic is discussed. The results indicated that for the considered case, $t = 1.5$ is the moment after which the time-dependent transient motion of the fluid can be approximated with the steady-state motion, described by the steady-state solution. It is clear that, after this value of time t the time-dependent transient solution can be neglected.

Keywords: Periodic wall; Transient flow; Third-grade fluid; Analytical solutions; Magnetohydrodynamic; Porous space.

Nomenclature

A_1, A_2	arbitrary constants
$\mathbf{A}_i\ (i = 1,2)$	Revlin-Ericksen tensors
B_0	applied magnetic field
c	constant wave speed
$\frac{d}{dt}$	material time derivative
$h(y)$	arbitrary function
H_n	He's polynomial
$\mathbf{I}$	identity tensor
$\mathbf{J} \times \mathbf{B}$	magnetic body force
k	constant wall velocity
K	permeability of the porous medium
$\mathbf{L}$	$\nabla \mathbf{v}$
L	linear operator
M	magnetic field
N	nonlinear operator
p	pressure gradient
$\mathbf{R}$	Darcy's resistance due to porous medium
t	time variable
u	velocity field
u_s	steady velocity
u_t	transient velocity
v_w	wall velocity
U_x	unknown velocity function
V_0	amplitude of wall oscillations
x, y	perpendicular distances
X_1	time translation
X_2	space translation
$X_1 - cX_2$	wave-front type travelling solutions
$\mathbf{v}$	velocity vector
$\alpha_1, \alpha_2, \beta_3$	material constants
μ	dynamic viscosity
ν	kinematic viscosity
ρ	fluid density
σ	electrical conductivity
τ	Cauchy stress tensor
ϕ	porosity of the porous medium
ω	frequency of the wall velocity
∇	$\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j}$

1. INTRODUCTION

Theoretical interest in the flow of third-grade fluid has increased substantially over the past few decades due to the occurrence of these fluids in industrial processes (Fakhar et al. (2008), Ellahi and Afzal (2009), Siddiqui et al. (2010), Danish et al. (2012), Hayat et al. (2013), Abdulhameed et al. (2014)). The fluid of third-grade is a subclass of the differential type fluid whose equations of motion are highly non-linear and higher order than the Navier-Stokes equations for Newtonian fluid. Because of the complexity of the governing equations for third-grade fluid, finding analytical solutions is not easy. Further, these solutions are very useful to provide a great insight on more complex flow situations. In addition, they serve as a measurement for checking the accuracies of numerical solutions and experimental data.

The magnetohydrodynamic flow through a porous medium has become an active area of research due to its applications in several technological processes. Among these processes are petroleum exploration/recovery, cooling of electronic equipment, catalyst, chromatography, etc. The magnetohydrodynamic flow through a porous medium due to an arbitrary profile of a plate occurs in many industrial processes such as acoustic streaming around an oscillating body and an unsteady boundary layer with fluctuation. Therefore, it has become subject of many discussion for a different kind of flow configurations (Bennecib et al. (2009), Devi and Ganga (2010), Hayat et al. (2010), Sharma and Khan (2010), Ahmad and Asghar (2011), Ali et al. (2012), Aziz and Aziz (2012), Aziz et al. (2012), Mohammed et al. (2012), Abdulhameed et al. (2013)).

To the best of the authors knowledge, the time-dependent transient magnetohydrodynamic flow of a third-grade fluid due to an oscillating plate in a porous space has not been studied before, and it is the main aim of this paper to study this problem. We make use of symmetry reduction method, such that the transient governing nonlinear partial differential equation is reduced to a nonlinear ordinary differential equation, which further solved analytically for the time-dependent transient in the form of wave-front type travelling solution. The nonlinear steady-state equation is solved using a modified version of He's homotopy perturbation method. The results indicated that the differences between the transient and steady-state solutions solidly depends on small values of the time t. For large values of t, the starting solution can be approximated with the steady-state solution. Further, during the course of computation, it was observed that the transient and steady-state solutions agree very well at large value of time when the ratio related to fluid parameters $\frac{\beta_*}{\beta} > 1$. Effects of pertinent parameters on the flow fields are analyzed and shown graphically.

2. PROBLEM FORMULATION

Consider the unsteady viscoelastic of an incompressible electrically conducting third-grade fluid occupying a porous half-space and bounded by an infinite plane wall situated in the $(x,y)-$plane system of Cartesian coordinate. The fluid motion is driven due to an oscillating wall. Fig. 1 shows the physical configuration. Initially, both the plane wall and the fluid are at rest. At time $t > 0$ the wall moves in $x-$direction with velocity $v_w(t)$. A constant magnetic field B_0 is applied in the $y-$direction and there is no external electric field. The induced magnetic field and pressure gradient are neglected.

