



Unsteady Natural Convective Flow over an Impulsively Started Semi-Infinite Vertical Plate in the Presence of Porous Medium with Chemical Reaction

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ABSTRACT

An investigation is carried out to analyze the effects of heat and mass transfer over an impulsively started vertical plate in the presence of porous medium with chemical reaction. The unsteady, non-linear, coupled partial differential equations are solved by implicit finite difference scheme of Crank Nicolson type. The influence of various parameters like Prandtl number, Schmidt number, first order chemical reaction on the velocity, temperature and concentration are analyzed. The local skin friction, local Nusselt number, local Sherwood number, average skin friction, average Nusselt number and average Sherwood numbers are investigated. It is observed that the velocity and concentration boundary layer decreases with increasing chemical reaction. An increase in the Schmidt number reduces the concentration boundary layer thickness.

Keywords: Heat transfer; Mass transfer; Porous medium; Chemical reaction; Finite difference.

NOMENCLATURE

c_{∞}'	ambient concentration	T	temperature
c_w'	concentration near the wall	Sc	Schmidt number
C	concentration	T_{∞}'	ambient temperature
D	mass diffusivity	T_w'	wall temperature
Gc	mass Grashof number	t	time
Gr	thermal Grashof number	u	velocity in x- direction
g	acceleration due to gravity	u_0	uniform velocity of the plate
k	permeability parameter	v	velocity in y-direction
k_c	chemical reaction parameter	ν	kinematic viscosity
Pr	The Prandtl number	α	thermal diffusivity

1. INTRODUCTION

Natural convection over a vertical plate in the presence of porous medium has attracted many researchers and industrialists due to its wide range of real time applications. The main aim of this research is to focus on the often antagonistic relationship between the two buoyancy effects that drive the flow. Thermal and mass convection in porous medium have generated increasing interest during the last decades because of its many engineering applications such as geothermal systems, thermal insulation, solid-matrix compact heat exchangers, use of fibrous materials in the insulation of buildings,

underground energy storage and transport systems, heating of ground water by vertical intrusive, polymer production, manufacturing of ceramic, packed-bed catalytic reactors, food processing, cooling of nuclear reactors, enhanced oil recovery, and the dispersion of chemical contaminants through water-saturated soil.

Siegel (1958) was the first to study the unsteady free convective flow past a semi-infinite vertical plate by integral method. Gebhart (1961) gave the approximate solution for the same problem. Cheng and Minkowycz (1972) presented the similarity solution to free convective flow in a porous medium adjacent to a vertical plate, where the wall

temperature is considered as a power function of distance from the leading edge. The laminar flow which arises in fluids due to the interaction of the force of gravity and density differences caused by the simultaneous diffusion of thermal energy and of chemical species was investigated by Gebhart and Pera (1971). A fundamental study of the phenomenon of natural convection heat and mass transfer near a vertical surface embedded in a fluid saturated porous medium was analyzed by Bejan and Khair (1985).

Soundalgekar (1979) presented an exact solution for an viscous, incompressible fluid flow past an impulsively started vertical plate with Laplace transform technique. Soundalgekar and Ganesan (1985) discussed the transient free convective flow past a semi-infinite vertical plate with mass transfer by implicit finite difference scheme. Raptis and Vlohas (1982) analyzed the unsteady two dimensional hydromagnetic free convective flow through a porous medium bounded by a vertical infinite plate. Raptis *et al.* (1987) applied the explicit finite difference scheme to investigate the free convective flow through the porous medium bounded by a semi-infinite vertical plate.

Neild and Bejan (1992) have tried to provide a user friendly introduction to the topics of convection in porous medium. Na and Pop (1983) presented the numerical solution for the free convection flow past a vertical semi-infinite vertical plate embedded in a highly saturated porous medium by allowing the plate to have a non-uniform temperature or a non-uniform heat flux distributions. An exact solution to the mass transfer effects past an impulsively started infinite vertical plate with constant mass flux was presented by Das *et al.* (1996). Unsteady natural convective flow of a moving infinite vertical plate in a radiative and chemically reactive medium in the presence of transverse magnetic field is investigated by Reddy *et al.* (2013).

