



Hall Effects on Flow past an Exponentially Accelerated Infinite Isothermal Vertical Plate with Mass Diffusion

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ABSTRACT

The effects of hall current and rotation on unsteady hydro magnetic free convection flow past an exponentially accelerated infinite vertical plate with uniform temperature and variable mass diffusion has been discussed. The flow is induced by a general time-dependent movement of the vertical plate, and the cases of ramped temperature and isothermal plates are studied. The governing partial differential equations have been derived for the velocity, temperature, concentration profiles by Laplace transform technique. The solutions that have been obtained are expressed in simple forms in terms of elementary function and complementary error function. Expressions for velocity, temperature and concentration fields are obtained. The obtained results are discussed with the effect of various parameters like Rotation parameter, Hall parameter, Hartmann number, Schmidt number, radiation parameter thermal Grashof number and mass Grashof number. The numerical values of primary and secondary velocities are displayed graphically. The temperature and concentration distributions are discussed numerically and presented through graphs.

Keyword: Hall effects; Mass diffusion; Isothermal; Accelerated; Exponential; Vertical plate.

1. INTRODUCTION

The Hall Effect is due to the nature of the current in a conductor. Current consists of the movement of many small charge carriers, typically electrons, holes, ions or all three. The applications of the effect of Hall current on the fluid flow with variable concentration have been seen in MHD power generators. The Hall Effect is due merely to the sideways magnetic force on the drifting free charges. Hall-magneto hydrodynamics (HMHD) takes into account this electric field description of magneto hydrodynamics. The most important difference is that in the absence of field line breaking, the magnetic field is tied to the electrons and not to the bulk fluid.

Magneto hydrodynamics is the study of the dynamics of **conducting fluids**. Examples of such fluids include plasmas, liquid metals, and salt water or electrolytes MHD has applications in three different subject areas, such as astrophysical, geophysical and engineering problems. In addition, MHD free convection flows have significant applications in the field of stellar and planetary magneto spheres, aeronautical plasma flows and electronics. Al-Odat, Damseh and Al-Azab (2006) investigated thermal boundary layer on an exponentially stretching continuous surface in the presence of magnetic field effect. Ahmed, *et al.*

considered radiation and mass transfer effects on MHD free convection flow past a vertical plate with variable temperature and concentration. Bala Anki Reddy *et al.* (2012), discussed radiation effects on MHD flow past an exponentially accelerated isothermal vertical plate with uniform mass diffusion in the presence of heat source. Chandrakala and Bhaskar (2011) investigated effects of heat transfer on flow past an exponentially accelerate vertical plate with uniform heat flux. Chandrakala *et al.*(2013), discussed radiation effects on flow past an exponentially accelerated vertical plate with uniform flux. Jonah Phillip *et al.*, discussed MHD rotating heat and mass transfer free convective flow past an exponentially accelerated isothermal plate with fluctuating mass diffusion. Kumar *et al.*(2011), investigated radiation effects on MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature in the presence of heat generation. Makinde *et al.*(2003), investigated unsteady free convection flow with suction on an accelerating porous plate. Muthucumaraswamy *et al.*(2010), discussed thermal radiation on linearly accelerated vertical plate with variable temperature and uniform mass flux. Muthucumaraswamy, Thamizhsudar *et al.* (2014), studied hall effects on MHD flow past an exponentially accelerated vertical plate in the presence of rotation. Okedoye and Lamidi (2009)

discussed analytical solution of mass transfer effects unsteady flow past an accelerated vertical porous plate with suction. Partha *et al.* (2006), investigated sores and dufour effects in a non-darcy porous medium. Pattnaik, Dash. Singh (2012), studied radiation and mass transfer effects on MHD free convection flow through porous medium past an exponentially accelerated vertical plate with variable temperature. Rajesh and Vijaya Kumar Varma (2009) discussed radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature.

Rajput and Sahu (2011), studied effects of rotation and magnetic field on the flow past an exponentially vertical plate with constant temperature. Sahin Ahmed studied modelling of Newtonian Hartmann flow through darcian porous medium adjacent to an accelerated vertical plate in a rotating system. Sami Ulhaq *et al.* (2013), discussed radiation and magneto hydrodynamics effects on unsteady free convection flow in a porous medium. Sanatan Das *et al.*(2013), investigated hall effects on unsteady free convection in a heated vertical channel in presence of heat generation. Seddeck (2001), studied thermal radiation and buoyancy effects on MHD free convective heat generating flow over an accelerating permeable surface with temperature dependent viscosity. Vijaya Kumar *et al.* (2011), discussed radiation effects on MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature in the presence of heat generation. Thamizhsudar *et al.* (2014), discussed Hall effects on magneto hydrodynamic flow past an exponentially accelerated vertical plate in a rotating fluid with mass transfer effects. The basic equations governing the flow field are partial differential equations and these have been reduced to a set of ordinary differential equations by applying suitable similarity transformations, and these have been solved numerically by using Laplace transform technique. The effects of Schmidt number, Grashof number, and radiation, convective heat transfer parameter on fluid velocity, temperature and concentration have been shown graphically.

2. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Here consider the unsteady hydro magnetic flow of a viscous in compressible fluid past an exponentially accelerated infinite vertical plate with variable temperature and uniform mass diffusion, in the presence of thermal radiation. The x' -axis is taken along the plate in the vertically upward direction and also the z' -axis is taken normal to the plate. A uniform transverse magnetic field is applied in a direction which is parallel to B_0 is applied in a direction which is parallel to y' - axis. Initially, the plate and the fluid are at the same temperature T'_∞ in the stationary condition with concentration level C'_∞ at all the points. At time

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2},$$

the plate and fluid are at the same temperature T'_∞ and concentration C'_∞ . At time $t' > 0$, the plate is exponentially accelerated with a prescribed velocity $u = u_0 \exp(at')$, in its own plane and at the same time the temperature of the fluid near the plate is raised linearly with time t and concentration level near the plate is also increased linearly with time. u_0 is the velocity of the plate.

The geometry of the problem is presented in Fig. 1. Since plate is of infinite extent in x' and y' directions and is electrically non-conducting, all physical quantities except pressure, depend on z' and t' only. Then under the usual Boussinesq's approximation the unsteady flow equations are momentum equation, energy equation, and diffusion equation respectively.

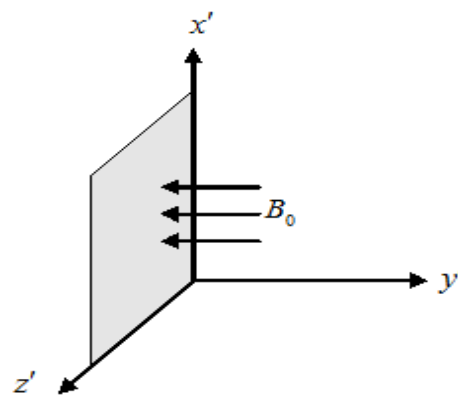


Fig. 1. Geometry of the Problem.

Equation of Momentum:

$$\frac{\partial u}{\partial t'} - 2\Omega'v = g \frac{\partial^2 u}{\partial z'^2} - \frac{1}{\rho} \frac{\partial p}{\partial x} + g + \frac{B_0}{\rho} j_y \quad (1)$$

$$\frac{\partial v}{\partial t'} + 2\Omega'u = g \frac{\partial^2 v}{\partial z'^2} - \frac{B_0}{\rho} j_x \quad (2)$$

Equation of Energy:

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q_r}{\partial z} \quad (3)$$

Equation of diffusion:

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z'^2} \quad (4)$$

where T' is the fluid temperature, C' is the concentration, t' is the time, D is the diffusion term.

Since there is no large velocity gradient here, the viscous term in Equation (1) vanishes for small and hence for the outer flow, beside there is no magnetic field along x -direction gradient, so we have

$$0 = -\frac{\partial p}{\partial x} - \rho_\infty g \quad (5)$$

By eliminating the pressure term from Equations (1) and (5), we obtain

$$\frac{\partial u}{\partial t'} - 2\Omega'v = g \frac{\partial^2 u}{\partial z^2} + (\rho_\infty - \rho)g + \frac{B_0}{\rho} j_y \quad (6)$$

The Boussinesq approximation gives

$$\rho_\infty - \rho = \rho_\infty \beta (T' - T'_\infty) + \rho_\infty \beta^* (C' - C'_\infty) \quad (7)$$

On using (7) in the equation (6) and noting that ρ_∞ is approximately equal to 1, the momentum equation reduces to

$$\frac{\partial u}{\partial t'} - 2\Omega'v = g \frac{\partial^2 u}{\partial z^2} + \frac{B_0}{\rho} j_y + g \beta (T' - T'_\infty) + g \beta^* (C' - C'_\infty) \quad (8)$$

where ρ is the density, g the kinematic viscosity, u and v are fluid velocity components, g the acceleration due to gravity, C_p the specific heat at constant pressure, q_r the radiative heat flux in the z - direction, k is thermal conductivity, β the volumetric coefficient of thermal expansion, β^* the volumetric coefficient of expansion with concentration.

The generalized Ohm's law, on taking Hall currents into account and neglecting ion-slip and thermo-electric effect, is

$$\vec{j} + \frac{\omega_e \tau_e}{B_0} (\vec{j} \times \vec{B}) = \sigma (\vec{E} + \vec{q} \times \vec{B}) \quad (9)$$

Where \vec{j} is the current density vector, \vec{q} the velocity vector, \vec{B} the magnetic field vector, \vec{E} the electric field vector, ω_e the cyclotron frequency, σ the electrical conductivity of the fluid and τ_e the collision time of electron.

