



# Flow Visualization of Internal Waves and Wakes of a Streamlined Body in a Stratified Fluid

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## ABSTRACT

The wake and internal waves of a moving three dimensional (3D) airfoil body in a stratified fluid has been investigated in a large stratified tank with a finite depth using movies of shadowgraphs of the flow fields. Typical Reynolds and Froude numbers of the flow varied between  $10^3$  and  $10^4$ , and 0.3 and 2 respectively. The flows are generated often by towing the body in a uniformly stratified flow, while limited cases are carried out with body stationary and the channel was in recirculating mode. For some experiments the density profile had a stepped like shape. The wake flow is often consisted of internal waves including random and coherent ones. Distortion of density fields was also observed ahead and above the body in cases where the Froude number was subcritical. Results show that as the Froude number ( $Fr=U/Nh$ , where  $U$  is the body relative velocity,  $N$  is buoyancy frequency and  $h$  is the thickness of the body) is increased, the flow undergoes from a subcritical narrow wake (for  $Fr<1$ ) to an internal waves dominated flow (for  $Fr\sim 1$ ) and then to a hydraulic jump with a turbulent wake with some mixing (for  $Fr>1$ ). Typical wavelength of the exited internal waves is increased with  $Fr$ , as the theory predicts. The wake of the flow for  $Fr>1.4$  appeared to collapse and some internal waves emission from it could be observed. Usually two types of internal waves, namely random small scale and large scale, more regular waves are observed.

**Keywords:** Stratified tank; Airfoil body; Wake; Internal lee waves; Flow shadowgraphs.

## NOMENCLATURE

$a$	a constant	$N$	buoyancy frequency
$C_g$	group velocity	Re	Reynolds number
$C_p$	phase velocity	$Ri=Fr^{-2}$	Richardson number
D	force coefficient in the x direction	t	time
$f=2\Omega\sin\varphi$	Coriolis parameter	$IT_i=f^{-1}$	inertial period
g	gravitational acceleration	x	travelled distance
H	fluid depth	$\alpha$	angle of waves spreading
h	body thickness	$\theta$	angle of wave number with horizontal
K	wave number vector	$\lambda$	wavelength
$K_h$	horizontal wave number	$\nu$	kinematic viscosity
$k_x, k_y, k_z$	components of wave number	$\gamma\rho(\rho_0)$	density (reference density)
L	length of the body	$\varphi$	latitude angle
$L_b$	size of the blocked region	$\omega$	internal wave angular frequency
n	the nth vertical mode		

## 1. INTRODUCTION

Flows around bodies moving in stratified fluids are found in nature and engineering and consist of large scale internal waves, random internal waves and turbulent mixing (Thorpe, 2005). Such flows are often found in atmosphere and ocean as these natural flows occur over uneven surfaces or as a

result of convective flows near stratified regions, as in the thermocline of ocean. Atmosphere and ocean often possess internal waves which appear to have periods near that of inertial periods ( $\sim f^{-1}$  where  $f$  is the Coriolis parameter, Garret and Munk, 1979). Interaction of moving submerged or surface vessels in stratified ocean can also generate flows that produce internal waves and turbulence (e.g.

Bonneton *et al.* 1993; Dotsenko and Savoskin, 1994). Hence understanding such flows are important in terms of finding the characters of internal waves and also their roles in energy transfer and turbulent mixing in stratified fluids.