In various industrial applications, a chemical reaction between a foreign mass and the fluid in which the plate is moving occurs. The chemical reaction can be classified as either heterogeneous or homogeneous process. This depends on whether they occur at an interface or as a single-phase volume. By considering the chemical reaction Das *et al.* (1994) applied the Laplace transform technique to solve the governing equations of homogeneous first order chemical reaction on the flow past an impulsively started vertical plate with constant heat flux. Loganathan and Golden Stepha (2012) analysed the effects of chemical reaction and mass transfer on flow of micropolar fluid past a moving porous plate with variable viscosity. A finite-difference solution of the transient natural convection flow of an incompressible viscous fluid past an impulsively started semi-infinite isothermal vertical plate with mass diffusion was presented by Muthucumarasamy and Ganesan (2002).

In the present work, unsteady, laminar, natural convective flow over an impulsively started vertical plate in the presence of porous medium with first order chemical reaction is investigated. A graphical

representation of velocity, temperature and concentration for various values of permeability parameter, chemical reaction parameter, Prandtl number, Schmidt number are presented. Also shear stress, the rate of heat transfer and mass transfer are analyzed.

2. MATHEMATICAL ANALYSIS

An unsteady laminar, two dimensional, viscous incompressible fluid flow past a moving vertical plate in the presence of porous medium is considered with chemical reaction and mass transfer. The physical model is shown in Fig.1. The x-axis is taken along the direction of vertical plate and y-axis is taken normal to the plate. The properties of the fluid are assumed to be constant. The Darcy's resistance term is considered with constant permeability of the porous medium. Initially the fluid and the plate are assumed to be of same temperature and concentration. Later, as time increases the plate is given an impulsive motion with constant velocity u_0 . The temperature and

concentration of the plate are raised to T_w' and C_w' respectively. The Boussinesq approximation is valid as long as a change in actual density is small. In general, any vertical boundary layer flow is driven by heat transfer ($|\beta\Delta T'| \gg |\beta^* \Delta c'|$) or by mass transfer ($|\beta^* \Delta c'| \gg |\beta\Delta T'|$) or by a combination of heat and mass transfer effects (Bejan 1985). In this problem, both the thermal diffusion and mass diffusion drive the flow, hence the values of thermal Grashof number and mass Grashof number are assumed to be equal.

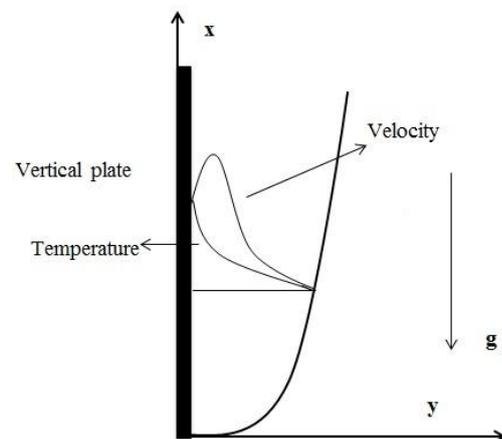


Fig. 1. Flow model and coordinate system.

Under these conditions and by usual Boussinesq's approximation the governing boundary layer equations are

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum:

$$\frac{\partial u}{\partial t'} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g \beta (T' - T_{\infty}') + g \beta^* (c' - c_{\infty}')$$

$$+ \nu \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{k'} u \quad (2)$$

Energy:

$$\frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} = \alpha \frac{\partial^2 T'}{\partial y^2} \quad (3)$$

Concentration:

$$\frac{\partial c'}{\partial t'} + u \frac{\partial c'}{\partial x} + v \frac{\partial c'}{\partial y} = D \frac{\partial^2 c'}{\partial y^2} - k_l (c' - c_{\infty}') \quad (4)$$

The appropriate boundary conditions for velocity, temperature and concentration are

$$t' \leq 0 \quad u = 0, \quad v = 0, \quad T' = T_{\infty}', \quad c' = c_{\infty}' \quad \text{for all } x \text{ and } y$$

$$t' > 0 \quad u = u_0, \quad v = 0, \quad T' = T_w', \quad c' = c_w' \quad \text{at } y = 0$$

$$u = 0, \quad v = 0, \quad T' = T_{\infty}', \quad c' = c_{\infty}' \quad \text{at } x = 0$$

$$u \rightarrow 0, \quad T' \rightarrow T_{\infty}', \quad c' \rightarrow c_{\infty}' \quad \text{as } y \rightarrow \infty \quad (5)$$