The equation (9) gives

$$j_x - m j_y = \sigma v B_0 \quad (10)$$

$$j_y + m j_x = -\sigma u B_0 \quad (11)$$

where $m = \omega_e \tau_e$ is the Hall parameter. Solving (10) and (11) for j_x and j_y , we have

$$j_x = \frac{\sigma B_0}{1+m^2} (v - mu) \quad (12)$$

$$j_y = \frac{\sigma B_0}{1+m^2} (u + mv) \quad (13)$$

On the use of (12) and (13), the momentum equations (8) and (2) become

$$\frac{\partial u}{\partial t'} = g \frac{\partial^2 u}{\partial z^2} + 2\Omega'v - \frac{\sigma B_0^2}{\rho(1+m^2)} (u + mv) + g \beta (T' - T'_\infty) + g \beta^* (C' - C'_\infty) \quad (14)$$

$$\frac{\partial v}{\partial t'} = g \frac{\partial^2 v}{\partial z^2} - 2\Omega'u + \frac{\sigma B_0^2}{\rho(1+m^2)} (mu - v) \quad (15)$$

Here u is the axial velocity (along the direction of the plate) and v is the transverse velocity (transverse to the main flow), the initial and boundary conditions are given by

$$\left. \begin{aligned} u = 0, v = 0, T' = T'_\infty, C' = C'_\infty, \\ t' \leq 0 \quad \forall z \\ u = u_0 \exp(at'), v = 0, t' > 0 \text{ at } z = 0 \\ T' = T'_w, C' = C'_w, t' > 0 \text{ at } z = 0 \\ u \rightarrow 0, v \rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \end{aligned} \right\} \quad (16)$$

as $z \rightarrow \infty$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial z} = -4a^* \sigma (T_\infty'^4 - T'^4) \quad (17)$$

It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature.

This is accomplished by expanding T'^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T_\infty'^3 T' - 3T_\infty'^4 \quad (18)$$

By using equations (17) and (18), equation (3) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z^2} + 16a^* \sigma T_\infty'^3 (T' - T_\infty') \quad (19)$$

Let us introducing the following non-dimensional quantities

$$U = \frac{u}{u_0}, V = \frac{v}{u_0}, Z = \frac{z u_0}{\gamma}, t = \frac{t' u_0^2}{\gamma}, \Omega = \frac{\Omega' \gamma}{u_0^2},$$

$$M^2 = \frac{\Omega \sigma B_0^2 \gamma}{2\rho u_0^2}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty},$$

$$\text{Pr} = \frac{\rho C_p}{k}, \text{Sc} = \frac{\nu}{D},$$

$$\text{Gr} = \frac{g \beta \gamma (T'_w - T'_\infty)}{u_0^3}, R = \frac{16a^* \sigma \gamma^2 T_\infty'^3}{k u_0^2},$$

$$\text{Gc} = \frac{g \beta^* \gamma (C'_w - C'_\infty)}{u_0^3}, a = \frac{a' \gamma}{u_0^2}$$

where Ω is the rotation parameter, Gr is the thermal Grashof number, Gc is the mass Grashof number, Pr the Prandtl number, Sc the Schmidt number, R the radiation parameter, t the dimensionless time, a the accelerating parameter u and v the dimensionless velocity, m - Hall

parameter, M - Hartmann number, C is the dimensionless concentration and θ is the dimensionless temperature.

Using these boundary conditions in above equations, we obtain the following dimensionless form of the governing equations:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial Z^2} + 2\Omega V - \frac{2M^2(U + mV)}{1+m^2} + G_r\theta + G_c C$$

(20)

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial Z^2} - 2\Omega U + \frac{2M^2(mU - V)}{1+m^2}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Z^2} - \frac{R}{Pr} \theta$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Z^2}$$

The boundary conditions for corresponding order are

$$\left. \begin{aligned} U = 0, V = 0 \text{ at } t \leq 0 \text{ for all } Z \\ t > 0, U = \exp(at'), V = 0 \text{ at } Z = 0 \\ U \rightarrow 0, V \rightarrow 0 \text{ as } Z \rightarrow \infty \end{aligned} \right\} \quad (24)$$

Now equations (20) & (21) and boundary conditions (24) can be combined to give

$$\frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial Z^2} - wF + G_r\theta + G_c C$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Z^2} - \frac{R}{Pr} \theta$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Z^2}$$

Where $F = U + iV$,

$$w = \frac{2M^2}{1+m^2} + 2i\left(\Omega - \frac{M^2 m}{1+m^2}\right)$$

The initial and boundary conditions in non-dimensional quantities are

$$\left. \begin{aligned} F = 0, \theta = 0, C = 0 \text{ for all } Z, t \leq 0 \\ t > 0, F = \exp(at'), \theta = 1, C = 1 \text{ at } Z = 0 \\ F \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \text{ as } Z \rightarrow \infty \end{aligned} \right\} \quad (28)$$

Exact solution for the fluid temperature and concentration of (26),(27) is expressed in the following form by taking inverse Laplace transform of solution as

$$C(Z, t) = \text{erfc}(\eta\sqrt{Sc}) \quad (29)$$

$$\theta(Z, t) = \frac{1}{2} \left[\begin{aligned} &\exp(2\eta\sqrt{bPrt})\text{erfc}(\eta\sqrt{Pr} \\ &+ \sqrt{bt}) + \exp(-2\eta\sqrt{bPrt}) \\ &\text{erfc}(\eta\sqrt{Pr} - \sqrt{bt}) \end{aligned} \right] \quad (30)$$