Many experimental and numerical works on this problem have been carried out and here some of the more relevant works are given. Stevenson *et al.* (1986) found experimentally, that a moving cylinder in a stratified region of a fluid generates internal waves that can be trapped in the region. Ray paths in the region were found to be in accord with ray theory. Voropayeva *et al.* (2003) used numerical simulations to show that wakes of a body in stratified media can lead to large amplitude internal waves. Hopfinger *et al.* (1991) used laboratory experiments to show that the flow behind a spherical body moving in stratified fluid is dominated with internal waves for  $Fr < 4$ , while flows with larger  $Fr$  create wakes with coherence structures and turbulence that can excite internal waves. This process is associated with the transfer of energy to internal waves and viscous dissipation rather than to mixing (e.g. Linden, 1979). Chomaz *et al.* (1993) have also extended the  $Fr$  number range of such flows (behind a spherical body moving in stratified fluid), finding 4 different regimes from purely 2D regime, dominated by waves to 3D flow with minor wave activity. Gilreath and Brandt (1985) have also considered the influence of the shape of the vertical density profile with small and large extensions compared to the size of a propelled body, on the wake of the moving body. The main result of this work is the strong coupling between the random internal waves and the wake of the body; also when the density gradient is large some solitary wave structure can be formed behind the body.

Mitkin and Chashechin (2000) and also Chashechin *et al.* (1995) have investigated the flow around a cylinder moving in a linear stratified flow with very small  $Fr$  that included viscous effects. Their results show that the blocked zone ahead of the moving body almost moves with the body. The modal structure of the internal wave field behind the body is associated with shear layers or laminar jet like structures. Such shear layers have also been observed in flows generated by plumes in confined stratified fluids (Wong and Griffiths, 2001, Griffiths and Bidokhti, 2008). The blocked region ahead of the moving body,  $L_b$ , is given by:  $[L_b / (DRiRe)]^{1/4} = 0.15$  and the associated speed of the blocked zone is about  $u/U = a(x/D)^{-3/4}$  with  $a$  is about 3 depending on  $U$ , the body speed with diameter  $D$  and  $Ri$  is  $Fr^{-2}$ .

Glushko *et al.* (1994) also studied turbulent wakes of spherical bodies in stratified fluids using a mathematical model by integrating numerically the governing equations, including turbulence. They found that at a time scale less than  $10N^{-1}$  the fluctuating velocities in the wake of the body are usually larger than the mean values. Chashechin and Mitkin (2004) also used high resolution visualizations of the flow behind a strip moving in its plan in a stratified fluid, showing that a speed

larger than a certain value, butterfly-like vortices appear in the wake of the body. Nicolaou *et al.* (1995) also considered internal waves generated by bodies accelerating in a thermocline that generate unsteady waves. These waves appear as V-shaped wedge with an angle  $\alpha$ , such that  $\alpha = \sin^{-1}(Fr^{-1})$ , behind the body in the plan view above the body (similar to surface waves generated by ship). They found the perturbation fields on the isopycnal surfaces around and behind the moving body. Free surface waves by moving hydrofoil just beneath the surface of water have also some similarity with such wave field patterns (Ghassemi and Kohansal, 2013).

Dotsenko and Savoskin (1994) studied internal waves generated by external forcing in a stratified fluid with constant depth using analytical 2D solution consisting of vertical modes. Resonance values of forcing frequency (atmospheric) could generate large amplitude waves in the ocean. Direct observations of internal waves in ocean have also shown that such waves in the ocean, generated by triad interactions can lead to formations of waves with near inertial periods ( $T_i = P^{-1} = (2\Omega \sin \phi)^{-1}$ , where  $\phi$  is the latitude angle) (Garret and Munk, 1979).

Ohya *et al.* (2013) considered flow of wakes behind cylinders in a stratified fluid with a finite depth, showing that the shedding frequency and two-dimensionality of the flow changes with increasing stratification. Abdilghanie and Diamessis (2013) also used nonlinear simulations to study the internal wave emitting turbulent wakes of such flows.

In the present work we report the wake and flow field behind a rather thick streamlined airfoil in a large stratified tank which is either have a continuous or a stepped density profile. The results are mainly acquired from flow visualizations of shadowgraphs for a range of Froude and Reynolds numbers.

