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Spectral Collocation Solution of MHD Stagnation-Point Flow in Porous Media with Heat Transfer

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Abstract

An efficient Collocation method based on the shifted Legendre polynomials is implemented for solving the Magnetohydrodynamic Hiemenz flow with variable wall temperature in a porous medium. In the presented method the need for guessing and correcting the initial values during the solution procedure is eliminated and by using the given boundary conditions of the problem a stable solution can be derived. Numerical results show influence of the Prandtl number, permeability parameter, Hartmann number and suction/blowing parameter on the velocity and temperature profiles. The skin friction coefficient and the rate of heat transfer given by the Spectral Collocation method are in good agreement with those of the previous studies.

Keywords: Magnetohydrodynamic (MHD); Hiemenz flow; Stagnation flow; Porous media; Shifted Legendre polynomials; Collocation method.

Nomenclature

B_0	externally imposed magnetic field in the y-direction
C_1, C_2	Shifted Legendre polynomial coeffcient
D	Operational matrix of derivative
f_w	suction or blowing parameter
K	permeability of the porous medium
L_m	legendre polynomial
M	Hartmann number
P	pressure
P_m	shifted Legendre polynomial
Pr	Prandtl number
T	temperature
U_∞	free-stream velocity
u, v	velocity components along x and y axes
x, y	cartesian coordinates
ψ	stream function
v_w	uniform surface suction/blowing
Ω	permeability parameter
ρ	density
σ	electrical conductivity
ν	kinematic viscosity
λ	exponent of wall temperature
Ψ	shifted Legendre polynomial vector

1. Introduction

Stagnation flows have many important engineering applications such as flows over the tips of rockets, aircrafts, submarines and oil ships. The problem of stagnation flow was studied by Hiemenz (1911) who demonstrated that the Navier-Stokes equations for the forced convection problem can be reduced to an ordinary differential equation of third order by means of a similarity transformation. Two dimensional flow near a stagnation point was investigated by Beard and Walters (1964). Schlichting and Bußmann (1943) investigate the effect of suction on the Hiemenz flow problem and gave the numerical results. Ariel (1994) presents approximate solution to the problem of uniform suction. Takhar and Ram (1994) have studied MHD forced and free convection flow of water at 4^oC through a porous medium in the presence of a uniform transverse magnetic field for the local similarity equations. Kechil and Hashim (2009) obtaine an approximate analytical solution for the MHD stagnation flow against a flat plate in porous media. The effect of uniform suction/blowing on heat transfer of MHD Hiemenz flow through porous media was solved numerically using the implicit finite-difference by Yih (1998). An analytical approximate solution based on DTM-Padé is presented for MHD stagnation-point flow in porous media with heat transfer by Rashidi and Erfani (2011). Erfani *et al.* (2010) have studied the steady stagnation flow toward an off-centered rotating disc by applying the combination of the differential transform method (DTM) and the Padé approximation. A Series solutions for unsteady laminar MHD flow near forward stagnation point of an impulsively rotating and translating sphere in presence of buoyancy forces is derived by Dinarvand *et al.* (2010). In Rashidi *et al.* (2011) a combination of differential transform method (DTM) and Padé approximant, called DTM-Padé, is employed to investigate entropy generation in magnetohydrodynamic (MHD) stagnation-point flow with heat transfer in a porous medium. Yildirim and Sezer (2012) have solved the steady two-dimensional laminar forced magnetohydrodynamic Hiemenz flow against a flat plate with variable wall temperature in a porous medium by using the homotopy perturbation method (HPM). Mabood and Khan (2014) have found an accurate analytic solution for MHD stagnation point flow in porous medium for different values of Prandtl number and suction/injection parameter. The effects of thermal radiation and viscous dissipation on a stagnation point flow and heat transfer over a flat stretching/shrinking surface in nanofluids was analyzed by Pal (2009). Mostafa and Waheed have investigated the effects of magnetic field and thermal radiation on a micropolar fluid flow near a stagnation point towards a moving surface (Mahmoud and Waheed (2012)). Mostafa et al. have studied stagnation-point flow of a nanofluid towards a stretching sheet Mustafa *et al.* (2011). Numerical simulation of MHD stagnation point flow and heat transfer of a micropolar fluid towards a heated shrinking sheet has studied by Ashraf and Bashir (2012). Hydromagnetic stagnation point flow of a viscous fluid over a stretching or shrinking sheet was investigated by Van Gorder *et al.* (2012). An analysis is carried out to study the steady two-dimensional stagnation-point flow of a micropolar fluid over a shrinking sheet in its own plane by Ishak *et al.* (2010).