Now introducing the following non-dimensional quantities

$$X = \frac{xu_0}{\nu}, \quad Y = \frac{yu_0}{\nu}, \quad U = \frac{u}{u_0}, \quad V = \frac{v}{u_0}, \quad t = \frac{t'u_0^2}{\nu},$$

$$T = \frac{T' - T_{\infty}'}{T_w' - T_{\infty}'}, \quad C = \frac{c' - c_{\infty}'}{c_w' - c_{\infty}'}, \quad Gr = \frac{\nu g \beta (T_w' - T_{\infty}')}{u_0^3},$$

$$Gc = \frac{\nu g \beta^* (c_w' - c_{\infty}')}{u_0^3}, \quad Pr = \frac{\nu}{\alpha}, \quad Sc = \frac{\nu}{D},$$

$$k = \frac{k'u_0^2}{\nu^2}, \quad k_c = \frac{k_l \nu}{u_0^2} \quad (6)$$

where k' is the permeability parameter and k_l is the chemical reaction parameter in dimensional form. The governing equations in non-dimensional form is given by

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (7)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = GrT + GcC + \frac{\partial^2 U}{\partial Y^2} - \frac{U}{k} \quad (8)$$

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial Y^2} \quad (9)$$

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - k_c C \quad (10)$$

The corresponding boundary conditions are

$$t \leq 0 \quad U = 0, \quad V = 0, \quad T = 0, \quad C = 0 \quad \text{for all } X \text{ and } Y$$

$$t > 0 \quad U = 1, \quad V = 0, \quad T = 1, \quad C = 1 \quad \text{at } Y = 0$$

$$U = 0, \quad V = 0, \quad T = 0, \quad C = 0 \quad \text{at } X = 0$$

$$U \rightarrow 0, \quad T \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } Y \rightarrow \infty \quad (11)$$

The local as well as average values of skin friction, Nusselt number and Sherwood number in non-dimensional form are as follows

$$\tau_x = - \left(\frac{\partial U}{\partial Y} \right)_{Y=0} \quad (12)$$

$$\bar{\tau}_x = - \int_0^1 \left(\frac{\partial U}{\partial Y} \right)_{Y=0} dX \quad (13)$$

$$Nu_x = - X \left(\frac{\partial T}{\partial Y} \right)_{Y=0} \quad (14)$$

$$\overline{Nu}_x = - \int_0^1 \left(\frac{\partial T}{\partial Y} \right)_{Y=0} dX \quad (15)$$

$$Sh_x = - X \left(\frac{\partial C}{\partial Y} \right)_{Y=0} \quad (16)$$

$$\overline{Sh}_x = - \int_0^1 \left(\frac{\partial C}{\partial Y} \right)_{Y=0} dX \quad (17)$$

3. NUMERICAL PROCEDURE

The two-dimensional, unsteady, coupled, non-linear partial differential equations (7) to (10) subject to the boundary conditions in equation (11) are discretized with Crank Nicolson implicit finite difference scheme which converges faster and is unconditionally stable. Depending on the boundary conditions in equation (11), the region of integration is decided as $X_{\max}=1$ and $Y_{\max}=14$, where Y_{\max} corresponds to $Y=\infty$. Here the subscript i designates the grid points in the direction of X , j designates the grid points in the direction of Y , and n along the t - direction. The equations at every internal nodal point for a particular i -level constitute a tridiagonal system. This system of tridiagonal matrix can be solved by applying Thomas algorithm (1969). Hence the values of U , V , T , and C are known at all nodal points in the region at $(n+1)^{\text{th}}$ time level. Computations are carried out for all the time levels until the steady state is reached. The scheme is proved to be unconditionally stable using the Von Neumann technique. The local truncation error for the scheme is $O(\Delta t^2 + \Delta Y^2 + \Delta X)$ and it approaches zero as Δt , ΔY and ΔX tends to zero. Stability and compatibility ensures the convergence of the scheme.

4. METHOD OF SOLUTION

The Crank Nicolson finite difference scheme for the governing equations (7) to (10) are given by

$$\frac{1}{4\Delta X} \left[U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^n - U_{i-1,j}^n + U_{i,j-1}^{n+1} - U_{i-1,j-1}^{n+1} + U_{i,j-1}^n - U_{i-1,j-1}^n \right] + \frac{1}{2\Delta Y} \left[V_{i,j}^{n+1} - V_{i,j-1}^{n+1} + V_{i,j}^n - V_{i,j-1}^n \right] = 0 \quad (18)$$