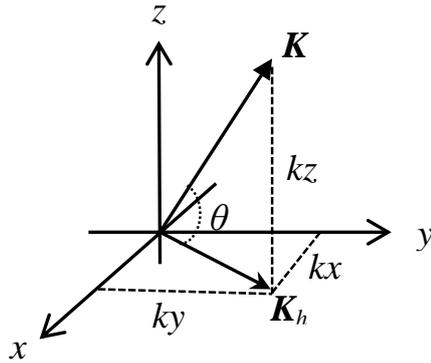
## 2. SOME THEORY

Internal waves generated by a body in a stratified fluid can be considered using linear theory (e.g. Lighthill 1978, Gill.1982). Dispersion relation for small amplitude waves in uniformly stratified fluid can be found for inviscid unbounded flows as:

$$\omega^2 = N^2 \left( \frac{K_h}{K} \right)^2 \quad \text{or} \quad \omega^2 = N^2 \cos^2 \theta \tag{1}$$

Where  $\omega$  is the frequency of the wave,  $N = \left( \frac{-g}{\rho_0} \frac{\partial \rho}{\partial z} \right)^{1/2}$  is the buoyancy frequency of the

stratified fluid,  $K_h = (k_x, k_y)$  is the horizontal wave number,  $K = (k_x, k_y, k_z)$  is the wave number vector and  $\theta$  is the angle between the wave number vector and horizontal (figure 1).



**Fig. 1. Wave number vector and its components.**

The phase velocity of the wave is  $C_p = \omega / K$  and its group velocity is  $C_g = d\omega / dK$  with their magnitude of  $(N/K)\cos \theta$  and  $(N/K)\sin \theta$  respectively, indicating that they are orthogonal. Also the vertical components of these are opposite to each other in direction. When the source of the waves moves with velocity  $U$ ,  $\omega = \omega_r + K \cdot U$ , hence with the relation to the body, phase lines are stationary as  $\omega$ , or  $C_p = 0$ . When the body is moving with velocity  $U$  in the  $-x$  direction (as these experiments)  $C_{pr} = U \cos \theta$  in the plane of  $y=0$  and the wavelength in  $x-z$  plane with  $UK=N$  is  $\lambda = 2\pi U / N$ . Then in terms of  $Fr$  we can write:

$$Fr = U / (Nh) = \lambda / (2\pi h) \quad \text{or} \quad \lambda = 2\pi h Fr \quad (2)$$

Where  $h$  is the body height or thickness. This indicates that the wavelengths of these waves are proportional to  $Fr$  and body thickness, in the domain of  $Fr$  when wave activities are large. Hopfinger *et al.* (1991) found that for a spherical body moving in a uniform stratified fluid this domain is  $Fr \sim 0.5-4$  for lee waves (behind the body) in the vertical plane.

For  $k_x$  of stationary waves (with respect of the body) with  $U \sim C_p$  and  $U < C_g$  ( $C_p$ ,  $C_g$  and  $U$  are the horizontal components of phase, group and fluid relative velocities.), hence disturbance cannot propagate upstream of the moving body. For such situation it can be shown that:

$$NH / (\pi U) = (n^2 - kx^{-2} H^2 / \pi)^{1/2} \quad (3)$$

Where  $H$  is the fluid depth and  $2H/n$  is the wavelength for the  $n$ th vertical mode in the vertical direction (Ohya *et al.*, 2013). Ohya *et al.* (2013) also showed that for stationary lee waves with  $kx \rightarrow 0$  we have

$$NH / (\pi U) = n \quad (4)$$

The wavelength of these waves is then given by:

$$\lambda = 2^{1/2} H / \left[ (NH / \pi U)^2 - n^2 \right]^{1/2} \quad (5)$$

This is similar to (2) but the vertical structure of the waves is quantized in term of the fluid depth. For  $n=0$

$$\lambda = 2^{1/2} H \pi Fr \quad (6)$$

where here  $Fr = (NH/U)^{-1}$ . Hence we could express the wavelength of the internal waves in terms of  $h$ , the thickness of the body or  $H$ , the fluid depth, if it is small enough.