The equations (25), (26), (27), subject to the boundary conditions (28), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\begin{aligned} F(Z, t) = &g_1 \left[\begin{aligned} &\exp(2\eta\sqrt{(w+a)t})\text{erfc}(\eta + \sqrt{(w+a)t}) + \\ &\exp(-2\eta\sqrt{(w+a)t})\text{erfc}(\eta - \sqrt{(w+a)t}) \end{aligned} \right] \\ &+ \left[\begin{aligned} &(A-B)\left(\frac{1}{2}\right)(\exp(2\eta\sqrt{wt})\text{erfc}(\eta + \sqrt{wt}) \\ &+ \exp(-2\eta\sqrt{wt})\text{erfc}(\eta - \sqrt{wt})) \end{aligned} \right] \\ &- \left[\begin{aligned} &Ag_2(\exp(2\eta\sqrt{(w+d)t})\text{erfc}(\eta + \sqrt{(w+d)t}) \\ &+ \exp(-2\eta\sqrt{(w+d)t})\text{erfc}(\eta - \sqrt{(w+d)t})) \end{aligned} \right] \\ &+ \left[\begin{aligned} &Bh(\exp(2\eta\sqrt{(w+e)t})\text{erfc}(\eta + \sqrt{(w+e)t}) \\ &+ \exp(-2\eta\sqrt{(w+e)t})\text{erfc}(\eta - \sqrt{(w+e)t})) \end{aligned} \right] \\ &+ \left[\begin{aligned} &Ag_2(\exp(2\eta\sqrt{P_r(b+d)t})\text{erfc}(\eta\sqrt{P_r} \\ &+ \sqrt{(b+d)t}) + \exp(-2\eta\sqrt{P_r(b+d)t}) \\ &\text{erfc}(\eta\sqrt{P_r} - \sqrt{(b+d)t})) \end{aligned} \right] \\ &- \left[\begin{aligned} &\frac{A}{2}(\exp(2\eta\sqrt{P_rbt})\text{erfc}(\eta\sqrt{P_r} + \sqrt{bt}) \\ &+ \exp(-2\eta\sqrt{P_rbt})\text{erfc}(\eta\sqrt{P_r} - \sqrt{bt})) \end{aligned} \right] \\ &- \left[\begin{aligned} &Bh(\exp(2\eta\sqrt{(Sc)et})\text{erfc}(\eta\sqrt{Sc} + \sqrt{et}) + \\ &\exp(-2\eta\sqrt{(Sc)et})\text{erfc}(\eta\sqrt{Sc} - \sqrt{et})) \end{aligned} \right] \\ &- B(\text{erfc}(\eta\sqrt{Sc})), \text{ where } \eta = \frac{z}{2\sqrt{t}} \end{aligned}$$

(31)

3. RESULTS AND DISCUSSION

The physical depth of the problem of flow past an exponentially accelerated infinite vertical plate and temperature with variable mass diffusion has been formulated and solved analytically. In order to investigate the effects of the heat sources when the plate moves with velocity $u = u_0 \exp(at')$ in its own plane, numerical calculations are carried out for different values of Rotation parameter, Hall parameter, Hartmann number, thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number, Radiation parameter and time. The value of the Prandtl number Pr is chosen to represent air ($Pr = 0.71$). The value of Schmidt number is chosen to represent water vapour ($Sc = 0.6$). The Thermal Grashof number Gr represents here the effects of the free convection currents, and receives positive, zero or negative values. The velocity distribution, temperature distribution and concentration distribution are studied in figures 2-19, while keeping the other parameters as constants.

In fig. 2, the concentration profile at time $t=0.2$ decreases with increase in the values of Sc . The temperature profiles are calculated for different values of thermal radiation parameter ($R=0.2, 0.2, 2.0, 5.0$) and time ($t=0.2, 0.6, 0.2, 0.2$) are shown in fig. 3. It is observed that the temperature increases with decreasing radiation parameter.

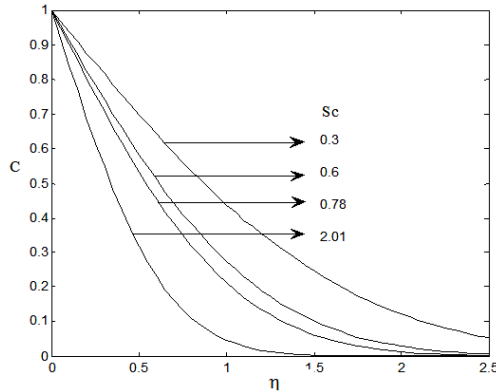


Fig. 2. Concentration profiles for different values of Sc .

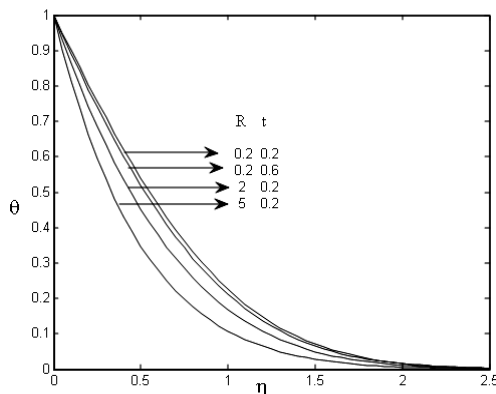


Fig. 3. Temperature profiles for different values of R and t .

