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MHD Homogeneous-Heterogeneous Reactions in a Nanofluid due to a Permeable Shrinking Surface

S. Mansur[1], A. Ishak[2†] and I. Pop[3]

[1]*Department of Mathematics and Statistics, Faculty of Science, Technology and Human Development, Universiti Tun Hussein Onn Malaysia, 86400 Parit Raja, Batu Pahat, Johor, Malaysia*
[2]*Centre for Modelling and Data Analysis, School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia*
[3]*Department of Mathematics, Babe-Bolyai University, 400084 Cluj-Napoca, Romania*

†*Corresponding Author Email: anuar_mi@ukm.edu.my*

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ABSTRACT

The MHD homogeneous-heterogeneous reaction in a nanofluid flow due to a permeable shrinking surface is studied. The bvp4c program in MATLAB is used to obtain the numerical solutions for several values of parameters such as suction parameter, magnetic parameter, nanoparticle volume fraction, heterogeneous reaction and homogeneous reaction rates. The results show that dual solutions exist and the magnetic parameter and the nanoparticle volume fraction widen the range of the solution domain. Suction parameter, magnetic parameter and nanoparticle volume fraction cause the skin friction coefficient to increase and the velocity to decrease. The concentration increases as the nanoparticle volume fraction increases but decrease as the homogeneous reaction rate and heterogeneous reaction rate increase.

Keywords: Magnetohydrodinamic; Homogeneous-heterogeneous reaction; Nanofluid; Shrinking sheet; Fluid mechanics.

NOMENCLATURE

a, b	concentrations of the chemical species
B_0	magnetic field strength
C_f	skin friction coefficient
c	constant
D_A, D_B	diffusion coefficients
f	dimensionless stream function
K	strength of the homogeneous reaction
k_c, k_s	constants
M	magnetic parameter
Re_x	local Reynolds number
S_c	Schmidt number
s	mass flux parameter
u, v	velocity components along the $x-$ and $y-$ directions, respectively
u_w	shrinking velocity
v_0	mass flux velocity
x, y	Cartesian coordinates along the surface and normal to it, respectively
δ	ratio of the diffusion coefficient
η	similarity variable
μ	dynamic viscosity
ν	kinematic viscosity
ρ	fluid density
σ	electric conductivity
τ_w	surface shear stress
φ	nanoparticle volume fraction
ψ	stream function

1. INTRODUCTION

Nanofluids are produced by dispersing nanometer-sized particles into the base fluids such as water, ethylene glycol and propylene glycol, which increase their thermal conductivities. The main characteristic of this fluid is the significant enhancement of the thermal properties of the base fluid: minimal clogging in flow passage and long term stability and homogeneity compared to those fluids containing micro- or milli-sized particles (Rahman *et al*. 2012, Xuan and Li 2000; Xuan and Roetzel 2000; Lee *et al*. 1999; Masuda *et al*. 1993). Tiwari and Das (2007) developed a model to analyze the behavior of nanofluids taking into account the solid volume fraction.

Many chemically reacting systems involve both homogeneous and heterogeneous reactions, such as in combustion, catalysis and biochemical systems. The interaction between the reactions is generally very complex due to the production and consumption of reactant species both within the fluid and on the catalytic surface occurring at different rates. Chambre and Acrivos (1956) studied an isothermal chemical reaction on a catalytic reactor in laminar boundary layer flow where they found the actual surface concentration without introducing unnecessary assumptions related to the reaction mechanism. Chaudhary and Merkin (1995a, b) constructed a simple isothermal model for homogeneous-heterogeneous reactions in boundary layer flow for both equal and different diffusivities for reactant and autocatalyst. The same model was then used by Merkin (1996) to study the flow over a flat surface. Khan and Pop (2010) studied the stagnation-point flow on an infinite permeable wall with a homogeneous and heterogeneous reaction. Bachok et al. (2011) and Kameswaran *et al*. (2013) studied the effect of homogeneous-heterogeneous reactions on a stretching sheet.

The boundary layer flow over a stretching sheet is significant in applications such as plastic extrusion, wire drawing and hot rolling (Fischer 1976). However, to complement the study of flow over a stretching sheet, Miklavčič and Wang (2006) studied the flow over a shrinking sheet in which they observed that the vorticity is not confined within a boundary layer and a steady flow cannot exist without exerting adequate suction at the boundary. As the studies of shrinking sheet garner considerable attention, this finding proves to be crucial to these researches. In response to Miklavčič and Wang (2006), numerous studies on these problems have been conducted by researches, such as Wang (2008), Fang *et al*. (2010), Bachok *et al*. (2011), Bhattacharyya *et al*. (2011), Zaimi *et al*. (2012), Roşca and Pop (2013), Sharma *et al*. (2014) and Mahapatra and Nandy (2014), among others. In addition, it should be mentioned that only several papers were published on the MHD flow and heat transfer in a nanofluid due to a stretching/shrinking surface. Thus, we mention here the papers by Turkyilmazoglu (2012), Sheikholeslami *et al*. (2012), Mansur *et al*. (2015) and Haroun *et al*. (2015).