### 3. EXPERIMENTAL METHOD

The experiments are carried out in a large channel with a working section of 1.5 m wide, 0.75 deep and 5 m long and has been explained in Bidokhti and Britter (2002). It is a multilayer system of which each layer can be circulated independently by a pump, while a density smoothing section diffuses the density steps between the layers. The density of the fluid is controlled by the amount of salt concentration in water. As the density is about a linear function of salt concentration which can change conductivity of water, hence the measured conductivity by a salinity sensor can readily give the density. Notice that the salt molecular diffusivity in water is about  $10^{-5} \text{ cm}^2 \text{ s}^{-1}$  which can help maintain the vertical density gradient.

There are nine operating layers (5 cm each), although only six layers are used here. Here for these experiments the tank was often used in towing mode, with some limited cases in recirculating mode. This was because in the recirculating mode, some random internal waves of background source can, to some extent, spoil the field of view around the body.

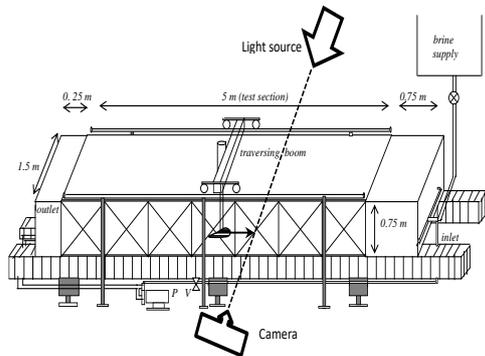
The body is a 3D streamlined slightly asymmetric (with an angle of attack of 2 degrees) airfoil, 5 cm thick, 20 cm long and 38 cm wide which is much smaller than the width of the channel; hence the flow around the body is actually three dimensional. The body was moved or placed along the center of the channel. The motion of the body along the channel or up and down, was controlled by a motorized traverse system under the control of a computer. The motors responsible for the motion in three directions are stepper motors; hence movement and positioning of the body or any other device on it could be done with good precisions.

Two forms of density profiles were used (linear and stepped profiles), although the overall density profile was used to calculate the buoyancy frequency, typically  $0.93 \text{ s}^{-1}$  and  $0.43 \text{ s}^{-1}$ . In order to change  $Re$  and  $Fr$ , the towing velocity of the body is often varied. Figure 2 shows the experimental set up (mainly taken from Bidokhti and Britter, 2002). The light source is a projector light at a long distance from the view section, as to provide parallel lighting to the field of the flow, generated by the body. The camera is also a high resolution video camera for capturing the flow images.

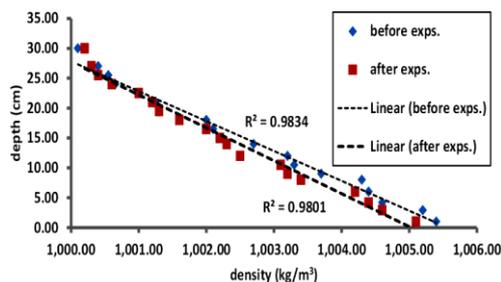
Figure 3 shows a typical vertical profile of the density with  $N=0.43 \text{ s}^{-1}$  before and after the experiments. Some discontinuities between the layers are observed, although a density smoothing section at the inlet diffuses the profile across the layers. The profile changes little after all the towing experiments and then  $N=0.42 \text{ s}^{-1}$ .

**Table 1 Operating conditions of all experiments, including wavelengths of IW generated**

Exp. No.	Fr	Re (based on L=20 cm)	Wavelength (cm)	Mode	Density Pro.
1	0.35	1480	None!	Towing	Cont.
1-1	0.40	1600	12	Towing	Cont.
2	0.60	2500	20	Towing	Cont.
3	0.86	3600	32	Towing	Cont.
4	1.14	4800	40	Towing	Cont.
5	1.30	5400	42	Towing	Cont.
6	1.26	5280	42	Towing	Cont.
7	1.02	4280	28	Towing	Step.
8	0.90	3760	28	Towing	Step.
9	0.73	3060	30	Towing	Cont.
10	1.00	4200	28	Towing	Cont.
11	1.50	6200	Turb.-wake	Towing	Cont.
12	1.20	5000	40	Towing	Cont.
13	0.97	9000	28	Circ.	Cont.
14	0.97	9000	28	Circ.	Cont.
15	0.55	2400	15	Up-down	Step.
16	1.03	4340	27	Towing	Step.
17	1.07	4500	28	Towing	Step.