In fig. 4 the primary velocity profile for different Ω has been presented and it is observed that the primary velocity U falls when Ω are increased. It is observed from fig. 5 secondary velocity with minimum for $\Omega = M^2m/(1+m^2)$ and increases as Ω increases. From fig. 6, it is clear that the primary velocity increases with decreasing values of the Hartmann number (M). Fig. 7. show that due to an increase in the Hartmann number M , the secondary velocity increases. Fig. 8 represents the velocity profiles for various values of m . It is observed that the primary velocity rises due to increasing value of the Hall parameter m . It is found that from fig. 9 due to an increase in the Hall parameter, m , there is rise in the secondary velocity components. In the fig. 10, it is observed that the primary velocity increases with increasing values of the thermal Grashof number or mass Grashof number. It is observed from fig. 11 that the velocity increases with increasing values of Gr, Gc . The effect of radiation parameter R on velocity profile is shown in fig. 12, 13. From fig. 12, it is clear that the primary velocity

increases with decreasing values of radiation parameter.

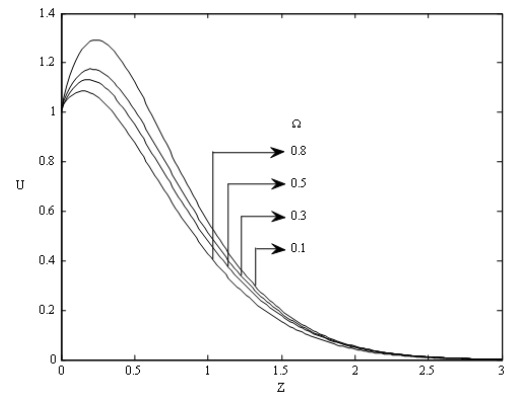


Fig. 4. Primary velocity profiles for several values of Ω when $Sc=0.6, Pr=0.71, a=0.1, t=0.2, M=0.5, m=0.5, Gr=5, Gc=5, R=5$

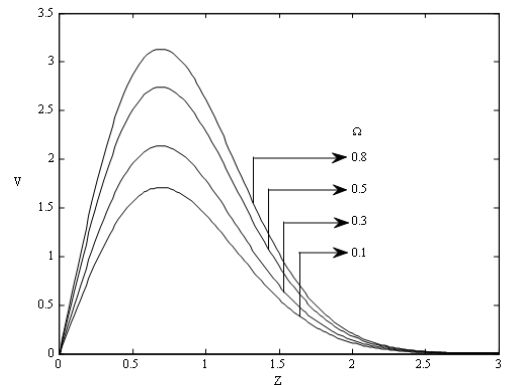


Fig. 5. Secondary velocity profiles for several values of Ω when $Sc=0.6, Pr=0.71, a=0.1, t=0.2, M=0.5, m=0.5, Gr=5, Gc=5, R=5$.

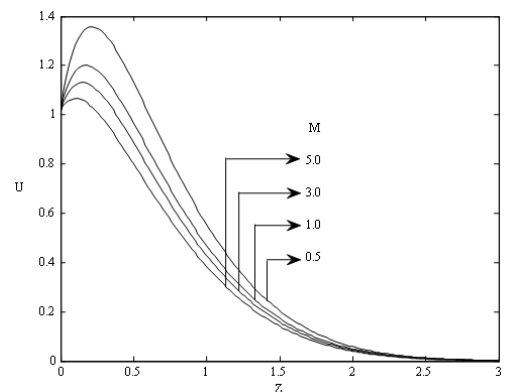


Fig. 6. Primary velocity profiles for several values of M when $Sc=0.6, Pr=0.71, a=0.1, t=0.2, \Omega=0.1, m=0.5, Gr=5, Gc=5, R=5$.

Fig. 13. show that due to an increase in the radiation parameter, the secondary velocity increases. In fig. 14. it is observed that the primary velocity increases with decreasing values of Schmidt number (Sc).

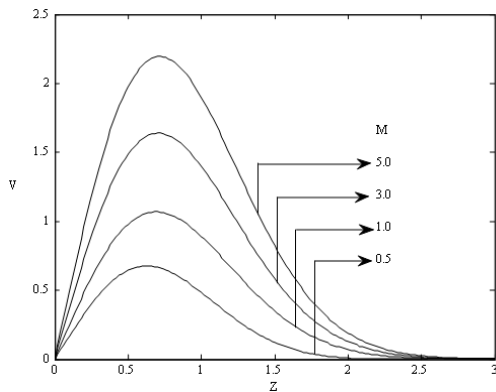


Fig. 7. Secondary velocity profiles for several values of M when $Sc=0.6, Pr=0.71, a=0.1, t=0.2, \Omega=0.1, m=0.5, Gr=5, Gc=5, R=5$.