Since the time of Fourier, orthogonal functions and polynomials have been used in the analytic study of differential equations and their applications for numerical solution of ordinary differential equations refer, at least, to the time of Lanczos (1938). It is well known that the eigenfunctions of certain singular Sturm-Liouville problems such as Legendre or Chebyshev orthogonal polynomials allow the approximation of functions $C^{\infty}[a,b]$ where truncation error approaches zero faster than any negative power of the number of basic functions used in the approximation, as that number (order of truncation N) tends to infinity. This phenomenon is usually referred to as spectral accuracy. The accuracy of derivatives obtained by direct, term-by-term differentiation of such truncated expansion naturally deteriorates Canuto *et al.* (1988), but for low-order derivatives and sufficiently high-order truncations this deterioration is negligible, For a given ordinary differential equation which is defined on the interval $[a,b]$, if solution function and coefficient functions are analytic on $[a,b]$, i.e. $C^{\infty}[a,b]$, spectral methods are very efficient and suitable (Mohammadi *et al.* (2011), Canuto *et al.* (1988), Babolian and Hosseini (2002)). The collocation approach appears to have been first used by Slater and by Kantorovic (1934) in specific applications. This approach is especially attractive whenever it applies to variable-coefficient and even nonlinear problems (Kamrani and Hosseini (2012)). Some major advantages of the collocation methods are as follow:

(i) Since no integration is required, the construction of the final system of equations is very efficient

(ii) The functions must be evaluated only at the collocation nodes in contrast to other methods

(iii) Computational cost of calculating nonlinear terms is reasonably low with good numerical accuracy.

The spectral collocation method has been applied for numerical solution of different kind of differential and integral equations. For example, it has been used for deriving approximate solution of Burgers-type equation (Khater *et al.* (2008)), stochastic Burgers equation (Kamrani and Hosseini (2012)), Navier-Stokes equations (Malik *et al.* (1985)), two-point boundary value problem in modelling viscoelastic flows (Karageorghis *et al.* (1988)), Poisson equation in polar and cylindrical coordinates (Chen et al. (2000)), Volterra integral equations (Chen and Tang (2010)), (Nemati *et al.* (2013)) compressible flow, two-dimensional and axisymmetric boundary layer problems (Pruett and

Streett (1991)), hypersonic boundary layer stability (Malik *et al.* (1985)), Helmholtz and variable coefficient equations in a disk (Bialecki and Karageorghis (2008)) and Burgers-Huxley equation (Darvishi *et al.* (2008)).

The Legendre polynomials Canuto *et al.* (1988) are well known family of orthogonal polynomials on the interval $[0,1]$ of the real line. These polynomials present very good properties in the approximation of functions. Therefore, Legendre polynomials appear frequently in several fields of Mathematics, Physics and Engineering. Spectral methods based on Legendre polynomials as basis functions for solving numerically differential equations have been used by many authors, (see for example Shidfar and Pourgholi (2006), Khalil and Khan (2014), Chang and Wang (1984), Saadatmandi and Dehghan (2010). Here, we have considered the steady two-dimensional laminar forced convection in MHD Hiemenz flow of an electrically conducting viscous fluid against a flat plate through porous media with variable wall temperature and uniform surface mass flux. A transverse magnetic field is applied and the fluid is assumed to have constant properties. The magnetic Reynolds number is assumed small and the induced magnetic field, the Hall effect and the viscous dissipation terms are neglected. The main goal of this paper is to find the approximate analytic solutions by using the Legendre polynomials and Spectral Collocation method.

The structure of the paper is as follows. In Section 2, the flow analysis and mathematical formulation are presented. In Sections 3 we introduce basic definition of the Legendre polynomials. In Section 4, we extend the application of the Legendre Spectral Collocation method to find the approximate solutions of nonlinear system derived by similarity solution of the MHD Hiemenz flow against a flat plate with variable wall temperature in a porous medium. Section 5 contains the results and discussion. The conclusions are summarized in Section 6.

2. PROBLEM STATEMENT AND MATHEMATICAL FORMULATION

Let us consider the effect of uniform suction or blowing rate on the steady two-dimensional laminar forced convection in MHD Hiemenz flow through porous media. The fluid is an electrically conducting incompressible viscous fluid. Following Raptis and Takhar model for the porous medium and introducing the boundary-layer approximation, the governing equations for the continuity, momentum and energy can be written as follows Yih (1998), Rashidi and Erfani (2011):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\nu}{K}u-\frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha^2\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where x and y are the coordinates along and normal to the flat plate, u and v are the components of the velocity in the x and y directions, respectively, P is the pressure, ρ is the density, ν is the kinematic viscosity, K is the permeability of the porous medium, σ is the electrical conductivity and B_0 is the externally imposed magnetic field in the y-direction. The magnetic Reynolds number is assumed to be small enough that the thickness of the magnetic boundary-layer is very large and the induced magnetic field effect is negligible compared with the applied magnetic field. The Hall effect and the viscous dissipation terms are also neglected. Also T is the temperature of the fluid and the porous medium which are in local thermal equilibrium (LTE) and α is the equivalent thermal diffusivity. The appropriate boundary conditions are introduced as

$$v=v_w,\ \ u=0,\ \ T=T_w=T_\infty+Ax^\lambda \quad as\ \ y=0,$$

$$u=U_\infty=Cx,\ \ T=T_\infty,\ \ as\ \ y\longrightarrow\infty, \tag{4}$$

where A is a constant, v_w is the uniform surface suction/blowing, λ is the exponent of wall temperature chosen 0 or 1 here, $U_\infty=Cx$ is the free-stream velocity and C is a positive number. In the free-stream Eq. (2) becomes

$$U_\infty\frac{dU_\infty}{dx}=-\frac{1}{\rho}\frac{\partial P}{\partial x}-\frac{\nu}{K}U_\infty-\frac{\sigma B_0^2}{\rho}U_\infty. \tag{5}$$