**Fig. 2. The density-stratified channel with the airfoil in place (Bidokhti and Britter, 2002).**



**Fig. 3. Typical density profiles before and after the experiments in the channel (for towing cases).**

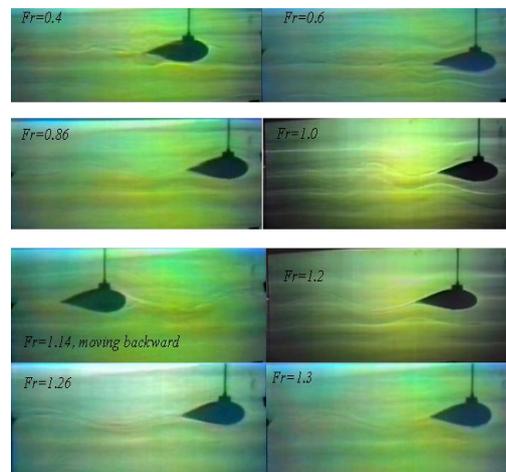
Typical ranges of these numbers are  $Re \sim 10^3 - 10^4$  ( $Re = UL/\nu$ , where  $L$  is the length of the body and  $\nu$  is the kinematic molecular viscosity of fluid, here it is  $10^{-2} \text{ cm}^2\text{s}^{-1}$ ) and  $Fr = U/(Nh) \sim 0.5 - 2$ , where  $h$  is the thickness of the body. The operating conditions of these experiments are shown in Table 1. Here typical observed wavelengths of internal waves (IW) generated by the body are also shown.

Here we present the results consisting mainly of shadowgraphs. Flow properties are extracted from the video movies which also recorded time with hundredth of a second.

#### 4. RESULTS AND DISCUSSION

##### 4.1 Towing Experiments

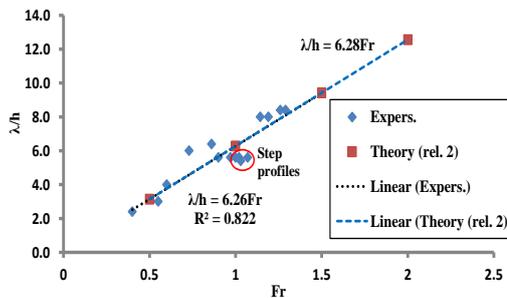
First the towing experiments with continuous stratification (often  $N = 0.43 \text{ s}^{-1}$ ) are presented, as they are more systematic and numerous. Figure 4 shows shadowgraphs of the flow around the body in towing mode with increasing  $Fr$ . It is clear that as  $Fr$  increases wave activity behind the body increases and the wavelength of the observed waves also increases. When  $Fr$  reaches around 1 the waves behind the body become marked, also longer waves are observed as  $Fr$  increases up to about 1.4.



**Fig. 4. Flow around the body moving left to right with different  $Fr$ , except one with  $Fr = 1.14$  in which it moves backwards.**

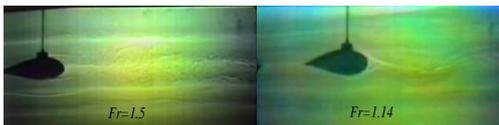
In one case when the body moves backwards with  $Fr=1.14$  the wave amplitude is also larger as the wake behind the body is thicker. Figure 5 is a plot of normalized wavelengths by the thickness of the body ( $\lambda/h$ ) against  $Fr$ . The increase is evident as the relations 2 or 6 also show.

The points for limited experiments with stepped density profiles, with  $Fr$  of about one, are also shown. The best fit through the experimental points are surprisingly close to the linear theory (relation 2).