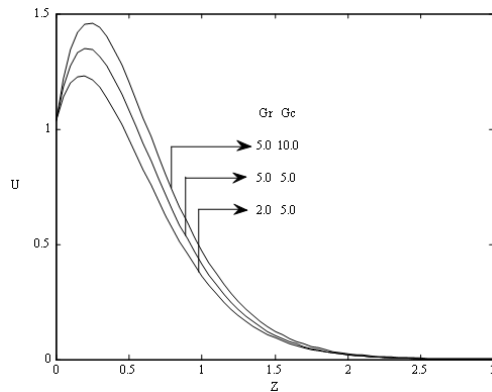


Fig. 10. Primary velocity profiles for several values of Gr and Gc when $Sc=0.6, Pr=0.71, a=0.1, t=0.2, \Omega=0.1, M=0.5, m=0.5, R=5$.

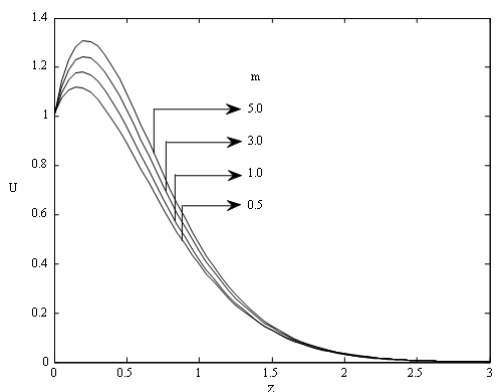


Fig. 8. Primary velocity profiles for several values of m when $Sc=0.6, Pr=0.71, a=0.1, t=0.2, \Omega=0.1, M=0.5, Gr=5, Gc=5, R=5$.

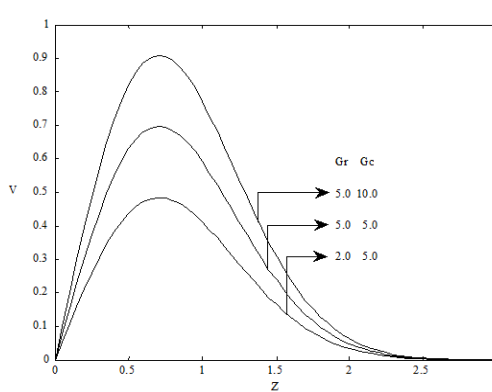


Fig. 11. Secondary velocity profiles for several values of Gr and Gc when $Sc=0.6, t=0.2, Pr=0.71, a=0.1, \Omega=0.1, M=0.5, m=0.5, R=5$.

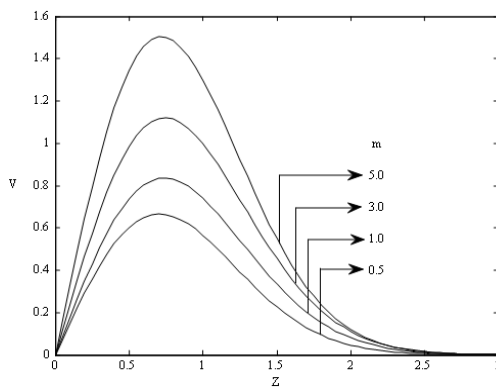


Fig.9. Secondary velocity profiles for several values of m when $Sc=0.6, Pr=0.71, a=0.1, t=0.2, \Omega=0.1, M=0.5, Gr=5, Gc=5, R=5$.

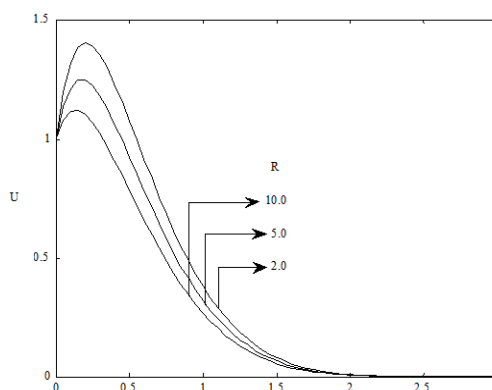


Fig. 12. Primary velocity profiles for several values of R when $Sc=0.6, Pr=0.71, a=0.1, t=0.2, \Omega=0.1, M=0.5, m=0.5, Gr=5, Gc=5$.

In fig.15.it is observed that the secondary secondary velocity increases with increasing values of Sc . The velocity profiles for different values of time t are presented in fig .16, 17. It is observed that the primary velocity and secondary velocity increases with increasing values of t . The velocity profiles for different ($a=0.2, 0.5, 0.8$), $Pr=0.71, R=5, t=0.2, \Omega=0.1, M=0.5, m=0.5, Gr=5, Gc=5, Sc=0.6$ are studied and presented in fig 18,19. It is evident from figures that the primary velocity and secondary velocity increases with increasing values of ‘ a ’.