Based on the above-mentioned literatures, the objective of this paper is to study the magneto hydrodynamic (MHD) homogeneous-heterogeneous reactions in a nanofluid due to a permeable shrinking surface which has not been studied before. MHD is the study of the dynamics of electrically conducting fluids such as plasmas, liquid metals and electrolytes which conforms to the nature of nanofluids. Numerical solutions are found to document the effects of magnetic parameter, suction parameter and nanoparticle volumetric fraction on the skin friction coefficient which will affect the concentration rate. To the best of our knowledge, the MHD homogeneous-heterogeneous reactions in a nanofluid have not been given any attention in the past. Therefore we focus to introduce the homogeneous-heterogeneous reactions in a nanofluid in the present paper. The results of this paper are, therefore, new and original.

2. BASIC EQUATIONS

Consider a steady MHD two-dimensional boundary layer flow of an incompressible nanofluid over a permeable shrinking sheet. A Cartesian coordinate system is used with the $x-$ axis along the sheet and the $y-$ axis normal to it. The fluid is a water based nanofluid containing copper (Cu) nanoparticles. The base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. It is assumed that a constant magnetic field of strength B_0 is applied perpendicular to the plate. It is also assumed that a simple homogeneous-heterogeneous reaction model exists as proposed by Chaudhary and Merkin (1995) in the following form:

$$A + 2B \to 3B, \quad rate \quad = k_c\, a\, b^2 \tag{1}$$

while on the catalyst surface we have the single, isothermal, first order reaction

$$A \to B, \quad rate = k_s\, a \tag{2}$$

where a and b are the concentrations of the chemical species A and B, k_c and k_s are the rate constants. We assume that both reaction processes are isothermal. Under these assumptions, the boundary layer equations governing the flow can be written as (Tiwari and Das 2007; Chaudhary and Merkin 1995),

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \frac{1}{\rho_{nf}}\left(\mu_{nf}\,\frac{\partial^2 u}{\partial y^2} - \sigma\, B_0^2\, u\right) \tag{4}$$

$$u\,\frac{\partial a}{\partial x} + v\,\frac{\partial a}{\partial y} = D_A\,\frac{\partial^2 a}{\partial y^2} - k_c\, a\, b^2 \tag{5}$$

$$u\,\frac{\partial b}{\partial x} + v\,\frac{\partial b}{\partial y} = D_B\,\frac{\partial^2 b}{\partial y^2} + k_c\, a\, b^2 \tag{6}$$

The boundary conditions for Eqs. (3) - (6) are given in the form

$$\begin{aligned} &v = v_w = v_0, \quad u = u_w = -c\,x, \\ &D_A\,\frac{\partial a}{\partial y} = k_s\, a, \quad D_B\,\frac{\partial b}{\partial y} = -k_s\, a \quad \text{at} \quad y = 0 \\ &u \to 0, \quad a \to a_0, \quad b \to 0 \quad \text{as} \quad y \to \infty \end{aligned} \tag{7}$$

where x and y are Cartesian coordinates measured along the surface and normal to it, respectively, u and v are the velocity components along the x and y axes, σ is the electric conductivity, D_A and D_B are respective diffusion coefficients and, c and a_0 area positive constants.

The quantity $v_0 < 0$ represents suction and $v_0 > 0$ represents injection. Further $\mu_{\rm nf}$ is the dynamic viscosity of the nanofluid and $\rho_{\rm nf}$ is the density of the nanofluid, which are given by (Oztop and Abu-Nada 2008)

$$\mu_{\rm nf} = \frac{\mu_{\rm f}}{(1\text{-}\varphi)^{2.5}}, \quad \rho_{\rm nf} = (1-\varphi)\rho_{\rm f} + \varphi\rho_{\rm s}, \tag{8}$$

where φ is the nanoparticle volume fraction, and $\rho_{\rm f}$ and $\rho_{\rm s}$ are the densities of the fluid and of the solid fractions, respectively.

