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Axisymmetric Stokes Flow past a Swarm of Porous Cylindrical Shells

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ABSTRACT

The problem of an axisymmetric Stokes flow for an incompressible viscous fluid past a swarm of porous cylindrical shells with four known boundary conditions as Happel's, Kuwabara's, Kvashnin's and Cunningham/Mehta-Morse's is tackled. The Brinkman equation is taken for fluid flow through the porous region and the Stokes equation for fluid region in their stream function formulation are used. Drag force experienced by the porous cylindrical shell within a cell is evaluated. The hydrodynamic permeability of the membrane built by the porous particles is also investigated. For different values of parameters, the variation of drag force and the hydrodynamic permeability are presented graphically and discussed.

Key words: Cell models; Brinkman equation; Modified Bessel's functions; Hydrodynamic permeability.

NOMENCLATURE

C_D	drag coefficient	r	radial component
F	drag force	T	stress tensor
$I_1(\sigma r)$ & $K_1(\sigma r)$	modified Bessel functions of first and second kinds of order one, respectively	$\mathbf{v}^{(i)}$	velocity vectors
		θ	transverse component
i	phase (region) 1or 2or 3	γ	particle volume fraction
l, m	dimensionless quantity	μ	viscosity coefficient
$p^{(i)}$	pressures	σ	permeability parameter
(r,θ,z)	cylindrical polar coordinates	$\psi^{(i)}$	stream function

1. INTRODUCTION

The fluid flow through porous media has been a topic of longstanding interest for researchers from last five decades, due to its numerous applications in bio-mechanics, physical sciences, chemical engineering, and industries etc. Several conceptual models have been developed for describing fluid flow in porous media as discussed in the classical book entitled 'Convections in Porous Media' by Neild and Bejan (2006). Henri Darcy proposed an empirical law which states that the rate of flow is proportional to pressure drop through a densely packed bed of fine particles, is one of the basic model that has been used extensively in the literature.

During nineteenth century after the Darcy's work, flow through porous media has been simulated by questions arising in practical problems. Brinkman (1947) proposed a modification of the Darcy's law for a porous medium which was assumed to be governed by a swarm of homogeneous spherical particles and provides an equation commonly known as Brinkman equation. Some exact stream function solutions for axisymmetric flow are given in the classical book entitled 'Low Reynolds Number Hydrodynamics' by Happel and Brenner (1991).

Happel (1958, 1959) and Kuwabara (1959) proposed cell models in which both particle and outer envelope are spherical/cylindrical. The Happel model assumes that the inner sphere –while at the center-moves with a constant velocity and fluid is at rest. Also, he used no-slip condition on the inner sphere, nil radial velocity and nil shear stress on the outer envelope. The Kuwabara model assumes that the inner sphere is stationary and that fluid passes through the unit cell. The following boundary conditions are imposed: nil radial and tangential velocity on the inner sphere/cylinder, velocity with axial component equal to a constant approach velocity on the outer envelope and nil vorticity on the outer envelope. Cunningham (1910) and Mehta-Morse (1975) assumed the uniform velocity condition on hypothetical cell to investigate flow through charged membrane. This assumption signifies the homogeneity of flow on the cell boundary. Kvashnin (1979) assumed the symmetry condition for velocity and proposed that the tangential component of velocity approaches extreme value on the cell surface along radial direction.

Pop and Cheng (1992) studied the problem of the steady, incompressible fluid flow past a circular cylinder embedded in a constant porosity medium based on the Brinkman model. They have obtained a closed form exact solution for the governing equation, which leads to an expression for the separation parameter. The streamlines and velocity profiles for flow past a circular cylinder embedded in a constant porosity medium with different particle diameter ratio are presented by them. Filippov *et al.* (2006) used the cell method to model the permeability of a membrane built from porous particles with a permeable shell. They investigated the influence of the porous shell on the total permeability by applying the Mehta-Morse boundary condition on the cell boundary.