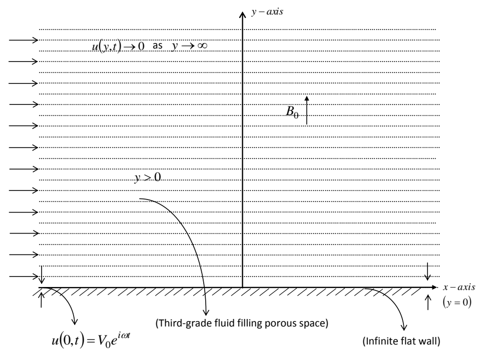


Fig. 1. The physical model configuration

The governing equations are:

$$\mathrm{div}(\mathbf{v}) = 0, \tag{1}$$

$$\rho \frac{d\mathbf{v}}{dt} = \mathrm{div}\tau + \mathbf{R} + \mathbf{J} \times \mathbf{B}, \tag{2}$$

where $\mathbf{v}$ is the velocity vector, ρ is the fluid density, $\frac{d}{dt}$ is the material time derivative, τ is the Cauchy stress tensor, $\mathbf{R}$ is the Darcy's resistance due to porous medium and $\mathbf{J} \times \mathbf{B}$ is the magnetic body force. The stress tensor, τ for a third-grade fluid is

$$\tau = -p\mathbf{I} + \mu \mathbf{A}_1 + \alpha_1 \mathbf{A}_2 + \alpha_2 \mathbf{A}_1^2 + \beta_3 \left(tr\mathbf{A}_1^2 \right) \mathbf{A}_1, \tag{3}$$

where $\mathbf{I}$ is the identity tensor, p is the pressure, μ is the dynamic viscosity, $\alpha_1, \alpha_2, \beta_3$ are the material constants and $\mathbf{A}_i\,(i = 1,2)$ are the Revlin-Ericksen tensors which are defined by

$$\begin{aligned} \mathbf{A}_1 &= \mathbf{L} + \mathbf{L}^T, \\ \mathbf{A}_n &= \frac{d}{dt}\mathbf{A}_{n-1} + \mathbf{A}_{n-1}\mathbf{L} + \mathbf{L}^T \mathbf{A}_{n-1}, \quad n > 1, \end{aligned} \tag{4}$$

where $\mathbf{L} = \nabla \mathbf{v}$.

In line with Davidson (2001) the magnetic Reynolds number is considered very small. It follows that the induced magnetic field produced by the fluid motion is negligible, the magnetic body force, $\mathbf{J} \times \mathbf{B}$, becomes $\sigma(\mathbf{v} \times B_0) \times B_0$ when imposed and induced electric fields are negligible and only the magnetic field, B_0, contributes to the current $\mathbf{J} = \sigma(\mathbf{v} \times B_0)$.

The Lorentz force on the last term of the right hand side of Eq. (2) becomes

$$\mathbf{J} \times \mathbf{B} = -\sigma B_0^2 \mathbf{v}, \tag{5}$$

where σ is the electrical conductivity.

The constitutive relationship between the pressure drop and the velocity for the unidirectional flow of a third grade fluid is

$$\frac{\partial p}{\partial x} = -\frac{\phi}{K}\left[\mu + \alpha_1 \frac{\partial}{\partial y} + 2\beta_3 \left(\frac{\partial u}{\partial y}\right)^2\right] u, \tag{6}$$

where K is the permeability of the porous medium, u is the velocity field and ϕ is the porosity of the porous medium. Using Eqs. (3-6) in Eq. (2), we obtains the governing equation for time-dependent transient flow:

$$\begin{aligned} \rho \frac{\partial u}{\partial t} = {} & \mu \frac{\partial^2 u}{\partial y^2} + \alpha_1 \frac{\partial^3 u}{\partial y^2 \partial t} + 6\beta_3 \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} \\ & -\frac{\phi}{K}\left[\mu + \alpha_1 \frac{\partial}{\partial t} + 2\beta_3 \left(\frac{\partial u}{\partial y}\right)^2\right] u \\ & -\sigma B_0^2 u. \end{aligned} \tag{7}$$

Eq. (7) will be solved subject to the boundary conditions as follows:

$$u(y,0) = h(y), \quad y > 0, \tag{8}$$
$$v_w(t) = u(0,t) = V_0 \exp(i\omega t), \tag{9}$$
$$u(y,t) \to 0 \text{ as } y \to \infty, \ t > 0, \tag{10}$$

where $h(y)$ is an arbitrary function, V_0 is the amplitude of wall oscillations, $\omega > 0$ is the frequency of the wall velocity and i is the imaginary unit. Using the wall velocity $v_w(t)$ given by Eq. (9), the cosine and sine oscillation can be obtained by taking the real and imaginary parts of the velocity field $u(y,t)$.