Eliminating $\frac{\partial P}{\partial x}$ between Eqs. (2) and (5), we have

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+U_\infty\frac{dU_\infty}{dx}-\frac{\nu}{K}\left(u-U_\infty\right)$$

$$-\frac{\sigma B_0^2}{\rho}\left(u-U_\infty\right). \tag{6}$$

The mathematical analysis of the problem is simplified by introducing the following similarity transforms Rashidi and Erfani (2011)

$$\eta=\sqrt{\frac{C}{\alpha}}y,\ \ f(\eta)=\frac{\psi}{\sqrt{C\alpha}}x,\ \ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{7}$$

where ψ is the stream function and is defined as

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}. \tag{8}$$

Substituting Eq. (7) into Eqs. (3) and (6), we obtain

$$Prf'''+ff''+(1-f'^2)+\Omega(1-f')$$

$$+M^2(1-f')=0, \tag{9}$$

$$f''+f\theta'-\lambda\theta f'=0, \tag{10}$$

subject to the boundary conditions

$$f(0)=f_w,\quad f'(0)=0,\quad f'(\infty)=1, \tag{11}$$

$$\theta(0)=1,\quad \theta(\infty)=0, \tag{12}$$

where $Pr=\frac{\nu}{\alpha}$ is the Prandtl number, $\Omega=\frac{\nu}{KC}$ is the permeability parameter, $M=\sqrt{\frac{\sigma B_0^2}{C\rho}}$ is the Hartmann number, $f_w=-\frac{v_w}{\sqrt{C\alpha}}$ is the suction or blowing parameter, $f_w<0$ represents blowing, $f_w>0$ represents suction and $f_w=0$ corresponds to an impermeable surface.

3. Shifted Legendre polynomials and their properties

The well known Legendre polynomials are defined on the interval and can be determined with the aid of the following recurrence formulae Canuto *et al.* (1988)

$$L_{m+1}(t)=\frac{2m+1}{m+1}\,t\,L_m(t)-\frac{m}{m+1}L_{m-1}(t),$$

$$m=1,\,2,\,3,\,..., \tag{13}$$

where $L_0(t)=1,\ L_1(t)=t$. In order to use Legendre polynomials on the interval $[0,1]$ we define the so-called shifted Legendre polynomials by introducing the change of variable $t=2x-1$. Let the shifted Legendre polynomials $L_m(t)$ are denoted by $P_m(x)$. Then $P_m(x)$ can be obtained as follows

$$P_{m+1}(x)=\frac{2m+1}{m+1}(2x-1)\,P_m(x)-\frac{m}{m+1}P_{m-1}(x),$$

$$m=1,\,2,\,3,\,..., \tag{14}$$

where $P_0(x)=1$ and $P_1(x)=2x-1$. The orthogonality condition for these polynomials is

$$\int_0^1 P_m(x)P_n(x)\,dx=\begin{cases}\frac{1}{2m+1} & for\ m=n,\\ 0 & for\ m\neq n.\end{cases} \tag{15}$$

A function $f(t)$ defined over $[0,1]$ may be expanded in the terms of shifted Legendre polynomials as

$$f(t)=\sum_{k=0}^{\infty}c_kP_k(t), \tag{16}$$

where $c_k=(f(t),P_k(t))$, in which (.,.) denotes the inner product. If the infinite series in Eq. (16) is truncated, then it can be written as

$$f(t)=\sum_{k=0}^{N}c_kP_k(t)=C^T\Phi(t), \tag{17}$$

where C and $\Phi(t)$ are $(N+1)$ vectors given by

$$C^T=[c_1,c_2,...,c_N], \tag{18}$$

$$\Phi(t)=[P_1(t),P_2(t),...,P_N(t)]. \tag{19}$$

In the next theorem we derived a relation between shifted Legendre polynomials and their derivatives that is very important for deriving the operational matrix of derivative for shifted Legendre polynomials.