Deo *et al.* (2010) studied the problem of slow viscous flow through an aggregate of concentric clusters of porous cylindrical particles with Happel boundary condition. They have used the stream function formulation of the Brinkman equation for the evaluation of the problem. Hydrodynamic permeability of membranes built up by porous cylindrical or spherical particles with impermeable core is investigated by Deo and his collaborators (2011). Vasin and Kharitonova (2011) studied the problem of an infinite uniform flow of liquid around the encapsulated spherical drop coated with the porous layer. They assume that the external liquid pass through the porous layer but is not mixed with the liquid appear in the internal cavity of the capsule. They evaluated the velocity and pressure distributions and hydrodynamic force acting on the capsule. Recently, Gupta and Deo (2013) studied the problem of axisymmetric Stokes flow of a micropolar fluid past a sphere coated with a thin, immiscible Newtonian fluid. They obtained the expression for the drag force experienced by the fluid-coated sphere and its variations for different parameters were presented graphically and discussed.

This work is concerns with the axisymmetric Stokes flow of an incompressible viscous fluid past a swarm of porous cylindrical shells with four known boundary conditions as Happel's, Kuwabara's, Kvashnin's and Cunningham/Mehta-Morse's. Drag force experienced by the porous cylindrical shell within a cell is evaluated. In addition to this, the hydrodynamic permeability of the membrane built by the porous particle has been evaluated. For different values of parameters, the variation of drag force and the hydrodynamic permeability are presented graphically and discussed.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Here, we consider an axisymmetric Stokes flow of an incompressible viscous fluid past a swarm of homogeneous porous cylindrical shells whose external and internal radii are $\tilde{b}$ and $\tilde{a}$ ($\tilde{b} > \tilde{a}$) respectively. Inner region of the shell is filled with incompressible Newtonian viscous fluid. Applying the cell method, we assume that the shell is enveloped by a hypothetical concentric cylinder of radius $\tilde{c}$ named as the outer surface of shell. We further assume that fluid approaches to the shell and passes through the porous cylinder perpendicular to the axis of cylinder ($\tilde{z}$ -axis) with velocity $\tilde{U}$ from left to right. The external region ($\tilde{b} \le \tilde{r} \le \tilde{c}$), the porous region ($\tilde{a} \le \tilde{r} \le \tilde{b}$) and the cavity region ($0 \le \tilde{r} \le \tilde{a}$), are designated by I, II and III, respectively.

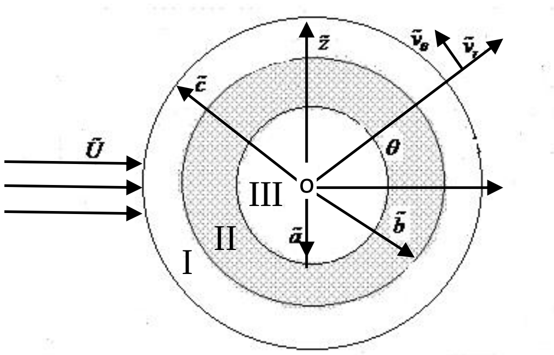

Fig. 1. Schematic of the physical model.

The governing equations for the creeping flow of an incompressible Newtonian viscous fluid, which lies in the outside of the porous cylindrical shell and in the cavity region, i.e. the regions I and III are governed by Stokes equations:

$$\tilde{\mu}_1 \tilde{\nabla}^2 \tilde{\mathbf{v}}^{(i)} = \tilde{\nabla} \tilde{p}^{(i)}, \qquad i = 1, 3. \tag{1}$$

The flow of fluid through the porous cylindrical shell (in the region II) is governed by Brinkman's equation:

$$\tilde{\mu}_2 \tilde{\nabla}^2 \tilde{\mathbf{v}}^{(2)} - \frac{\tilde{\mu}_1}{\tilde{k}} \tilde{\mathbf{v}}^{(2)} = \tilde{\nabla} \tilde{p}^{(2)}, \tag{2}$$

where superscripts $i = 1, 2$ and 3 denote the external region, the porous region and the cavity region, respectively; $\tilde{\mu}_1$ is the viscosity of the clear fluid, $\tilde{\mu}_2$ is the effective viscosity in the porous region; $\tilde{k}$ is the permeability of the porous region; $\tilde{\mathbf{v}}^i, \tilde{p}^i, i = 1, 2, 3$ are the velocity vectors and pressures in abovementioned regions. The viscosity coefficients $\tilde{\mu}_1$ and $\tilde{\mu}_2$ are assumed to be constant and taken equal.

The equations of continuity for incompressible fluids must be satisfied in all the three regions:

$$\tilde{\nabla}.\tilde{\mathbf{v}}^{(i)}=0\text{ ,i=1,2,3.}\tag{3}$$

The cylindrical polar coordinate system $(\tilde{r},\theta,\tilde{z})$ with origin at the center of the cylindrical particle and $\tilde{z}$ - axis along the axis of the cylinder, is used.