Introducing the quantities

$$y^* = \frac{V_0}{\nu} y, \quad u^* = \frac{u}{V_0}, \quad t^* = \frac{V_0^2}{\nu} t, \quad \omega^* = \frac{\omega \nu}{V_0^2},$$
$$\beta_3^* = 2\beta_3 \frac{V_0^4}{\nu^3}, \ \alpha_1^* = \alpha_1 \left(\frac{V_0}{\nu}\right)^2, \ \phi^* = \frac{\phi \nu^2}{K V_0^2},$$
$$M^2 = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, \tag{11}$$

we obtain the non-dimensional initial-boundary values problem (after dropping the $*$ notation)

$$\begin{aligned} \frac{\partial u}{\partial t} = {} & \mu_* \frac{\partial^2 u}{\partial y^2} + \alpha_* \frac{\partial^3 u}{\partial y^2 \partial t} + \beta \left(\frac{\partial u}{\partial y}\right)^2 \frac{\partial^2 u}{\partial y^2} \\ - {} & \beta_* u \left(\frac{\partial u}{\partial y}\right)^2 - (\phi_* + M_*^2)\, u = 0, \end{aligned} \tag{12}$$

subject to

$$u(y,0) = h(y), \quad y > 0, \tag{13}$$
$$v_w(t) = u(0,t) = \exp(i\omega t), \tag{14}$$
$$u(y,t) \to 0 \text{ as } y \to \infty, \ t > 0, \tag{15}$$

where

$$\begin{aligned} \mu_* &= \frac{1}{(1+\alpha_1\phi)}, \quad \alpha_* = \frac{\alpha_1}{(1+\alpha_1\phi)}, \\ \beta &= \frac{3\beta_3}{(1+\alpha_1\phi)}, \quad \beta_* = \frac{\beta_3\phi}{(1+\alpha_1\phi)}, \\ \phi_* &= \frac{\phi}{(1+\alpha_1\phi)}, \quad M_*^2 = \frac{M^2}{(1+\alpha_1\phi)}. \end{aligned} \tag{16}$$

3. SOLUTION TECHNIQUE

The flow equation presented in the previous section is strongly nonlinear and exhibit no closed-form solutions. It will be interesting to reduce the governing equations of the present problem to a form that can be solved to a closed-form. A special case of the present problem that exhibits exact or closed-form solution is the problem of time-dependent transient flow. The nonlinear steady-state equation is approximated using a modified version of He's homotopy perturbation method. The accuracy of the modified version of He's homotopy perturbation solutions for the velocity field is achieved by comparing with the exact solutions for the transient flow.

From Eq. (12) the dimensionless velocity, u can be expressed respectively as

$$u(y,t) = u_s(y) + u_t(y,t), \tag{17}$$

where u_s is the steady-state velocity and u_t is the time-dependent transient part. Note that, if we allow $t \to \infty$, we obtain the steady-state solutions.

3.1 Steady-state solution

Substituting Eq. (17) into (12), the resulting steady-state equation and boundary conditions for this special problem can be written as

$$\mu_* \frac{d^2 u_s}{dy^2} + \beta \left(\frac{du_s}{dy}\right)^2 \frac{d^2 u_s}{dy^2} - \beta_* u_s \left(\frac{du_s}{dy}\right)^2 - \left(\phi_* + M_*^2\right) u_s = 0, \tag{18}$$

with the boundary conditions

$$u_s(0) = k, \quad u_s(y) = 0 \text{ as } y \to \infty, \tag{19}$$

where k represent constant wall velocity.

To construct an approximate analytical solution of Eq. (18) subject to (19), a modified version of the He's homotopy perturbation technique is evoked.

According to the He's homotopy perturbation method He (2005), Eq. (18) satisfied by the velocity field $u_s(y)$ is decomposed into a linear part $L(u_s)$ and a non-linear part $N(u_s)$ and is written in the form

$$L(u_s(y)) + N(u_s(y)) = 0. \tag{20}$$

We introduce the linear operator L in the form

$$L = \frac{d^2}{dy^2} + \frac{d}{dy}, \tag{21}$$

thus

$$L(u_s(y)) = \left(\frac{d^2}{dy^2} + \frac{d}{dy}\right) u_s(y). \tag{22}$$

Write $L(u_s(y))$ in form of series

$$\sum_{i=0}^{\infty} L(u_{s_i}(y)) = \sum_{i=0}^{\infty} \left(\frac{d^2}{dy^2} + \frac{d}{dy}\right) u_{s_i}(y), \tag{23}$$

while the nonlinear operator N by Eq. (20) can be decomposed as He's polynomial as follows

$$N(u_s(y)) = \sum_{i=0}^{\infty} H_i. \tag{24}$$

Using Eqs. (20), (23) and (24), we could write

$$\sum_{i=0}^{\infty} u_{s_i}(y) = \sum_{i=0}^{\infty} \left(\frac{d^2}{dy^2} + \frac{d}{dy}\right) u_{s_i}(y) + \sum_{i=0}^{\infty} H_i. \tag{25}$$