Theorem 1. *Let $P_m(x)$ be the shifted Legendre polynomials into $[0,1]$ and $P_m'(x)$ be derivative of $P_m(x)$ with respect to x, then we have*

$$P_m'(x)=2\sum_{\substack{k=0\\ k+m\ odd}}^{m-1}(2k+1)P_k(x). \tag{20}$$

Proof. Consider the Legendre expansion of a function $u(x)$ as

$$u(x)=\sum_{k=0}^{\infty}\hat{u}_kL_k(x), \tag{21}$$

then $u'(x)$ can be represented as Canuto *et al.* (1988)

$$u'(x)=\sum_{k=0}^{\infty}\hat{u}_k^{(1)}L_k(x), \tag{22}$$

where

$$\hat{u}_k^{(1)}=(2k+1)\sum_{\substack{p=k+1\\ p+k\,odd}}^{\infty}\hat{u}_p,\quad k\geq 0. \tag{23}$$

Now, by taking $u(x)=L_m(x)$ in Eq. (2) we have $\hat{u}_m = 1$ and $\hat{u}_i = 0$ for $i \neq m$, consequently

$$\hat{u}_k^{(1)} = \begin{cases} 2k+1 & m+k \ \textit{is odd} \ k \leq m-1 \\ 0 & o.w, \end{cases} \tag{24}$$

as a result Eq. (23) becomes

$$L'_m(x) = \sum_{\substack{k=0 \\ m+k \ odd}}^{m-1} (2k+1) L_k(x), \tag{25}$$

by substituting $x = 2t-1$ in Eq. (26) we have

$$P'_m(t) = 2 \sum_{\substack{k=0 \\ m+k \ odd}}^{m-1} (2k+1) P_k(t), \tag{26}$$

and this proves the desired result. □

Theorem 2. *Let $\Psi(t)$ be the Legendre polynomial vector defined as*

$$\Psi(t) = [P_0(t), P_1(t), ..., P_N(t)], \tag{27}$$

the derivative of this vector can be expressed by

$$\frac{d\Psi(t)}{dt} = D\Psi(t), \tag{28}$$

which D is $(N+1)\times(N+1)$ matrix and its (i,j)-th element is defined as below

$$D_{i,j} = \begin{cases} 2(2j-1) & j = 1, ..., i-1 \ \textit{and} \ (i+j) \ \textit{odd}, \\ 0 & o.w. \end{cases}$$

Proof. By using shifted Legendre polynomial into $[0,1]$ the i-th element of vector $\Psi(t)$ in Eq. (27) can be written as

$$\Psi_i(t) = P_{i-1}(t), \tag{29}$$

by differentiation with respect to t in (29) we have

$$\frac{d\Psi_i(t)}{dt} = P'_{i-1}(t), \tag{30}$$

now by substituting Eq. (26) into (30) we get

$$\frac{d\Psi_i(t)}{dt} = 2 \sum_{\substack{j=0 \\ j+i \ odd}}^{i-2} (2j+1) P_j(x), \tag{31}$$

this equation can be expanded in shifted Legendre polynomials as

$$\frac{d\Psi_i(t)}{dt} = 2 \sum_{\substack{j=1 \\ j+m \ odd}}^{i-1} (2j-1) \Psi_j(x), \tag{32}$$

from (32) we conclude that

$$\frac{d\Psi(t)}{dt} = D\Psi(t), \tag{33}$$

and this leads to desired results. □

Corollary 1. *By using Eq. (28) the operational matrix for n-th derivative can be derived as*

$$\frac{d^n\Psi(x)}{dx^n} = D^n\Psi(x), \tag{34}$$

where D^n is the n-th power of matrix.

4. METHOD OF SOLUTION

Consider the coupled nonlinear differential equations (9) and (10) subject to boundary conditions (11) and (12) . By using change of variable

$$t = \frac{\eta}{\eta_\infty}, \quad g(t) = f(t\eta_\infty), \quad \vartheta(t) = \theta(t\eta_\infty), \tag{35}$$

we have the following nonlinear differential systems in the interval $[0,1]$,

$$Pr\frac{d^3g}{dt^3} + \eta_\infty g\frac{d^2g}{dt^2} + \left(\eta_\infty^3 - \eta_\infty\left(\frac{dg}{dt}\right)^2\right)$$
$$+\Omega\left(\eta_\infty^3 - \eta_\infty^2\frac{dg}{dt}\right) + M^2\left(\eta_\infty^3 - \eta_\infty^2\frac{dg}{dt}\right) = 0, \tag{36}$$

$$\frac{d^2\vartheta}{dt^2} + \eta_\infty g\frac{d\vartheta}{dt} - \lambda\eta_\infty\vartheta\frac{dg}{dt} = 0, \tag{37}$$

subject to boundary conditions

$$g(0) = f_w, \quad g'(0) = 0, \quad g'(1) = \eta_\infty, \tag{38}$$

$$\theta(0) = 1, \quad \theta(1) = 0. \tag{39}$$

Now we expand the unknown function $f(t)$ and $\vartheta(t)$ by the shifted Legendre polynomial into interval $[0,1]$ as

$$g(t) = C_1^T\Phi(t), \quad \vartheta(t) = C_2^T\Phi(t), \tag{40}$$

where C_1 and C_2 are the unknown shifted Legendre polynomial coefficient vectors defined in (18). By using the operational matrix derived in (28) we get