Introducing the dimensionless variables and constants as follows:

$$\gamma=\frac{\tilde{b}^2}{\tilde{c}^2},\ m=\frac{\tilde{c}}{\tilde{b}}=\frac{1}{\sqrt{\gamma}},\ l=\frac{\tilde{a}}{\tilde{b}},\ r=\frac{\tilde{r}}{\tilde{b}},\ \nabla=\tilde{\nabla}\tilde{b},$$

$$v=\frac{\tilde{v}}{\tilde{U}},\ p=\frac{\tilde{p}}{\tilde{p}_o},\ \tilde{p}_o=\frac{\tilde{U}\tilde{\mu}_1}{\tilde{b}},\ \sigma^2=\frac{\tilde{b}^2}{\tilde{k}}.$$

The Eqs. (1) and (2) in dimensionless form can be written as

$$\nabla^2\mathbf{v}^i=\nabla p^i,\qquad i=1,3,\tag{4}$$

$$\nabla^2\mathbf{v}^2-\sigma^{(2)}\mathbf{v}^{(2)}=\nabla p^2\ .\tag{5}$$

The equation of continuity (3) for axisymmetric, incompressible viscous fluid in cylindrical coordinates in dimensionless form in all three regions can be written as:

$$\frac{\partial}{\partial r}(rv_r^{(i)})+\frac{\partial}{\partial\theta}(v_\theta^{(i)})=0,\ \ i=1,2,3,\tag{6}$$

where $v_r^{(i)}$ and $v_\theta^{(i)}$ are components of velocities in the directions of r and θ, respectively.

Introducing the stream functions $\psi^{(i)}(r,\theta)$, satisfying equations of continuity, in all the three regions can be defined as

$$v_r^{(i)}=\frac{1}{r}\frac{\partial\psi^{(i)}}{\partial\theta},\ v_\theta^{(i)}=-\frac{\partial\psi^{(i)}}{\partial r}.\tag{7}$$

Furthermore, in the dimensionless form the expressions for tangential and normal stresses $T_{r\theta}^{(i)},\ T_{rr}^{(i)}$, $i=1,2,3$ respectively, can be find by the relation

$$T_{r\theta}^{(i)}=[\frac{1}{r}\frac{\partial v_r^{(i)}}{\partial\theta}+r\frac{\partial(v_\theta^{(i)}/r)}{\partial r}]\ ,\tag{8}$$

$$T_{rr}^{(i)}=-p^{(i)}+2\frac{\partial v_r^{(i)}}{\partial r}\ .\tag{9}$$

Also, the pressures may be obtained in all the three regions by integrating the following relations respectively as

$$\frac{\partial p^{(i)}}{\partial r}=\frac{1}{r}\frac{\partial}{\partial\theta}\left(\nabla^2\psi^{(i)}\right),\quad i=1,3,\tag{10}$$

$$\frac{\partial p^{(i)}}{\partial\theta}=-r\frac{\partial}{\partial r}\left(\nabla^2\psi^{(i)}\right),\ i=1,3,\tag{11}$$

$$\frac{\partial p^{(2)}}{\partial r}=\frac{1}{r}[\frac{\partial}{\partial\theta}\left(\nabla^2\psi^{(2)}\right)-\sigma^2\frac{\partial\psi^{(2)}}{\partial\theta}]\tag{12}$$

$$\frac{\partial p^{(2)}}{\partial\theta}=-r[\frac{\partial}{\partial r}\left(\nabla^2\psi^{(2)}\right)-\sigma^2\frac{\partial\psi^{(2)}}{\partial r}].\tag{13}$$

3. SOLUTION OF THE PROBLEM

Eliminating the pressure from Eqs. (4) and (5) by taking curl both sides, and using Eq. (7) we obtained the following fourth order, linear partial differential equation in terms of the stream function as

$$\nabla^4\psi^{(i)}=\nabla^2(\nabla^2\psi^{(i)})=0\text{ ,i = 1, 3,}\tag{14}$$

$$\begin{aligned}&\nabla^4\psi^{(2)}-\sigma^2\nabla^2\psi^{(2)}\\&=\nabla^2(\nabla^2-\sigma^2)\psi^{(2)}=0,\end{aligned}\tag{15}$$

where the Laplacian operator ∇^2 is :