$$\begin{aligned} & \frac{U_{i,j}^{n+1} - U_{i,j}^n}{\Delta t} + \frac{U_{i,j}^n}{2\Delta X} \left[U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^n - U_{i-1,j}^n \right] \\ & + \frac{V_{i,j}^n}{4\Delta Y} \left[U_{i,j+1}^{n+1} - U_{i,j-1}^{n+1} + U_{i,j+1}^n - U_{i,j-1}^n \right] \\ & = Gr \left[\frac{T_{i,j}^{n+1} + T_{i,j}^n}{2} \right] + Gc \left[\frac{C_{i,j}^{n+1} + C_{i,j}^n}{2} \right] + \\ & \frac{1}{2(\Delta Y)^2} \left[U_{i,j+1}^{n+1} - 2U_{i,j}^{n+1} + U_{i,j-1}^{n+1} + U_{i,j+1}^n \right. \\ & \left. - 2U_{i,j}^n + U_{i,j-1}^n \right] - \frac{1}{k} \left[\frac{U_{i,j}^{n+1} + U_{i,j}^n}{2} \right] \end{aligned} \quad (19)$$

$$\begin{aligned} & \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta t} + \frac{U_{i,j}^n}{2\Delta X} \left[T_{i,j}^{n+1} - T_{i-1,j}^{n+1} + T_{i,j}^n - T_{i-1,j}^n \right] + \\ & \frac{V_{i,j}^n}{4\Delta Y} \left[T_{i,j+1}^{n+1} - T_{i,j-1}^{n+1} + T_{i,j+1}^n - T_{i,j-1}^n \right] \\ & = \frac{1}{2Pr(\Delta Y)^2} \left[T_{i,j+1}^{n+1} - 2T_{i,j}^{n+1} + T_{i,j-1}^{n+1} + T_{i,j+1}^n \right. \\ & \left. - 2T_{i,j}^n + T_{i,j-1}^n \right] \end{aligned} \quad (20)$$

$$\begin{aligned} & \frac{C_{i,j}^{n+1} - C_{i,j}^n}{\Delta t} + \frac{U_{i,j}^n}{2\Delta X} \left[C_{i,j}^{n+1} - C_{i-1,j}^{n+1} + C_{i,j}^n - C_{i-1,j}^n \right] \\ & + \frac{V_{i,j}^n}{4\Delta Y} \left[C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1} + C_{i,j+1}^n - C_{i,j-1}^n \right] = \\ & \frac{1}{2Sc(\Delta Y)^2} \left[C_{i,j+1}^{n+1} - 2C_{i,j}^{n+1} + C_{i,j-1}^{n+1} + C_{i,j+1}^n - 2C_{i,j}^n + C_{i,j-1}^n \right] \\ & - k_c \left[\frac{C_{i,j}^{n+1} + C_{i,j}^n}{2} \right] \end{aligned} \quad (21)$$

These equations (18) to (21) at every nodal point for a particular i^{th} level constitute a tridiagonal system of equations. In order to get the physical insight into the problem the numerical values of U, V, T and C are computed for different values of thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number, permeability parameter and chemical reaction parameter. Knowing the velocity, temperature and concentration it is interesting to calculate the skin friction coefficient, local Nusselt number, local Sherwood number, average skin friction, average Nusselt number and average Sherwood number. Five point approximations is used to approximate the derivatives in local skin friction, local Nusselt number and local Sherwood number. Newton Cote's formula is used to calculate the average skin friction, average Nusselt number and average Sherwood number.

5. RESULTS AND DISCUSSION

A numerical study of natural convective flow past an impulsively started vertical plate in the presence

of porous medium with chemical reaction is analyzed. For the discussion $Pr = 0.73$ and $Pr = 7$ are considered which corresponds to air and water respectively. Schmidt numbers are chosen to be $Sc=0.60, 0.94, 2$ which corresponds to water vapor, Carbon dioxide and Ethyl benzene respectively. The governing partial differential equations are solved by semi-implicit finite difference scheme of Crank Nicolson type. The various results are obtained to study the influence of permeability, chemical reaction, Schmidt number, thermal Grashof number, mass Grashof number and Prandtl number on velocity, temperature and concentration. Also the effects on local skin friction, local Nusselt number, local Sherwood number, average skin friction, average Nusselt number and average Sherwood number are discussed.