The recurrence relation are defined as follows:

$$\begin{cases} u_{s_0} = \left(\frac{d^2}{dy^2} + \frac{d}{dy}\right) u_{s_0}(y), \\ u_{s_1} = \left(\frac{d^2}{dy^2} + \frac{d}{dy}\right) u_{s_1}(y) + H_0, \\ u_{s_{n+1}} = \left(\frac{d^2}{dy^2} + \frac{d}{dy}\right) u_{s_{n+1}}(y) + H_n, \\ n = 1, 2, \ldots \end{cases} \tag{26}$$

where, the He's polynomial [Ghorbani (2009)], H_n, is defined as

$$H_n(u_{s_0}, \ldots, u_{s_n}) = \frac{1}{n!} \left[\frac{\partial^n}{\partial p^n} N\left(\sum_{k=0}^{\infty} p^k u_{s_k}\right)\right]_{p=0}, \quad n = 0, 1, 2, \ldots \tag{27}$$

Using the recurrence Eq. (26), Eq. (18) subject to the boundary condition (19) form a set of system of differential equation as follows:

$$\begin{cases} \frac{d^2 u_{s_0}}{dy^2} + \frac{du_{s_0}}{dy} = 0, \\ u_{s_0}(0) = k, \quad u_{s_0}(\infty) = 0, \\ \frac{d^2 u_{s_1}}{dy^2} + \frac{du_{s_1}}{dy} = H_0, \\ u_{s_1}(0) = 0, \quad u_{s_1}(\infty) = 0, \\ \frac{d^2 u_{s_2}}{dy^2} + \frac{du_{s_2}}{dy} = H_1, \\ u_{s_2}(0) = 0, \quad u_{s_2}(\infty) = 0, \end{cases} \tag{28}$$

where

$$H_0 = \mu_* \frac{d^2 u_{s_0}}{dy^2} + \beta \left(\frac{du_{s_0}}{dy}\right)^2 \frac{d^2 u_{s_0}}{dy^2} - \beta_* u_{s_0} \left(\frac{du_{s_0}}{dy}\right)^2 - \left(\phi_* + M_*^2\right) u_{s0}, \tag{29}$$

$$\begin{aligned} H_1 = {} & \mu_* \frac{d^2 u_{s_1}}{dy^2} - \left(\phi_* + M_*^2\right) u_{s_1} \\ & + \beta \left[\left(\frac{du_{s_0}}{dy}\right)^2 \frac{d^2 u_{s_1}}{dy^2} + 2 \frac{du_{s_0}}{dy} \frac{d^2 u_{s_0}}{dy^2} \frac{du_{s_1}}{dy}\right] \\ & - \beta_* \left[u_{s_1} \left(\frac{du_{s_0}}{dy}\right)^2 + 2 \frac{du_{s_0}}{dy} \frac{d^2 u_{s_0}}{dy^2} \frac{du_{s_1}}{dy}\right] \end{aligned} \tag{30}$$

The solution of the above system is

$$u_{s_0} = k e^{-y}, \tag{31}$$

$$\begin{aligned} u_{s_1} = {} & \frac{1}{6} e^{-3y} k \left\{k^2 (\beta - \beta_*) + 6 y e^{2y} (-\mu + \phi_*) \right. \\ & \left. + e^{2y} \left[6 M^2 y - k^2 (\beta - \beta_*)\right]\right\}, \end{aligned} \tag{32}$$

$$
\begin{aligned}
u_{s_2} &= \frac{1}{360} e^{-5y} k \left\{ 3k^4 \left(15\beta^2 - 22\beta\beta_* + 7\beta_*^2\right) \right. \\
&+ e^{4y}[-180M^4 y(2+y) \\
&+ k^4(-15\beta^2 + 6\beta\beta_* + 9\beta_*^2) + 180y(\mu_* - \phi_*) \\
&\times [(-2+y)\mu_* - (2+y)\phi_*] \\
&- 20M^2[k^2(-5\beta + 3y\beta - \beta_* - 3y\beta_*) \\
&+ 18y(y\mu_* - 2\phi_* - y\phi_*)] \\
&+ 20k^2[\beta(3(-3+y)\mu_* + (5-3y)\phi_*) \\
&+ \beta_*(-3(-1+y)\mu_* + \phi_* + 3y\phi_*)]] \\
&- 10e^{2y}k^2[3k^2(\beta - \beta_*)^2 \\
&+ 2M^2((5-9y)\beta + \beta_* + 9y\beta_*) \\
&+ 2(\beta(9(-1+y)\mu_* + (5-9y)\phi_*) \\
&+ \left. \beta_*((3-9y)\mu_* + \phi_* + 9y\phi_*))]\right\}. \qquad (33)
\end{aligned}
$$