$$\frac{dg}{dt} = C_1^T D\Phi(t), \quad \frac{d^2g}{dt^2} = C_1^T D^2\Phi(t),$$

$$\frac{d^3g}{dt^3} = C_1^T D^3\Phi(t), \tag{41}$$

$$\frac{d\vartheta}{dt} = C_2^T D\Phi(t), \quad \frac{d^2\vartheta}{dt^2} = C_2^T D^2\Phi(t), \tag{42}$$

substituting Eqs. (41) and (42) into (36) and (37), we obtain

$$PrC_1^T D^3\Phi(t) + \eta_\infty \left(C_1^T\Phi(t)\right)\left(C_1^T D\Phi(t)\right) +$$

$$(\eta_\infty^3 - \eta_\infty \left(C_1^T D\Phi(t)\right)^2) + \Omega\left(\eta_\infty^3 - \eta_\infty^2 C_1^T D\Phi(t)\right)$$

$$+M^2\left(\eta_\infty^3 - \eta_\infty^2 C_1^T D\Phi(t)\right) = 0, \tag{43}$$

$$C_2^T D^2\Phi(t) + \eta_\infty \left(C_1^T\Phi(t)\right)\left(C_2^T D\Phi(t)\right)$$

$$-\lambda\eta_\infty \left(C_1^T D\Phi(t)\right)\left(C_2^T\Phi(t)\right) = 0. \tag{44}$$

Moreover, boundary conditions (38) and (39) result

$$C_1^T\Phi(0) = f_w,\ C_1^T D\Phi(0) = 1,\ C_1^T D\Phi(1) = \eta_\infty, \tag{45}$$

$$C_2^T\Phi(0) = 1, \quad C_2^T\Phi(1) = 0, \tag{46}$$

to find the approximate solution of the nonlinear system (36) and (40), we use the typical Collocation method and collocate Eq. (4.) at $(M-2)$ different points and Eq. (44) at $(M-1)$ different points in the interval $[0,1]$. For choosing suitable Collocation points, we use the first roots of shifted Legendre $P_{M+1}(t)$. These equations together 5 equations in (45) and (46) generate 2(M + 1) nonlinear equations. The well-known Newton-Raphson have been used for approximate solution of derived nonlinear systems. After finding the solution of this nonlinear systems we obtain unknown vectors C_1 and C_2. By substituting these vectors in Eq. (40) the solution functions $g(t)$ and $\vartheta(t)$ can be derived. Using change of variable in (35), the solution of the nonlinear system (9) and (10) can be approximated.

5. Numerical Results

In this section, the nonlinear system (9) and (10) subject to the boundary conditions (11) and (12) have been solved analytically by using the Legendre collocation method (LCM) presented in Section 4. A suitable domain truncation value for η_∞ is determined experimentally. Usually near suitable values η_∞ the results do not change significantly and the accuracy of the results is insensitive to the values of η_∞. All numerical results are derived by using Maple 17 with 20 digits precision.

The non-dimensional velocity profiles $f'(\eta)$ and temperature profiles $\theta(\eta)$ for various parameters M and $Pr = 1, \lambda = \Omega = 0$ are shown in Figs. 1-6. As this figures show for impermeable surface and suction/blowing cases when M increases, the velocity and temperature profiles increases. Figs. 7 and 8 show that an increment in the Prandtl number Pr decreases the velocity profile but increases the temperature profile. Figs. 9 and 10 illustrates the effect of the porosity of the medium Ω on the velocity and temperature. From this figure it can be concluded that the increment in the permeability parameter results in drastic changes in veloc-

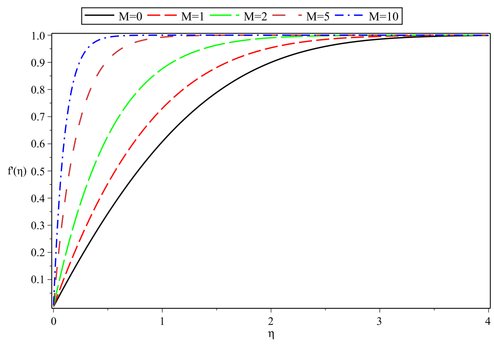

Fig. 1. The non-dimensional velocity profiles $f'(\eta)$ for $f_w = -1, Pr = 1, \lambda = \Omega = 0$ and various parameters M.

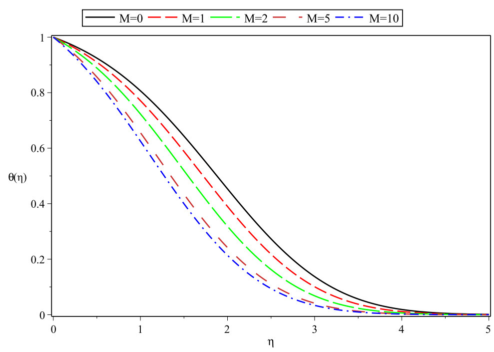

Fig. 2. The temperature profiles $\theta(\eta)$ for $f_w = -1, Pr = 1, \lambda = \Omega = 0$ and various parameters M.