In order to ascertain the accuracy of the numerical results, the present study is compared with the exact solution available in the literature. In the absence of porous medium and considering the chemical reaction parameter as zero, the velocity profile for $Gr = 2, Gc = 5, Pr = 0.71$ and $Sc = 0.16$ are compared with the exact solution of Soundalgekar (1979) at $t = 0.2$ in Fig. 2 and they are found to be in good agreement.

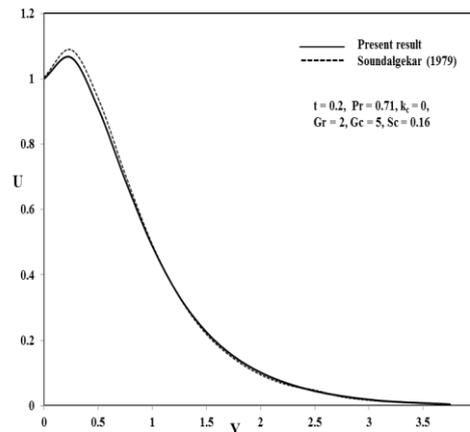


Fig. 2. Comparison of velocity profile.

In Fig. 3 and 4, the velocity variation for different chemical reaction parameter and permeability are presented. An increase in the chemical reaction parameter decreases the velocity boundary layer thickness. This is because of loss of energy in the particles of the species. Also the velocity profile for air is higher when compared to water. The heat diffusion becomes slow due to an increase in Prandtl number which in turn reduces the velocity of the flow. It is also observed that the time taken to reach the steady state is reduced with increasing values of chemical reaction parameter.

In Fig. 4, velocity boundary layer increases with increasing permeability values. It is observed that the velocity boundary layer grows in the direction of the vertically moving plate and gradually reduces to the free stream boundary conditions. This is due to the Darcy's law which relates the flow rate and the fluid properties to the pressure gradient applied

to the porous medium. Hence there is an increase in velocity boundary layer thickness.

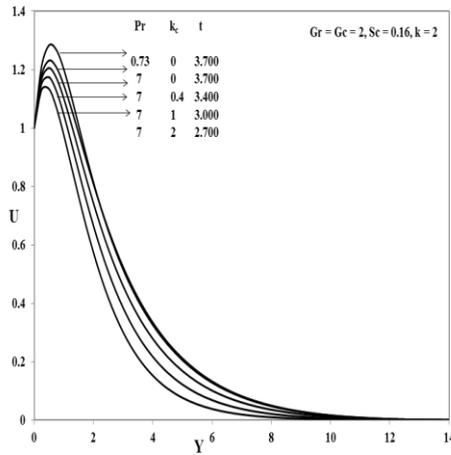


Fig. 3. Effects of k_c on velocity profile.

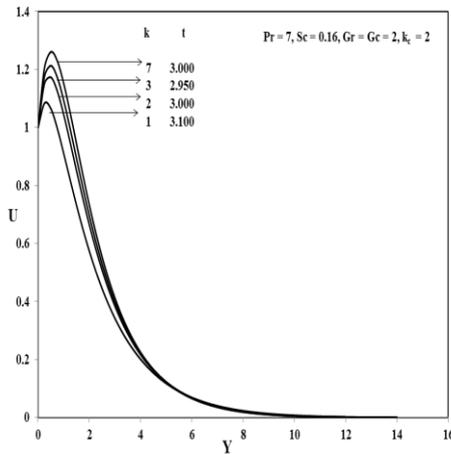


Fig. 4. Effects of permeability on velocity profile.

In Fig. 5 and 6, velocity profile for Prandtl number and Schmidt numbers are presented. An increase in the Prandtl number decreases the velocity boundary layer thickness. It is also observed that irrespective of the variation in the Prandtl number the steady state is common for all the Prandtl numbers. In Fig.6, velocity profile for various values of Schmidt number is plotted. The velocity boundary layer increases with decreasing values of Sc . The time taken to reach the steady state increases with increasing values of the Schmidt number. This describes that the contribution of mass diffusion to the buoyancy force increases the velocity significantly.

In Fig. 7 and 8, effects of Permeability and Prandtl number on thermal boundary layer are shown. The thermal boundary layer is reduced with increasing values of permeability. The permeability being the measure of the ability of a porous material allows the fluids to pass through it. When this parameter is increased there will be a free flow of fluid through it. Hence there is a reduction in the thermal boundary layer.