If we have

$$u_s = \sum_{i=0}^{n} u_{s_i}, \qquad (34)$$

then the second order solution is obtained by substituting Eqs. (31-33) into Eq. (34) for $n = 2$.

3.2 Time-dependent transient solution

The unsteady equation given by Eq. (12) is reduced to ordinary differential equations using symmetry approach, which further solved analytically in the form of wave-front type travelling wave solutions with constant wave speed $c(c > 0)$.

Consider an invariant solution using the operator X, in the form

$$X = X_1 - cX_2, \qquad (35)$$

where $X_1 = \frac{\partial}{\partial t}$ (time translation) and $X_2 = \frac{\partial}{\partial y}$ (space translation)

The characteristic curves of Eq. (35) is

$$\frac{dy}{c} = \frac{dt}{1} = \frac{du}{0}, \qquad (36)$$

where invariant solution is as follows:

$$u_t(y,t) = U(x), \text{ where } x = y + ct. \qquad (37)$$

Substituting Eq. (37) into Eq. (12), we deduce to a third-order nonlinear ordinary differential equation for $U(x)$ along certain curves in the y,t plane

$$c\frac{dU}{dx} = \mu_* \frac{d^2U}{\partial x^2} + \alpha_* c \frac{d^3U}{dx^3} + \beta \left(\frac{dU}{dx}\right)^2 \frac{d^2U}{dx^2} - \beta_* U \left(\frac{dU}{dx}\right)^2 - \left(\phi_* + M_*^2\right) U. \qquad (38)$$

Considering the solution of Eq. (38) as a function of

$$U(x) = A_1 \exp(i\omega t + A_2 x), \qquad (39)$$

where A_1 and A_2 are constants to be determined, and substituting Eq. (39) into Eq. (38) and equating the exponent of e^0 and $e^{2(i\omega t + Bx)}$ we obtain

$$e^0 : \mu_* A_2^2 + \alpha_* c A_2^3 - cA_2 - \left[\phi_* + M_*^2\right] = 0, \qquad (40)$$

$$e^{2Bx} : \beta A_1^2 A_2^4 - \beta_* A_1^2 A_2^2 = 0. \qquad (41)$$

Constants A_1 and A_2 are determined through Eqs. (40) and (41) respectively as

$$A_1 = 1, \qquad A_2 = \pm\sqrt{\frac{\beta_*}{\beta}}. \qquad (42)$$

Substituting A_2 in Eq. (40), we obtain

$$\mu_* \left(\frac{\beta_*}{\beta}\right) - \alpha_* c \left(\frac{\beta_*}{\beta}\right)^{\frac{3}{2}} + c \left(\frac{\beta_*}{\beta}\right)^{\frac{1}{2}} - \left(\phi_* + M_*^2\right) = 0. \qquad (43)$$

Assuming that condition (43) holds, $U(x)$ can be written as

$$U(x) = \exp\left[i\omega t - \sqrt{\frac{\beta_*}{\beta}} x\right]. \qquad (44)$$

Hence, the exact solution for $u_t(y,t)$ which satisfy the conditions (13)-(15) is

$$u_t(y,t) = \exp\left[i\omega t - \sqrt{\frac{\beta_*}{\beta}}(y + ct)\right]. \qquad (45)$$

From Eq. (43), the speed wave propagation c toward the wall in the $y-$direction is given by

$$c = \frac{\mu_* \left(\frac{\beta_*}{\beta}\right) - \left(\phi_* + M_*^2\right)}{\alpha_* \left(\frac{\beta_*}{\beta}\right)^{\frac{3}{2}} - \left(\frac{\beta_*}{\beta}\right)^{\frac{1}{2}}}. \qquad (46)$$

The solution above is to the best of the present authors' knowledge, the first known solution of the transient MHD flow in a porous space when an oscillation infinite plate was is considered. For zero oscillation rate, $\omega = 0$, the solution is given by Aziz et al. (2012).