**Fig. 5. Variations of the normalized wavelength with  $Fr$ . The points circled are for stepped density profiles.**

This indicates that for these experiments the scaling is  $h$  rather than  $H$ , the fluid depth. Similar results have also been acquired by Hopfinger *et al.* (2004). When  $Fr$  becomes larger than 1.4, the wake of the body, especially when it moves in a reversed direction, becomes turbulence and for some time downstream the turbulent wake collapses. Figure 6 is an example of this type of flow in which a case with  $Fr=1.14$  with lee waves and slight mixing, for comparison, is also shown.

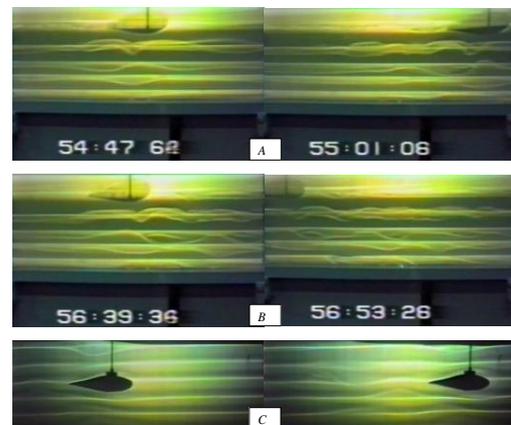


**Fig. 6. The turbulent wake for a case in which the body moves in reversed direction with  $Fr=1.5$ , and for a case with  $Fr=1.14$  with internal lee wave. The body moves from right to left.**

At about time of  $Nt=Nx/U\sim 8.0$ , where  $x$  is the distance travelled by the body, the wake which is initially about  $2h$  (10 cm) seems to collapse to about half of its initial size. Some sign of internal waves emission from the wake is also observed as Abdilghanie and Diamassis (2013) have also found, although the turbulence in the wake appears to be transformed to a random internal wave field.

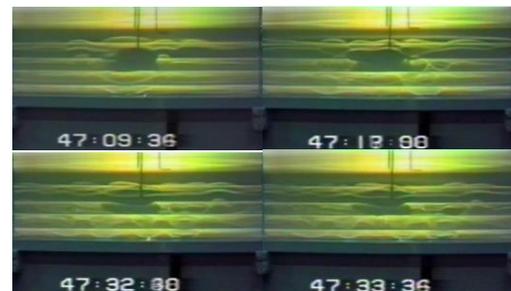
In some experiments the density profile is stepped and here we show some visualizations of the flow due to the body movements with this type of configuration. Figure 7 shows the waves generated by the body moving near the top and at the mid depth of the stratified channel. It is interesting that the movement of the body near the surface generates internal waves well away from the body with typical wavelengths of order of the length of the body, down to the bottom of the channel. While

the moving stationary wave patterns (with respect of the body) with the body near the mid depth of canal is more regular and more confined to the central part of the channel.



**Fig. 7. Wave patterns generated by the moving body A) left to right with  $Fr=1.03$ , B) right to left with  $Fr=1.07$ , body near the top (in the top layer); and C) left to right with  $Fr=0.9$  body in the middle.**

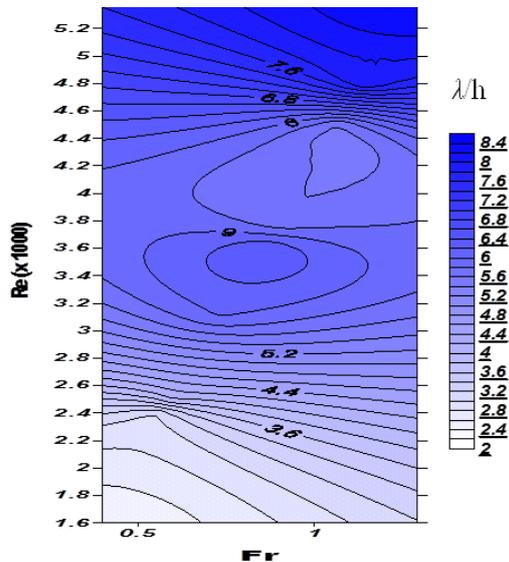
Figure 8 shows the waves generated by the body moving up and then down once with a vertical displacement of 5 cm with an average speed of 1.2 cm/s. A collection of waves is generated while propagating away in vertical and horizontal directions. The angle of between the propagation direction and horizontal is about 60 degrees (corresponding to  $\cos \theta = \omega/N \sim U/Nh \sim 0.5$ ). Reflection of the waves from the bottom is also observed.