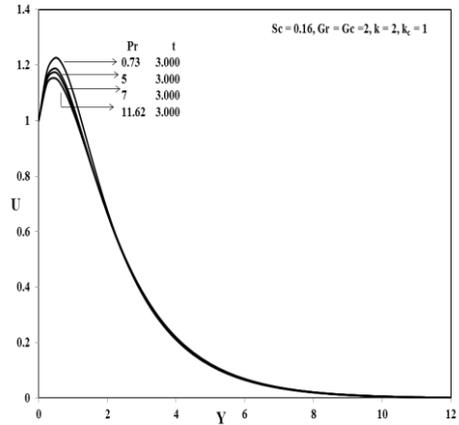


Fig. 5. Effects of Pr on velocity profile.

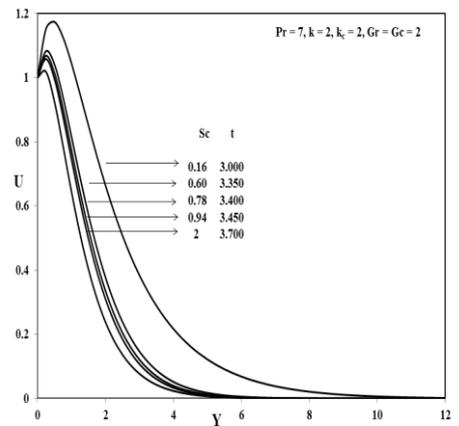


Fig. 6. Effects of Sc on velocity profile.

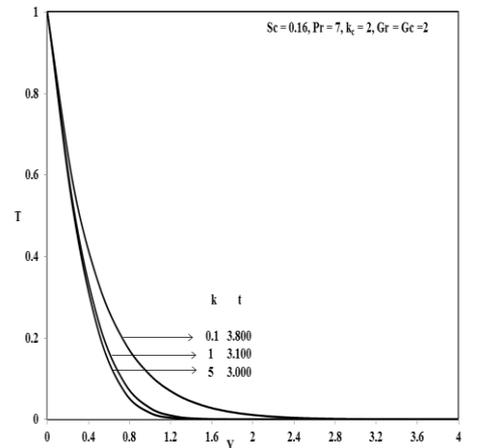


Fig. 7. Effects of k on temperature profile.

In Fig. 8, temperature profile for various values of Prandtl number is plotted. An increase in the Prandtl number reduces the thermal boundary layer thickness. As Prandtl number is the characteristic of the relationship between the momentum boundary layer thickness to the thermal boundary layer thickness, the thermal boundary layer is reduced by increasing the Prandtl number. Hence there is a very slow heat diffusion process.

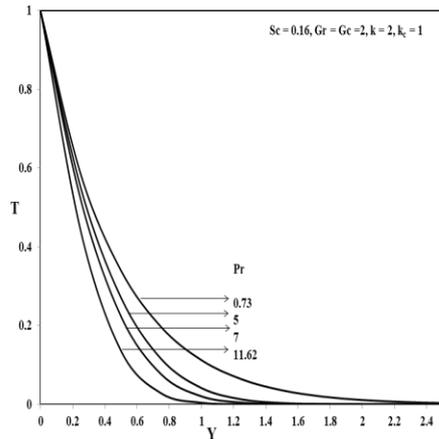


Fig. 8. Effects of Pr on temperature profile.

In Fig. 9, 10 and 11, Concentration boundary layer for various effects of permeability, chemical reaction and Schmidt numbers are presented. By increasing the permeability parameter the concentration boundary layer decreases. Also, the time taken to reach the steady state decreases with increasing values of permeability. In Fig. 10, the concentration decreases with increasing values of chemical reaction parameter. The chemical reaction reduces the local concentration thereby increasing its concentration gradient and its flux. Thus it is observed that there is a fall in concentration due to the increasing values of chemical reaction parameter.

In Fig. 11, effects of Schmidt number on concentration profile is shown. Concentration boundary layer decreases with increasing values of Schmidt number. As Schmidt number is the ratio of the momentum diffusivity to species diffusivity, it physically relates the relative thickness of the viscous boundary layer and concentration boundary layer. Hence there is an increase in the concentration boundary layer with decreasing Sc. The time taken to reach the steady state also increases with decreasing values of Schmidt number.