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}.$$

The suitable solutions of the Eqs. (14) and (15), by using the method of separation of variables can be expressed in the form as

$$\begin{aligned}\psi^{(1)}(r,\theta)=&[A_1r+\frac{B_1}{r}+C_1r^3\\&+D_1r\ln r]\sin\theta,\end{aligned}\tag{16}$$

$$\begin{aligned}\psi^{(2)}(r,\theta)=&[A_2r+\frac{B_2}{r}+C_2I_1(\sigma r)\\&+D_2K_1(\sigma r)]\sin\theta,\end{aligned}\tag{17}$$

$$\psi^{(3)}(r,\theta)=[A_3r+C_3r^3]\sin\theta,\tag{18}$$

where $I_1(\sigma r)$ and $K_1(\sigma r)$ are the modified Bessel functions of order one of the first and second kinds, respectively (Abramowitz and Stegun 1972). $A_1,B_1,C_1,D_1,A_2,B_2,C_2,D_2,A_3$ and C_3 are the arbitrary constants which can be obtained by applying boundary conditions.

Expressions for velocity components, stress components and pressures:

The velocity components, stress components and pressures are obtained by using the Eqs. (7)-(13) and Eqs. (16)-(18), which are given below as:

$$v_r^{(1)}=[A_1+\frac{B_1}{r^2}+C_1r^2+D_1\ln(r)]\cos\theta,\tag{19}$$

$$\begin{aligned}v_\theta^{(1)}=&-[A_1-\frac{B_1}{r^2}+3C_1r^2\\&+D_1(1+\ln(r))]\sin\theta,\end{aligned}\tag{20}$$

$$\begin{aligned}v_r^{(2)}=&[A_2+\frac{B_2}{r^2}+\frac{C_2}{r}I_1(\sigma r)\\&+\frac{D_2}{r}K_1(\sigma r\}]\cos\theta,\end{aligned}\tag{21}$$

$$v_\theta^{(2)} = -[A_2 - \frac{B_2}{r^2} + C_2\{\sigma I_0(\sigma r) - \frac{1}{r} I_1(\sigma r)\} + D_2\{\sigma K_0(\sigma r) + \frac{1}{r} K_1(\sigma r)\}]\sin\theta \quad (22)$$

$$v_r^{(3)} = [A_3 + C_3 r^2]\cos\theta , \quad (23)$$

$$v_\theta^{(3)} = -[A_3 + 3C_3 r^2]\sin\theta . \quad (24)$$

The components of stresses can be written in form as given below:

$$T_{rr}^{(1)} = -4[\frac{B_1}{r^3} + C_1 r - \frac{D_1}{r}]\cos\theta , \quad (25)$$

$$T_{r\theta}^{(1)} = -4[\frac{B_1}{r^3} + C_1 r]\sin\theta , \quad (26)$$

$$T_{r\theta}^{(2)} = -[\frac{4B_2}{r^3} + C_2\{(\sigma^2 + \frac{4}{r^2})I_1(\sigma r) - \frac{2\sigma}{r} I_0(\sigma r)\} + D_2\{(\sigma^2 + \frac{4}{r^2})K_1(\sigma r) + \frac{2\sigma}{r} K_0(\sigma r)\}]\sin\theta, \quad (27)$$

$$T_{rr(2)} = [\sigma^2 r A_2 - (\sigma^2 + \frac{4}{r^2})\frac{B_2}{r} + C_2\{\frac{2\sigma}{r} I_0(\sigma r) - \frac{4}{r^2} I_1(\sigma r)\} - D_2\{\frac{2\sigma}{r} K_0(\sigma r) + \frac{4}{r^2} K_1(\sigma r)\}]\cos\theta. \quad (28)$$

The pressures in all the three regions are given below as:

$$p^{(1)} = [8C_1 r - \frac{2}{r} D_1]\cos\theta \quad (29)$$

$$p^{(2)} = \sigma^2[-A_2 r + \frac{1}{r} B_2]\cos\theta \quad (30)$$

$$p^{(3)} = 8C_3 r\cos\theta. \quad (31)$$

Boundary Conditions:

The boundary conditions at the surfaces of the cylinder can be taken as follows:

(i) Continuity of velocity components:

At $r = 1(\tilde{r} = \tilde{b})$;

$$v_r^{(1)} = v_r^{(2)} , \; v_\theta^{(1)} = v_\theta^{(2)} \quad .(32)$$

At $r = l \; (\tilde{r} = \tilde{a})$;

$$v_r^{(2)} = v_r^{(3)} , v_\theta^{(2)} = v_\theta^{(3)} . \quad (33)$$

(ii) Continuity of stress components:

At $r = 1(\tilde{r} = \tilde{b})$;

$$T_{r\theta}^{(1)} = T_{r\theta}^{(2)} , \; T_{rr}^{(1)} = T_{rr}^{(2)} \quad (34)$$

At $r = l \; (\tilde{r} = \tilde{a})$

$$T_{r\theta}^{(2)} = T_{r\theta}^{(3)} , \; T_{rr}^{(2)} = T_{rr}^{(3)} \quad (35)$$

(iii) The boundary conditions on the outer cell boundary, $r = m \; (\tilde{r} = \tilde{c})$.

Applying the four boundary conditions, which are used by Happel's, Kuwabara's, Kvashnin's and Cunningham/Mehta-Morse's in their models on the outer cell boundary. All the four models assume continuity of the radial component of the liquid velocity on the outer cell surface $r = m \; (\tilde{r} = \tilde{c})$:

$$v_r^{(1)}(m, \theta) = \cos\theta . \quad (36)$$

According to the Happel's model, the tangential stress vanishes on the cell boundary $r = m \; (\tilde{r} = \tilde{c})$:

$$T_{r\theta}^{(1)}(m, \theta) = 0 . \quad (37)$$

According to the Kuwabara's model, the curl of velocity (vorticity) vanishes on the cell boundary $r = m \; (\tilde{r} = \tilde{c})$:

$$\nabla^2 \psi^{(1)}(m, \theta) = 0 . \quad (38)$$

According to the Kvashnin's model, on the cell boundary $r = m \; (\tilde{r} = \tilde{c})$:

$$\frac{\partial v_\theta^{(1)}}{\partial r} = 0 . \quad (39)$$

According to the Cunningham/Mehta-Morse's model, the condition on the cell boundary $r = m \; (\tilde{r} = \tilde{c})$:

$$v_\theta^{(1)}(m, \theta) = -\sin\theta . \quad (40)$$

Using these above boundary conditions (Eqs. (32)-(36)) and one from Eqs. (37)-(40), we obtained the constants $A_1, B_1, C_1, D_1, A_2, B_2, C_2, D_2, A_3$ and C_3 which are cumbersome, so they are not mentioned here.

Evaluation of drag force and hydrodynamic permeability of membrane:

On integration of the normal and tangential stresses over the porous cylindrical shell of radius $\tilde{b}$ in a cell gives the experienced drag force per unit length $\tilde{F}$ which is given below as:

$$\tilde{F} = \int_0^{2\pi} (\tilde{T}_{\tilde{r}\tilde{r}}^{(1)} \cos\theta - \tilde{T}_{\tilde{r}\theta}^{(1)} \sin\theta)_{\tilde{r}=\tilde{b}} \, \tilde{r} \, d\theta = F \tilde{\mu}_1 \tilde{U} \quad (41)$$

where, $F = \int_0^{2\pi} (T_{rr}^{(1)} \cos\theta - T_{r\theta}^{(1)} \sin\theta)_{r=1} \, d\theta$ (42)

is the force in dimensionless form.

Substituting the values from Eqs. (25) and (26) in Eq. (42) and integrating, we obtain the force as :

$$F = 4\pi D_1 . \quad (43)$$

Also, the drag coefficient C_D can be found as:

$$C_D = \frac{\tilde{F}}{(1/2)\rho\tilde{U}^2 2\tilde{b}} = \frac{4\pi D_1}{R_e}, \quad (44)$$

where $\mathrm{Re} = \frac{2\tilde{b}\tilde{U}}{\nu}$ is the Reynolds number and $\nu = \frac{\tilde{\mu}_1}{\rho}$ being the kinematic viscosity of the fluid.

Hydrodynamic permeability of a membrane is defined as the ratio of the uniform flow rate $\tilde{U}$ to the cell gradient pressure $\tilde{F}/\tilde{V}$:

$$\tilde{L}_{11} = \frac{\tilde{U}}{\tilde{F}/\tilde{V}} \quad (45)$$

where $\tilde{V} = \pi\tilde{c}^2$ is the volume of the cell per unit length.