**Fig. 8. Waves around the body when it moves first up then down once for 5 cm with a speed of 1.2 cm/s, so  $Fr=0.55$ .**

In order to see the role of  $Re$  as well as  $Fr$  in the wave field, contours of  $\lambda/h$  was plotted with these numbers. This is shown in Fig. 9 and shows that the wavelength of IWs, with a first order approximation, increases with  $Re$  and  $Fr$  for their ranges in these experiments. Some non-linearity is observed for  $Fr>1$  and  $Re>4000$  that may correspond to the Comaz *et al.* (1993) observations for the second and higher order regimes (4 regimes were found in their work for experiments on a moving sphere in a stratified fluid), as the three-dimensionality of the flow increases. At about  $Fr\sim 1$  and  $Re\sim 4000$  there seems to be a decrease and then an increase of wavelength. This may be partly due

to inclusion of some cases with stepped density profiles as they could have finite number of modal structures.



**Fig. 9. The variations of normalized wavenumbers of IWs with Fr and Re.**

As these experiments are done for a rather streamlined body, the flow is often dominated by internal waves for the range of Fr used.

#### 4.2 Recirculating Experiments

Figure 9 shows two limited cases in which the body is fixed (at two different positions) while the fluid is recirculating (recirculating mode) with continuous density profile. Again internal waves are evident behind the body. Some dye was also used to reveal the flow pattern (small scale turbulence of dye is due to its positive buoyancy as it is lighter than fluid in the channel which is salt stratified).



**Fig. 10. Flow around the body in the recirculating mode with Fr=1.0. For the case on the right, some dye was injected over the body (with some small scale turbulence probably due to it being slightly buoyant) to show the flow more clearly.**

As in this type of experiments there were some background small scale internal waves that slightly blurred the field of view, only two experiments are carried out. Apart from blurring effects of the small scale random internal waves, coherent wave structure due to the flow of stratified fluid over the stationary body, with length scale of order of the body length are observed. The waves appear to be more distinct in the lee side of the body and especially below the mid depth, towards the bottom of the channel.

## 5. CONCLUSION

Results of some experiments of the movements of a 3D airfoil in a large stratified channel have been presented. The stratification is often rather linear with some limited cases with stepped profiles. Although the experiments are more qualitative, but the towing experiments indicate that as the Froude number increases the internal lee wave's activities increase and also the horizontal wavelength of the waves is almost a linear function of the Froude number, as the theory indicates, and the scaling for the wavelength is the thickness of the body. For larger Fr than about 1.4 the wake of the body becomes turbulent and seems to collapse after a period of  $Nt \sim 8$  ( $t=x/U$ ,  $x$  is the distance travelled by the body). The transformation of turbulence into small scale random internal waves, during this process is also evident.

Some experiments were also done with stepped profiles showing that moving body near the top of the stratified fluid emitted a collection of waves with wavelengths of order of the length of the body. While the case when the body is at the mid depth, a more regular wave pattern moved with the body, with the same speed of the body.

These experiments were part of the experience operating a large stratified water channel facility. Other experiments with much more mixing (grid generated turbulence in stratified flows) were also carried out to show that the stratification could be maintained for appropriate times. Hence, these moving body experiments with wave activity could be done with no problem, while keeping the stratification rather unchanged, especially for towing experiments.