The rate of heat transfer and mass transfer between the plate and the fluid, molecular concentration respectively. It is found by using the non-dimensional quantities, the Nusselt number Nu and the Sherwood number Sh . The Nusselt number is defined as negative gradient of the temperature. The Sherwood number is negative gradient of concentration.

$$Nu = -\left(\frac{d\theta}{dZ}\right)_{Z=0} \quad (32)$$

From equations (30) and (32) we get Nusselt number as $Nu = \frac{\sqrt{Pr}}{\sqrt{\pi t}} [1 + (\sqrt{b\pi t}) \operatorname{erf}(\sqrt{bt})]$

From the concentration field, now we study Sherwood number (rate of change of mass transfer) it is given in non dimensional form as

$$Sh = -\left(\frac{dC}{dZ}\right)_{Z=0} \quad (33)$$

From equations (29) and (33) we get Sherwood number as

$$Sh = \frac{\sqrt{Sc}}{\sqrt{\pi t}}$$

Table 1 Sherwood number –Sh

Sc	t	Sh
0.3	0.2	0.6912
0.16	0.2	0.5048
0.78	0.2	1.1145
0.6	0.2	0.9774
2.01	0.2	1.789
0.6	0.4	0.6912
0.6	0.6	0.5644

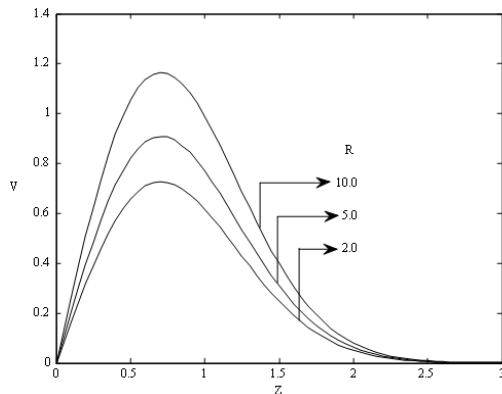


Fig.13. Secondary velocity profiles for several values of R when Sc=0.6, a=0.1, t=0.2, Pr=0.71, Ω=0.1, M=0.5, m=0.5, Gr=5, Gc=5.

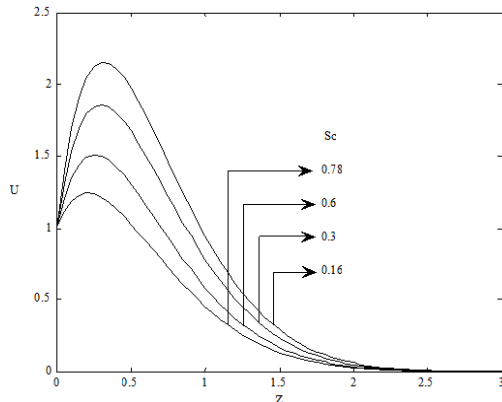


Fig. 14. Primary velocity profiles for several values of Sc when Pr=0.71, R=5, a=0.1, t=0.2, Ω=0.1, M=0.5, m=0.5, Gr=5, Gc=5.

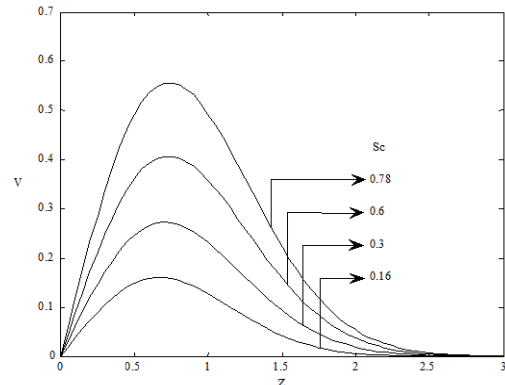


Fig. 15. Secondary velocity profiles for several values of Sc when Pr=0.71, R=5, a=0.1, t=0.2, Ω=0.1, M=0.5, m=0.5, Gr=5, Gc=5.

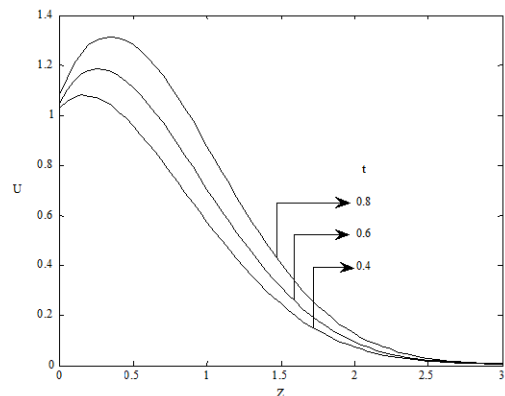


Fig. 16. Primary velocity profiles for several values of t when Pr=0.71, R=5, a=0.1, Sc=0.6, Ω=0.1, M=0.5, m=0.5, Gr=5, Gc=5.

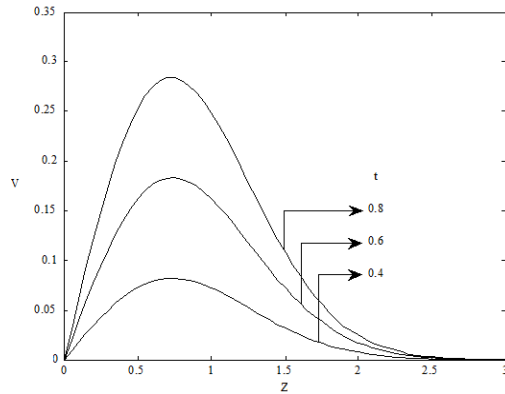


Fig. 17. Secondary velocity profiles for several values of t when Pr=0.71, R=5, a=0.1, Sc =0.6, Ω=0.1, M=0.5, m=0.5, Gr=5, Gc=5.