Substituting the value of $\tilde{F}$, using Eq. (43) in Eq. (41) and value of $\tilde{V}$ from above in Eq. (45), we can find as

$$\tilde{L}_{11} = \frac{\tilde{b}^2}{4\gamma\tilde{\mu}_1 D_1} = L_{11}\frac{\tilde{b}^2}{\tilde{\mu}_1} \quad (46)$$

where $L_{11} = \frac{1}{4\gamma D_1}$ is the dimensionless hydrodynamic permeability of a membrane.

RESULTS AND DISCUSSION

In this section, we discuss the variation of hydrodynamic permeability of a membrane and $\mathrm{Re}C_D$ on permeability parameter and particle volume fraction for all four cell models. From Fig.-2, it is seen that $\mathrm{Re}C_D$ with particle volume fraction γ for the value of $l = 0.5$ and $\sigma = 25$, increases slightly up to $\gamma < 0.3$ and then increases rapidly with γ for all four models.

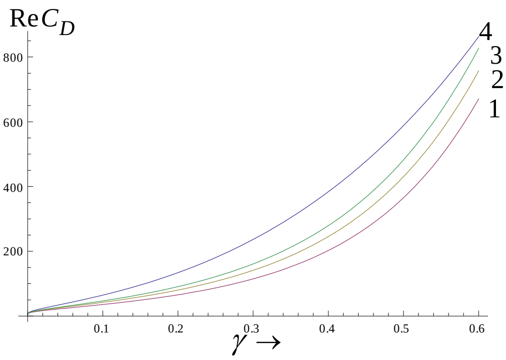

Fig. 2. Variation of $\mathrm{ReC_D}$ with particle volume fraction γ for l = 0.5 an σ = 25 for different models: 1-Happel, 2Kvashnin, 3-Kuwabara and 4-Cuningham/Mehta Morse.

Fig.-3 shows the variation of $\mathrm{Re}C_D$ with permeability parameter σ at $l = 0.5$ and $\gamma = 0.4$ for the different models, Happel's, Kvashnin's, Kuwabara's and Cunningham/Mehta-Morse's. We observe that $\mathrm{Re}C_D$ increases with increasing permeability parameter σ .

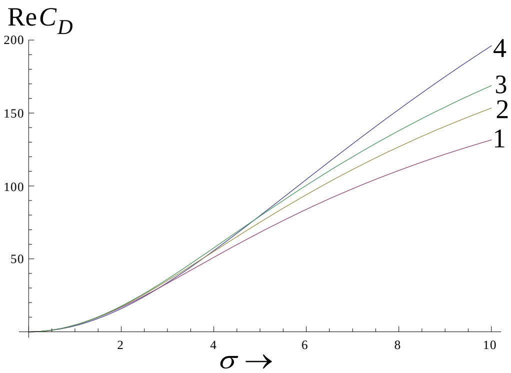

Fig. 3. Variation of $\mathrm{ReC_D}$ with permeability parameter σ for *l = 0.5* and γ = 0.4 for different models: 1-Happel, 2-Kvashnin, 3-Kuwabara and 4-Cuningham/Mehta-Morse.

For the low value of permeability parameter σ ($\sigma < 3$), $\mathrm{Re}C_D$ increases slowly for all models, and after $\sigma > 3$ the value of $\mathrm{Re}C_D$ increases rapidly with σ . It is seen that the effect of is to reduce the drag on the porous cylinder i.e. for highly permeable porous shells of the particles the drag on the porous cylinder is lower. In both the Figs. 2 and 3, the value of $\mathrm{Re}C_D$ is highest for Mehta-Morse's model and lowest for Happel's model.

On analyzing the effect of dimensionless permeability of the membrane with particle volume fraction γ from Figs.-4 and 5, we observe that the hydrodynamic permeability at low particle volume fraction, all four cell models agree. At $\gamma \rightarrow 0$, L_{11} increases unboundedly and for $\gamma \rightarrow 1$ hydrodynamic permeability tends to zero. In this case the hydrodynamic permeability is slightly higher for Happel's model and lower for Cunningham/Meta-Morse's model. Other models show a similar variation with particle volume fraction.