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## REFERENCES

- Abdilghanie, A. M. and P. J. Diamessis (2013). The internal gravity waves emitted by a stably stratified turbulent wake. *J. Fluid Mech.* 720, 104-139.
- Bidokhti, A. A. and R. Britter (2002). A large stratified shear flow water channel facility, *Exp. in Fluids* 33, 281-287.
- Bonneton, P., J. M. Chomaz and E. J. Hopfinger (1993). Internal waves produced by the turbulent wake of a sphere moving horizontally in a stratified fluid. *J. Fluid Mech.* 254, 23-40.
- Chashechkin, Yu. D. and V. V. Mitkin (2004). A

- Visual study on flow pattern around the strip moving uniformly in a continuously stratified fluid. *Journal of Visualization* 7(2), 127-134.
- Chashechkin, Yu. D., E. V. Gumennik and E. Ya. Sysoeva (1995). Transformation of a density field by a three dimensional body moving in a continuously stratified fluid. *Journal of Applied Mechanics and Technical Physics* 36 (1), 19-29.
- Chomaz, J. M., P. Bonneton and E. J. Hopfinger (1993). The structure of the near wake of a sphere moving horizontally in a stratified fluid. *J. Fluid Mech.* 254, 1-21.
- Dotsenko, S. F. and V. M. Savoskin (1994). Generation of internal waves by moving non-stationary disturbances in a real stratified ocean. *Journal of Phys. Oceanogr.* 5(5), 335-347.
- Garret, C. and W. Munk (1979). Internal waves in the ocean. *Ann. Rev. Fluid Mech.* 11, 339-369.
- Ghassemi, H. and M. R. Kohansal (2013). Wave generated by the NACA4412 hydrofoil near free surface. *Journal of Applied Fluid Mechanics* 6(1), 1-6.
- Gill, A. (1982). *Atmosphere Ocean Dynamics*, Academic Press.
- Gilreath, H. E. and A. Brandt (1985). Experiments on the generation of internal waves in a stratified fluid. *AIAA Journal* 23(5), 693-700.
- Glushko, G. S., A. G. Gumilevskii and V. I. Polezhaev (1994). Evolution of the turbulent wakes of spherical bodies in stably stratified media. *Fluid Dynamics* 29(1), 10-16.
- Griffiths, R and A. A. Bidokhti (2008). Interleaving intrusions produced by internal waves: a laboratory experiment. *J. Fluid Mech.* 602, 219-239.
- Hopfinger, E. J., J. B. Flor, J. M. Chomaz and P. Bonneton (1991). Internal waves generated by a moving sphere and its wake in a stratified fluid. *Exp. In Fluids* 11(4) 255-261.
- Lighthill, J. (1978) *Waves in Fluids*. Cambridge University Press.
- Linden, P. F. (1979). Mixing in stratified fluids. *Geophysical & Astrophysical Fluid Dynamics* 13(1), 3-23.
- Mitkin, V. V. and Yu. D. Chashechkin (2000). Experimental Investigation of the Velocity Field near a Cylinder in a Continuously Stratified Fluid. *Fluid Dynamics* 35(5), 642-651.
- Nicolaou, D., J. E. R. Garman and T. N. Stevenson (1995). Internal waves from a body accelerating in a thermocline. *Applied Scientific Research.* 55, 171-186.
- Ohya, Y., T. Uchida and T. Nagai (2013). Near Wake of a Horizontal Circular Cylinder in Stably Stratified Flows. *Open Journal of Fluid Dynamics* 3(4), 311-320.
- Stevenson, T. N., D. Kanellopoulos and M. Constantinides (1986). The phase configuration of trapped internal waves from a body moving in a thermocline. *Applied Scientific Research* 43(2), 91-105.
- Thorpe, S. A. (2005). *The turbulent ocean*, Cambridge University Press.
- Voropayeva, O. F., N. P. Moshkin and G. G. Chernykh (2003). Internal Waves Generated by Turbulent Wakes in a Stably Stratified Medium. *Doklady Physics* 48(9), 517-521.
- Wong, A. B. D., R. W. Griffiths and G. O. Hughes (2001). Shear layers driven by turbulent plumes. *J. Fluid Mech.* 434, 209- 241.