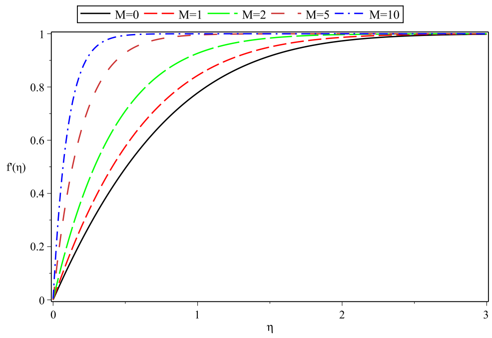

Fig. 3. Non-dimensional velocity profiles $f'(\eta)$ for $f_w = 0$, $Pr = 1, \lambda = \Omega = 0$ and various parameters M.

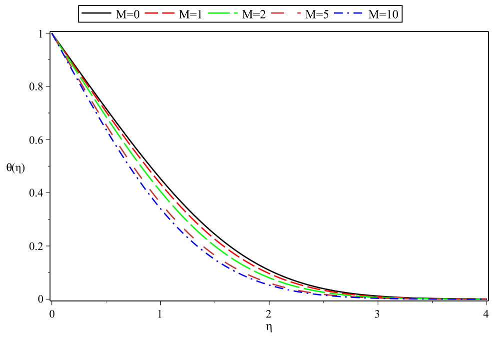

Fig. 4. Temperature profiles $\theta(\eta)$ for $f_w = 0$, $Pr = 1, \lambda = \Omega = 0$ and various parameters M.

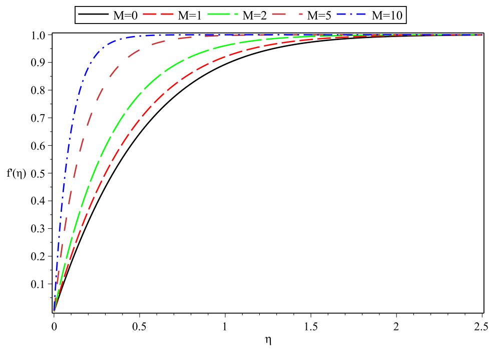

Fig. 5. Non-dimensional velocity profiles $f'(\eta)$ for $f_w = 1$, $Pr = 1, \lambda = \Omega = 0$ and various parameters M.

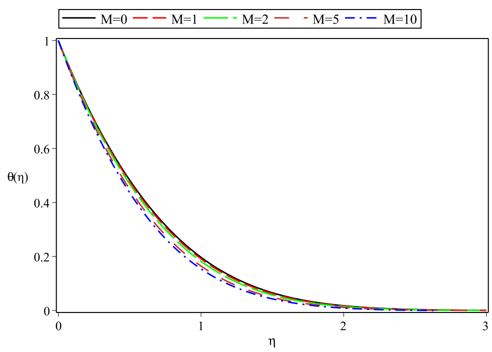

Fig. 6. Temperature profiles $\theta(\eta)$ for $f_w = 1$, $Pr = 1, \lambda = \Omega = 0$ and various parameters M.

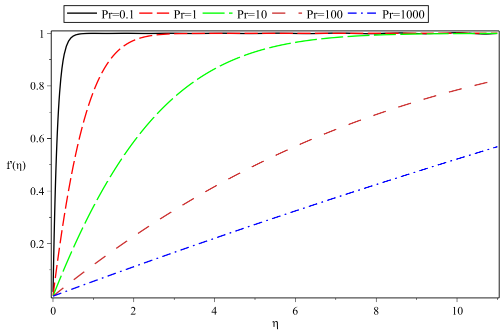

Fig. 7. Non-dimensional velocity profiles $f'(\eta)$ for $M = f_w = \lambda = \Omega = 0$ and various parameters Pr.

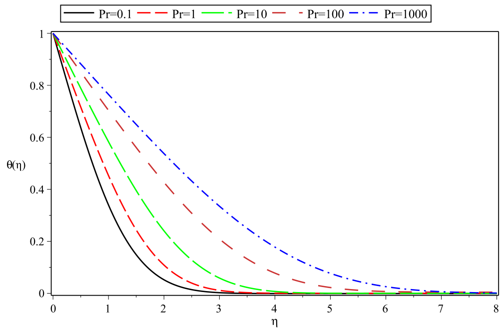

Fig. 8. Temperature profiles $\theta(\eta)$ for $M = f_w = \lambda = \Omega = 0$ and various parameters Pr.

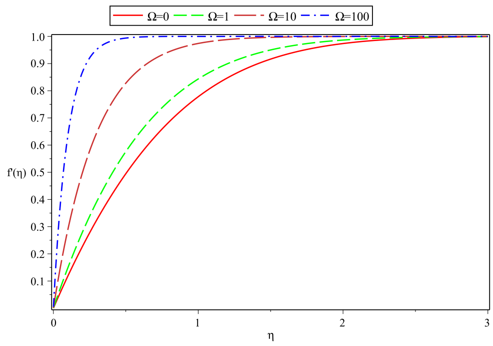

Fig. 9. Non-dimensional velocity profiles $f'(\eta)$ for for $Pr = 1, M = f_w = \lambda = 0$ and various parameters Ω.