Figs.-6 and 7 show that the hydrodynamic permeability decreases with σ, i.e. for highly permeable porous shells of the particles, the hydrodynamic permeability of the membrane is higher. The hydrodynamic permeability is highest for Happel's model and lowest for Cunningham/Meta-Morse's model. The dependence of hydrodynamic permeability for a porous medium built by cylindrical particles matches the earlier results reported in [11].

There are many physical situations in which the flow of a viscous fluid through a swarm of porous cylindrical particles arises, such as fluidization flow in packed bed, filtration, in petroleum reservoirs, etc. Hence the results of this paper are applicable to study the membrane filtration problemsor flow of fluids through a sandy or earthen soil (like bank of rivers).

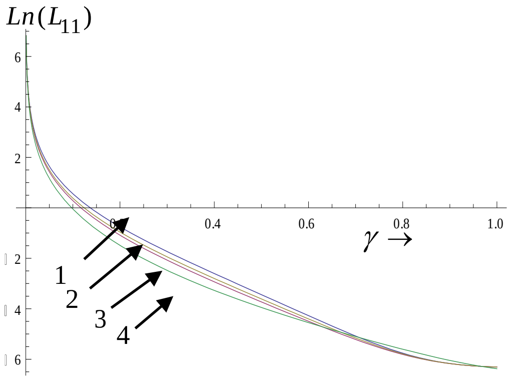

Fig. 4. Variation of natural logarithm dimensionless hydrodynamic permeability with cylindrical particle volume fraction γ for $l = 0.5$ and $\sigma = 30$ for different models: 1-Happel, 2-Kvashnin, 3-Kuwabara and 4-Cuningham/Mehta-Morse.

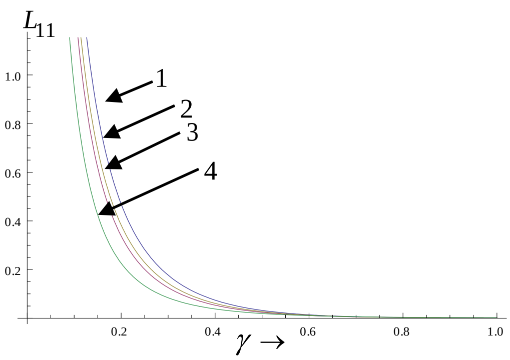

Fig. 5. Variation of dimensionless hydrodynamic permeability with cylindrical particle volume fraction γ for $l = 0.5$ and $\sigma = 30$ for different models: 1-Happel, 2-Kvashnin, 3-Kuwabara and 4-Cuningham/Mehta-Morse.

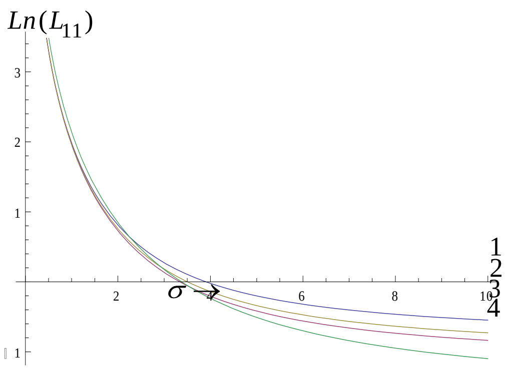

Fig. 6. Variation of natural logarithm dimensionless hydrodynamic permeability with σ for $l = 0.5$ and $\gamma = 0.2$ for different models: 1-Happel, 2-Kvashnin, 3-Kuwabara and 4-Cunningham/Mehta-Morse.

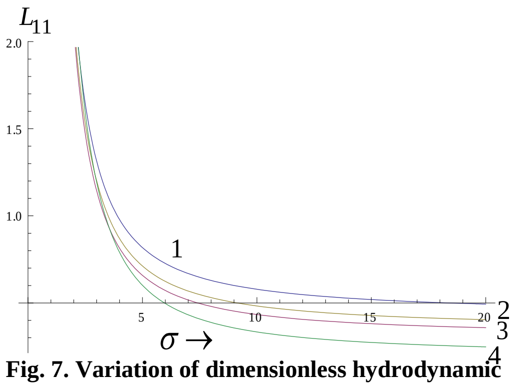

Fig. 7. Variation of dimensionless hydrodynamic permeability with σ for $l = 0.5$ and $\gamma = 0.2$ for different models: 1-Happel, 2-Kvashnin, 3-Kuwabara and 4-Cunningham/Mehta-Morse.

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