4. ANALYSIS OF RESULTS

The steady-state velocity, given by Eq. (34) are shown graphically for various pertinent parameters in Figs. 2-4. Figs. 2 and 3 show the effects of the fluid parameters β_* and β. It is observed from this figures that β_* and β have the opposite behavior on the velocity field. As noted, the fluid velocity increases for increasing values of β_* whereas it decreases for increasing values of β. Fig. 4 demonstrates the effects of porosity of the porous medium parameter ϕ_* on fluid velocity. It is found from Fig. 4 that fluid velocity increases on increasing porosity of the porous medium parameter ϕ_* in the boundary layer region.

The starting velocity $u(y,t)$ is written as the sum of the steady-state solution $u_s(y,t)$ given by Eq. (34) and the transient solution $u_t(y,t)$ given by Eq. (45). Fig. 5 shows the starting and steady state velocity profiles for different values of time t. Since $\lim_{t\to\infty} u_t(y,t) = 0$, the time-dependent transient solution can be neglected for large values of time t. When taking large values of the time t, the profiles corresponding to the starting solutions become identical with the profiles corresponding to the steady-state solutions. In the considered case, $t = 1.5$ is the moment that the motion of the fluid can be approximated with the steady-steady permanent motion, described by the steady state solution. It is clear that, after this value of time t the transient solution can be neglected.

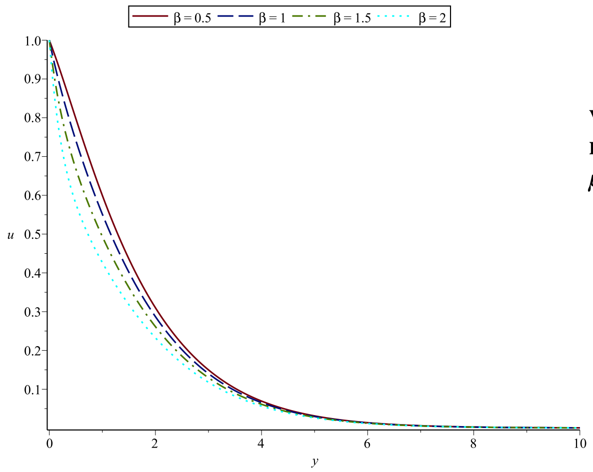


Fig. 2. Profiles of the flow velocity with different values of the material parameter β when $\beta_* = 0.5, M_* = 1$ $\phi_* = 0.5, k = 1, \mu_* = 0.5, \alpha_* = 0.5$ are fixed

The time-dependent transient velocity, given by Eq. (45), for various physical parameters are shown graphically in Figs. 6-8. In all these figures, frequency series of the flow velocity are shown for both cosine and sine oscillations of the plate. Fig. 6

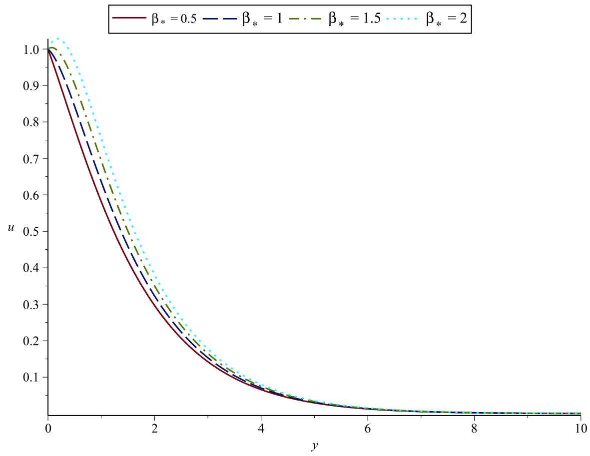


Fig. 3. Profiles of the flow velocity with different values of the material parameter β_* when $\beta = 1$, $M_* = 1$ $\phi_* = 0.5$, $k = 1$, $\mu_* = 0.5$, $\alpha_* = 0.5$ are fixed

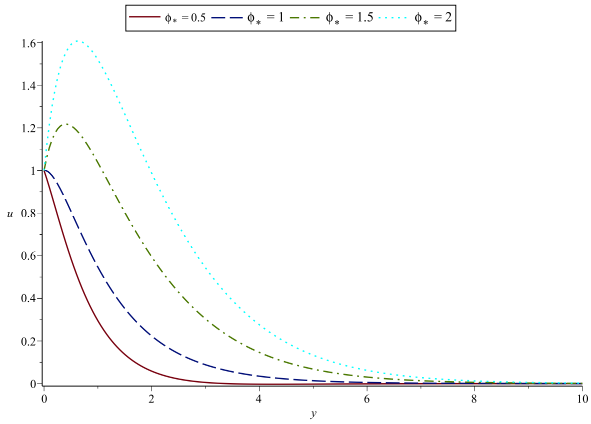


Fig. 4. Profiles of the flow velocity with different values of the porosity of the porous medium parameter ϕ_* when $\beta = 1.5, \beta_* = 1, M_* = 0.5, k = 1,$ $\mu_* = 0.5$, $\alpha_* = 0.5$ are fixed