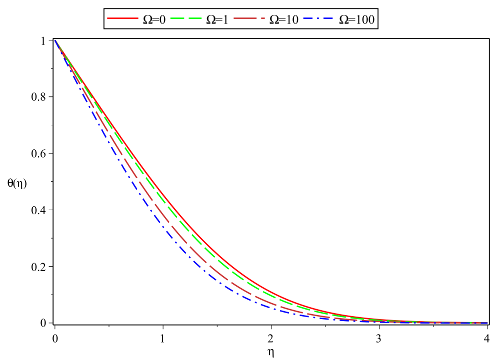

Fig. 10. Temperature profiles $\theta(\eta)$ for for $Pr = 1, M = f_w = \lambda = 0$ and various parameters Ω.

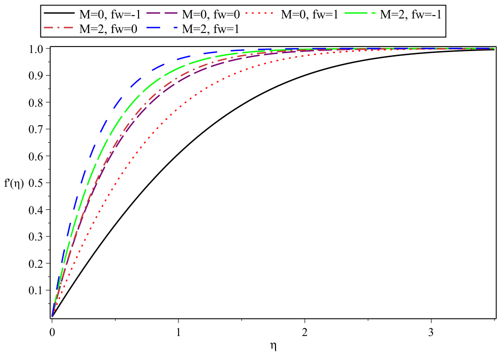

Fig. 11. Non-dimensional velocity profiles $f'(\eta)$ for $Pr = 1, \lambda = \Omega = 0, M = 0, 2$ and $f_w = -1, 0, 1$.

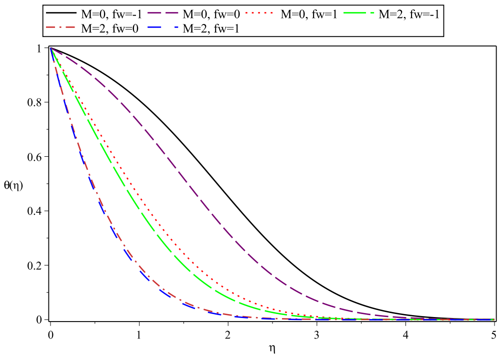

Fig. 12. Temperature profiles $\theta(\eta)$ for $Pr = 1, \lambda = \Omega = 0, M = 0, 2$ and $f_w = -1, 0, 1$.

ity profiles, but little variation in the temperature profiles. Figs. 11 and 12 displays the effect of suction/injection parameter f_w and magnetic parameter M. As M or f_w increases, the velocity profile increases and the thermal boundary layer thickness decreases. The magnetic field has a pronounced effect on the temperature distribution for injection, while its influence can be neglected in the case of suction. The influence of the Prandtl number Pr and f_w on the velocity and temperature profiles are plotted in Figs. 13 and 14. As f_w increases from injection to suction, the velocity profiles $f'(\eta)$ increases and the temperature profile $\theta(\eta)$ decreases. Fig. 15 displays the effect of suction/injection parameter and the wall temperature exponent λ. As the flow problem is uncoupled from the thermal problem, changes in the values of λ will not affect the fluid velocity. It can be seen that the temperature decreases and the thermal boundary layer becomes thin as the wall temperature exponent increases. To see the accuracy of the solutions, the residual errors of the approximate solution are plotted in Figs. 16. and 17.

In order to verify the results of this study, the results have been compared with previously published numerical ones of Refs. Kechil and

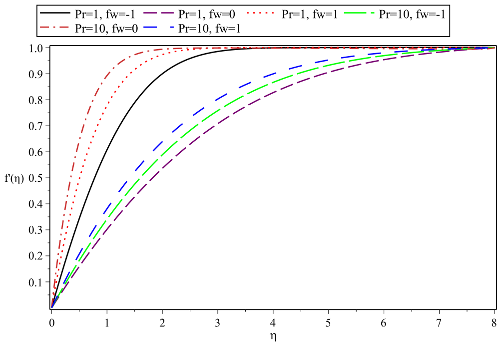

Fig. 13. Non-dimensional velocity profiles $f'(\eta)$ for $\lambda = \Omega = M = 0, Pr = 1, 10$ and $f_w = -1, 0, 1$.

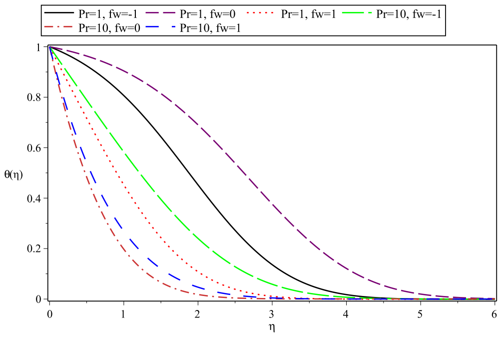

Fig. 14. Temperature profiles $\theta(\eta)$ for $\lambda = \Omega = M = 0, Pr = 1, 10$ and $f_w = -1, 0, 1$.