Table 2 Nusselt number for air at R=5

t	Nu
0.2	3.09056
0.4	2.94847
0.6	2.84173
0.8	2.76595

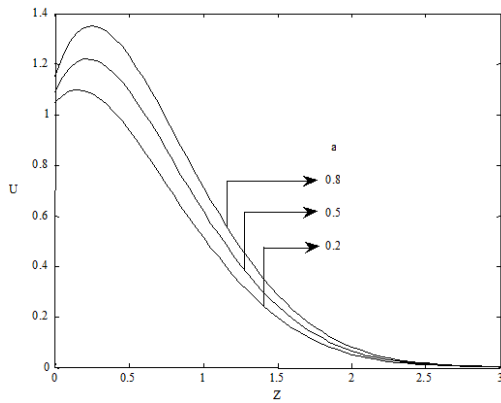


Fig. 18. Primary velocity profiles for several values of 'a' when $Pr=0.71$, $R=5$, $t=0.2$, $\Omega=0.1$, $M=0.5$, $m=0.5$, $Gr=5$, $Gc=5$, $Sc=0.6$.

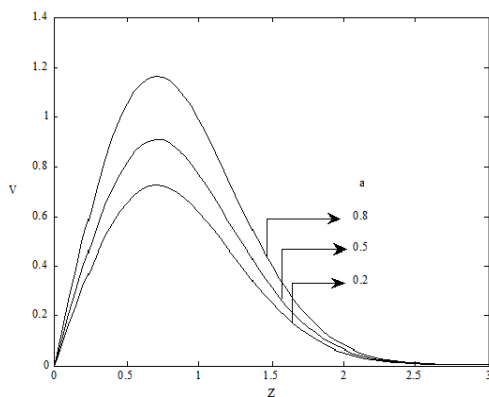


Fig. 19. Secondary velocity profiles for several values of 'a' when $Pr=0.71$, $R=5$, $\Omega=0.1$, $M=0.5$, $m=0.5$, $Gr=5$, $Gc=5$, $Sc=0.6$, $t=0.2$.

The Nusselt number and Sherwood number has been

discussed for a range of physical parameters. The values of Sherwood number are tabulated in table 1 for different time Schmidt number (Sc) and time t . Sherwood number increases when the values of Sc increase at $t=0.2$ but it decreases when the values time increase at $Sc=0.6$. The values of Nusselt number are tabulated in table 2 for different time t . Nusselt number increases when the values of time increase.

4. CONCLUSION

An analytic solution of radiation effects on exponentially accelerated infinite vertical plate in the presence of variable temperature and with variable mass diffusion has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique and computed for different parameters using MATLAB. The effects of temperature and concentration profiles, axial velocity and secondary velocity for different parameters like Rotation parameter, Hall parameter, Hartmann number, accelerating parameter, radiation parameter, mass Grashof number, thermal Grashof number, Schmidt number, and time are presented graphically. The

concentration increases with decreasing values of the Schmidt number. The concentration increases with decreasing values of the Schmidt number. The temperature increases with decreasing radiation parameter. The axial velocity rises due to increasing value of the Hall parameter, accelerating parameter, thermal Grashof number and mass Grashof number and time. The axial velocity u falls when Ω are increased, the velocity increases with decreasing values of the Hartmann number, the radiation parameter. Transverse velocity increases as Ω increases, due to an increase in the Hartmann number M , the Hall parameter, m , accelerating parameter, the radiation parameter R , Gr , Gc , Sc and t .

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APPENDIX

$$A = \frac{G_r}{(1 - P_r)d}; \quad B = \frac{G_c}{(Sc - 1)e}; \quad b = \frac{R}{P_r};$$

$$d = \frac{bP_r - w}{1 - P_r}; \quad e = \frac{w}{Sc - 1};$$

$$g_1 = \frac{\exp(at)}{2}; \quad g_2 = \frac{\exp(dt)}{2}; \quad h = \frac{\exp(et)}{2}$$

In order to get the physical insight into the problem, the numerical values of F have been computed from (20). While evaluating this expression, it is observed that the argument of the error function is complex and, hence, we have separated it into real and imaginary parts by using the following formula:

$$erf(a + ib) = erf(a) + \frac{\exp(-a^2)}{2a\pi} [1 - \cos(2ab) + i \sin(2ab)] + \frac{2 \exp(-a^2)}{\pi} \sum_{n=1}^{\infty} \frac{\exp(-\frac{n^2}{4})}{n^2 + 4a^2} [f_n(a, b) + ig_n(a, b)] + \epsilon(a, b)$$

Where

$$f_n = 2a - 2a \cosh(nb) \cos(2ab) + n \sinh(nab) \sin(2ab)$$

and

$$g_n = 2a \cosh(nb) \sin(2ab) + n \sinh(nab) \cos(2ab)$$

$$|\epsilon(a, b)| \approx 10^{-16} |erf(a + ib)|$$