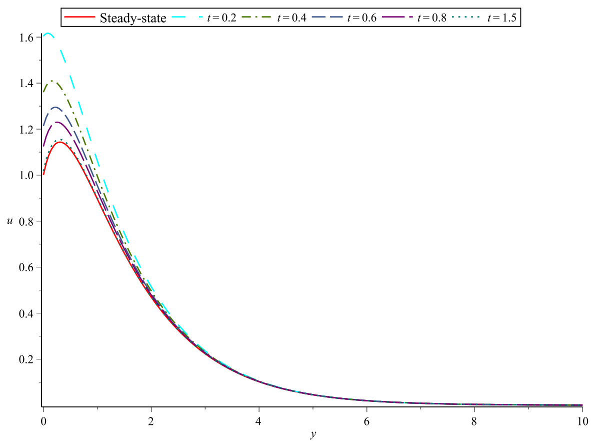
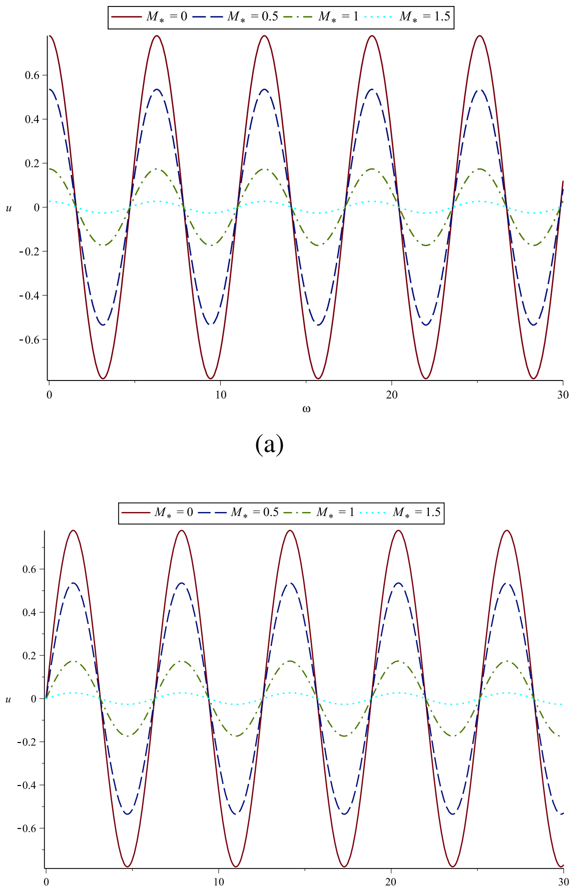


Fig. 5. Profiles of the starting and steady-state flow velocity with different values of the time t when when $\omega = 0.5$, $k = 1$, $\beta_* = 2.5$, $\beta = 1.5$, $M_* = 0.5, \phi_* = 1, \mu_* = 0.5, \alpha_* = 0.5$ are fixed

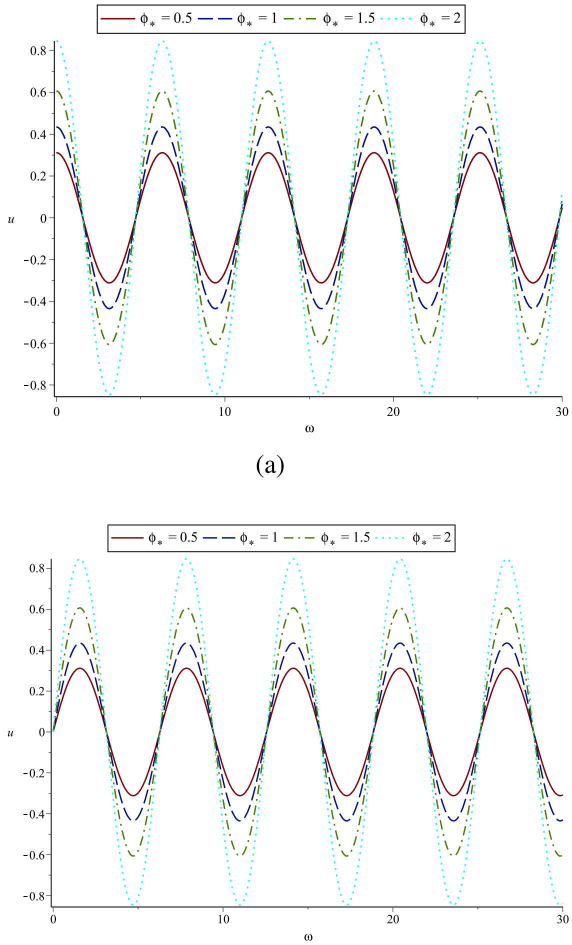


(a)