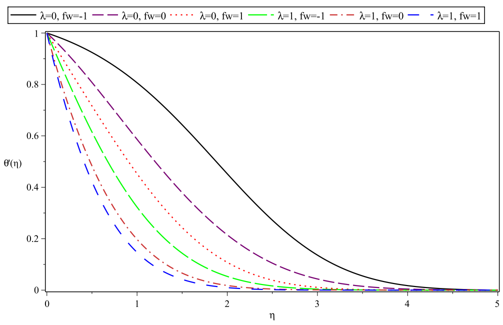

Fig. 15. The temperature profiles $\theta(\eta)$ for $\Omega = M = 0, Pr = 1, \lambda = 0, 1$ and $f_w = -1, 0, 1$.

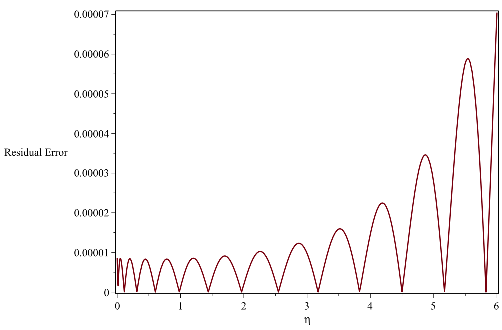

Fig. 16. Residual error for Eq. (9) with $\lambda = \Omega = M = 0, Pr = 1$ and $f_w = 1$.

Table 1 Numerical values of $f''(0)$ for $\Omega = 0, Pr = f_w = 1$ and various values of M

M	LCM	(Kechil and Hashim 2009)	(Yih 1998)
0	1.889339	1.884684	1.889314
1	2.202943	2.201433	2.202940
2	2.920113	2.920114	2.920111
5	5.676830	5.676830	5.676830
10	10.588367	10.588367	10.588367

Table 2 Numerical values of $-\theta'(0)$ for $Pr = 1, \lambda = \Omega = 0, M = 0,1,2$ and $f_w = -1,0,1$

4*M	$f_w = -1$			$f_w = 0$			$f_w = 1$		
(lr)2-4 (lr)5-7 (lr)8-10	0	1	2	0	1	2	0	1	2
(lr)1-10 LCM	0.116756	0.140003	0.173123	0.570455	0.595341	0.634129	1.323692	1.338059	1.364461
(Yih 1998)	0.116752	0.140002	0.173124	0.570465	0.595346	0.634132	1.323691	1.338060	1.364466
(Kechil and Hashim 2009)	0.11677	0.14000	0.17312	0.57035	0.59539	0.63418	1.32368	1.33804	1.36446

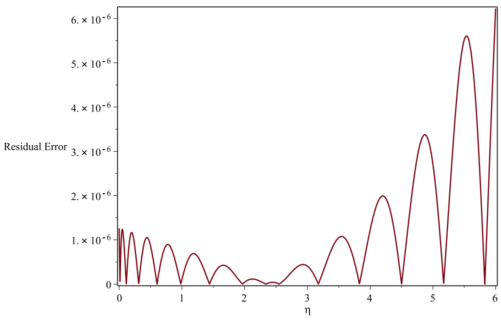

Fig. 17. Residual error for Eq. (10) with $\lambda = \Omega = M = 0, Pr = 1$ and $f_w = 1$.

Hashim (2009) and Yih (1998). Table1 shows a comparison of the numerical results of $f''(0)$ for the case of $\Omega = 0, Pr = f_w = 1$ and $M = 0,1,2,5,10$ with those of Refs. Kechil and Hashim (2009) and Yih (1998). Moreover, the numerical results of $\theta'(0)$ compared with those of Refs. Kechil and Hashim (2009) and Yih (1998) in Table 2. The obtained results in Tables 1 and 2 demonstrate the reliability and efficiency of the proposed LCM.

6. Conclusion

The shifted Legendre polynomial and its operational matrix of derivatives are employed obtain an approximate analytical solution for the MHD stagnation flow against a flat plate in porous media. In the proposed method, the requirement of to guessing the initial condition $f''(0)$ and $\theta'(0)$ in order to start the solution which is required in the conventional shooting methods is eliminated. The effect of the suction/injection parameter, the Hartmann number and the permeability parameter on velocity and temperature profiles were studied. The numerical results reveal that the velocity and the temperature profiles increse when the suction/injection parameter, the Hartmann number and the permeability parameter increses. Moreover, an increment in the Prandtl number, decreases the velocity profiles and increases the temperature profiles. By increasing the wall temperature exponent, the temperature profiles increases. The results of the present study, in a special case, were compared with the published numerical ones to verify them and excellent agreement was obtained.

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