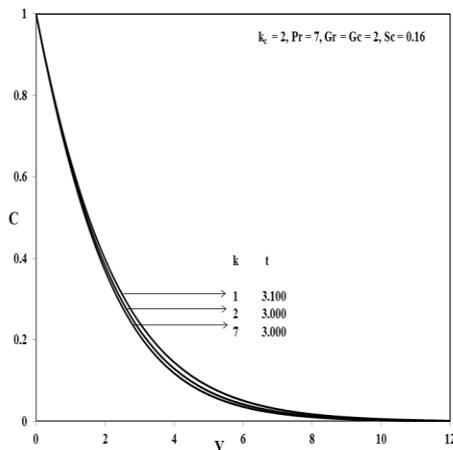


Fig. 9. Effects of k on concentration profile.

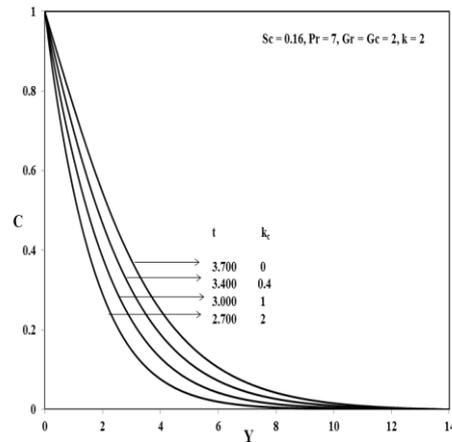


Fig. 10. Effects of k_c on concentration profile.

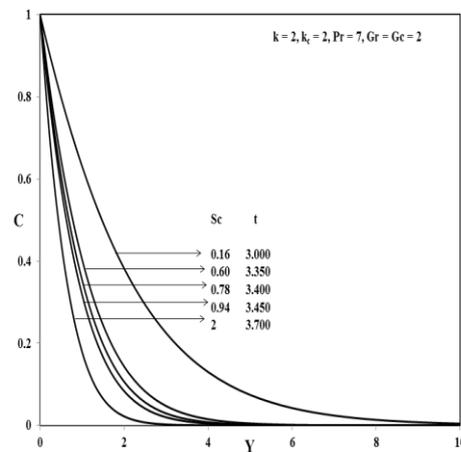


Fig. 11. Effects of Sc on concentration profile.

Fig. 12 and 15, Local skin friction and average skin friction are presented for various values of chemical reaction parameter. A reduction in the chemical reaction parameter decreases the local skin friction coefficient. The presence of chemical reaction reduces the velocity boundary layer. This deceleration in the velocity of the flow reduces the shear stress along the wall which leads to a decrease in the local skin friction. The average skin friction decreases with increasing values of chemical reaction parameter. This decrease is due to the fact that, the velocity of the fluid decreases with increasing values of chemical reaction parameter as shown in fig. 3. The deceleration in the velocity reduces the wall friction and hence there is a decrease in the rate of shear stress.

Fig. 13 and 16, Local Nusselt number and average Nusselt number for various values of chemical reaction parameter are presented. It is observed that an increase in the chemical reaction parameter reduces the local Nusselt number. Also, the average Nusselt number decreases with increasing chemical reaction parameter. It is observed that at initial times, for air the average Nusselt number is constant at each time level for various chemical

reaction parameters. This shows that initially the heat transfer is only by conduction.

increases with increasing values of chemical reaction. The chemical reaction parameter induces the rate of mass transfer.

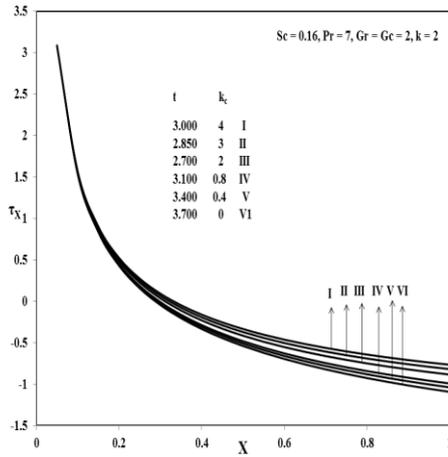


Fig. 12. Effects of k_c on local skin friction coefficient.

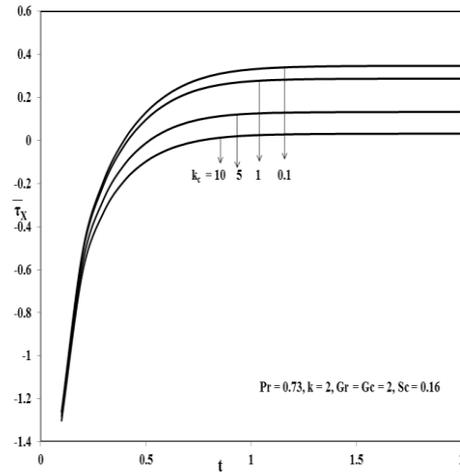


Fig. 15. Effects of k_c on average skin friction.