(b)

Fig. 6. Frequency series of the flow velocity with different values of the magnetic field parameter M_* **when** $\beta = 1.5$, $\beta_* = 1$, $y = 0$, $\phi_* = 0.5$, $t = 1$, $\mu_* = 0.5$, $\alpha_* = 0.5$ **are fixed (a) Cosine oscillation and (b) Sine oscillation**

shows the influence of magnetic field on the time series of the flow velocity. It is revealed from Fig. 6 that the frequency series of the flow velocity decreases on increasing magnetic parameter M_* in the boundary layer region for both types of oscillations. So, the higher values of M_*, the more prominent is the reduction in oscillating velocity. Fig. 7 illustrates the influences porosity of the porous medium parameter ϕ_* on fluid oscillating velocity. It is observed that the velocity amplitude increases with an increasing in porous medium parameter ϕ_* for both types of oscillations. As noted, the effects of ϕ_* on the time-dependent transient velocity profiles are the same as previous for the steady-state velocity profiles. Fig. 8 displayed the time series of the flow velocity for different distances from the plate. As can be seen, the velocity amplitude decreases rapidly with the increase of the distance from the plate while the flow of the third grade fluid oscillates in the whole domain approximately in phase with the driving phase movement.



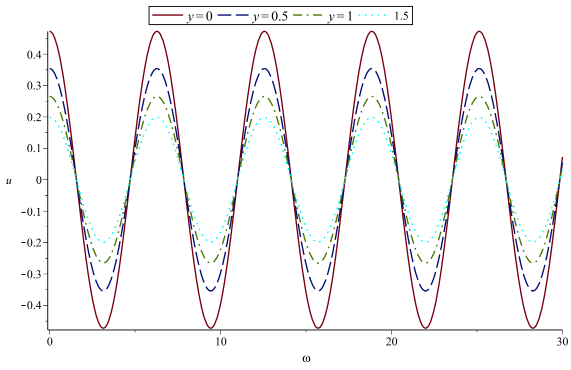
(a)

(b)

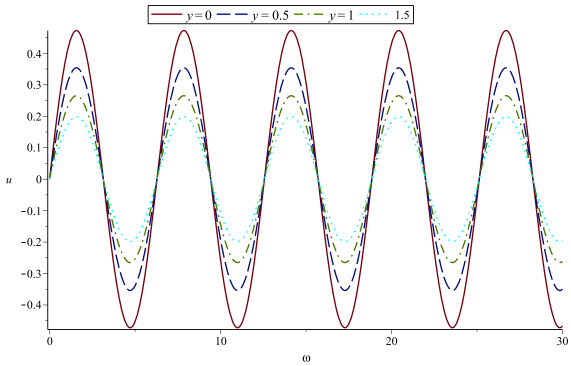
Fig. 7. Frequency series of the flow velocity with different values of the porosity of the porous medium parameter ϕ_* **when** $\beta = 0.5$, $\beta_* = 2.5$, $y = 0$, $M_* = 0.5$, $t = 1$, $\mu_* = 0.5$, $\alpha_* = 0.5$ **are fixed (a) Cosine oscillation and (b) Sine oscillation**

5. CONCLUSION

In this work, analytical solutions are obtained for the time-dependent transient as well as the steady-state flow induced by an oscillating profile of infinite wall with uniform magnetic field, located in a porous medium. The nonlinear steady-state equations are solved analytically using a modified version of He's homotopy perturbation method, and the transient equations are solved using symmetry reductions technique. Furthermore, in the present analysis, the results for the time-dependent transient and steady-state velocity are plotted and discussed. The results show that the variation of the starting and steady-state solutions mainly depends on small values of the time. For the large values of the time, the two solutions are identical. The period can determined before the transient solution vanishes. The results also show that the effects of the fluid material parameters exert great influence



(a)



(b)

Fig. 8. Frequency series of the flow velocity with different values of the distances from the plate y **when** $\beta = 1.5$, $\beta_* = 0.5$, $M_* = 1$ $\phi_* = 0.5$, $t = 1$, $\mu_* = 0.5$, $\alpha_* = 0.5$ **are fixed (a) Cosine oscillation and (b) Sine oscillation**

on the general flow pattern, by enhancing or decelerating the fluid flow.

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