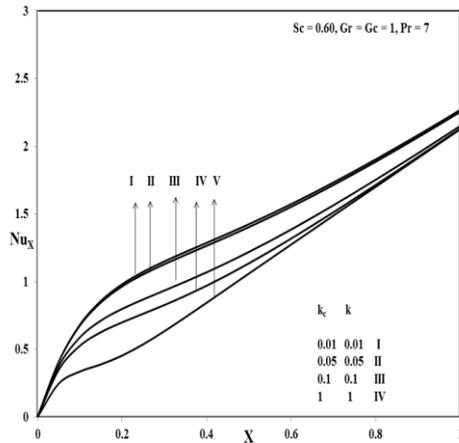


Fig. 13. Effects of k_c on local Nusselt number.

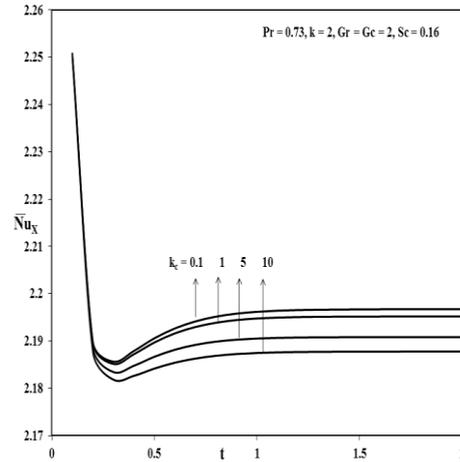


Fig. 16. Effects of k_c on average Nusselt number.

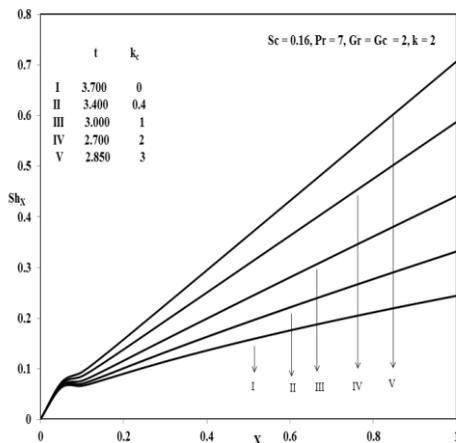


Fig. 14. Effects of k_c on local Sherwood number.

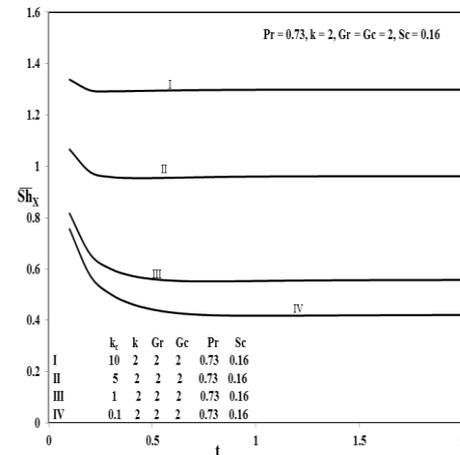


Fig. 17. Effects of k_c on average Sherwood number.

In Fig. 14 and 17, Local Sherwood number and Average Sherwood number for various values of chemical reaction parameter is presented. The local Sherwood number and average Sherwood number

6. CONCLUSION

In the present analysis, a numerical study of natural convective flow past an impulsively started vertical plate in the presence of porous medium with chemical reaction is analyzed. The non-dimensional governing partial differential equations are discretized by finite difference scheme of Crank Nicolson type. The results are summarized as follows.

1. An increase in chemical reaction, Prandtl number and Schmidt number decreases the velocity. But an increase in permeability increases the velocity boundary layer.
2. An increase in permeability and Prandtl number reduces the thermal boundary layer thickness.
3. An increase in chemical reaction, Prandtl number and Schmidt number decreases the velocity. But an increase in permeability increases the velocity boundary layer.
4. An increase in permeability and Prandtl number reduces the thermal boundary layer thickness.
5. An increase in chemical reaction and Schmidt number reduces the concentration boundary layer where as a reverse process is seen for increasing values of permeability.

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