We define now the following similarity variables

$$\begin{aligned} &\eta = (c/\nu_f)^{1/2} y, \quad \psi = (c\nu_f)^{1/2} x f(\eta), \\ &a = a_0 g(\eta), \quad b = a_0 h(\eta) \end{aligned} \tag{9}$$

where ψ is the stream function, which is defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which satisfies the continuity Eq. (3). Substituting (9) into Eqs. (4), (5) and (6), we obtain the following ordinary differential equations:

$$\frac{1}{(1-\varphi)^{2.5}} f''' + (1-\varphi+\varphi(\rho_s/\rho_f))(f f'' - f'^2) - M f' = 0 \tag{10}$$

$$\frac{1}{Sc} g'' + f g' - K g h^2 = 0 \tag{11}$$

$$\frac{\delta}{Sc} h'' + f h' + K g h^2 = 0 \tag{12}$$

and the boundary conditions (7) become

$$\begin{aligned} &f(0) = S, \quad f'(0) = -1, \quad g'(0) = K_s g(0), \\ &\delta h'(0) = -K_s g(0) \\ &f'(\eta) \to 0, \quad g(\eta) \to 1, \quad h(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{13}$$

The non-dimensional constants in equations (10) - (13) are the magnetic parameter M, the Schmidt number Sc, the measure of the strength of the homogeneous reaction K, the ratio of the diffusion coefficient δ and the mass flux parameter S, which are defined as

$$\begin{aligned} &M = \frac{\sigma B_0^2}{c\rho_f}, \quad Sc = \frac{\nu_f}{D_A}, \quad K = \frac{K_c a_0^2}{c}, \\ &\delta = \frac{D_B}{D_A}, \quad S = -\frac{v_0}{(c\nu_f)^{1/2}} \end{aligned} \tag{14}$$

where $s > 0$ for suction and $s < 0$ for injection or withdrawal of the fluid.

In most applications, we expect the diffusion coefficients of chemical species A and B to be of a comparable size. This leads us to make further assumption that the diffusion coefficients D_A and D_B are equal, i.e., to take $\delta = 1$ (Chaudhary and Merkin 1995a). In this case we have from (13)

$$g(\eta) + h(\eta) = 1 \tag{15}$$

Thus, Eqs. (11) and (12) reduce to

$$\frac{1}{Sc} g'' + f g' - K g(1-g)^2 = 0 \tag{16}$$

and is subjected to the boundary conditions

$$g'(0) = K_s g(0), \quad g(\eta) \to 1 \quad \text{as} \quad \eta \to \infty \tag{17}$$

The physical quantity of interest is the skin friction or shear stress coefficient C_f, which is defined as

$$C_f = \frac{\tau_w}{\rho_f u_w^2} \tag{18}$$

where τ_w is the shear stress along the surface of the sheet and is given by

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{19}$$

Using (9), (18) and (19), we obtain

$$\mathrm{Re}_x^{1/2} C_f = \frac{1}{(1-\varphi)^{2.5}} f''(0) \tag{20}$$

where $\mathrm{Re}_x = u_w(x)\,x/\nu_f$ is the local Reynolds number.

3. RESULTS AND DISCUSSION

MHD homogeneous-heterogeneous reaction in a nanofluid due to a permeable shrinking surface is studied by considering copper-water nanofluid. The system of ordinary differential equations (10) and (16) subject to the boundary conditions (13) and (17) were solved numerically using bvp4c solver in MATLAB for some values of physical parameters *S, K, K_s, M* and φ. The values of the density of copper ρ_s and water ρ_f are taken to be equal to 8933 and 997.1, respectively (Oztop and Abu-Nada 2008). A comparison of the obtained results with those reported by Fang and Zhang (2009) and Sajid and Hayat (2009) is shown in Table 1, which shows a very good agreement, so that we are confident that the present results are accurate.

Table 1 Comparison of the values of $f''(0)$ with previously published results

φ	M	S	Sajid and Hayat (2009)	Fang and Zhang (2009)	Present Results
0	0	2	-	1.00000	0.99980
		3	-	2.61803	2.61803
		4	-	3.73205	3.73205
	4	1	2.30277	2.30277	2.30277
		2	-	3.00000	3.00000
		3	-	3.79128	3.79128
0.1	0	2	-	-	2.10421
		3	-	-	3.77446
		4	-	-	5.25763
	4	1	-	-	2.16304
		2	-	-	3.27691
		3	-	-	4.51519

Figures 1 and 2 illustrate the variation of the reduced skin friction coefficient $C_f \, \mathrm{Re}_x^{1/2}$ with the suction parameter S for different values of the magnetic parameter M when the nanoparticle volume fraction parameter $\phi = 0.1$ (Fig. 1) and the variation of $C_f \, \mathrm{Re}_x^{1/2}$ with S for different values of ϕ when $M = 0.1$ (Fig. 2). These figures show that there are double (first and second) solutions for Eqs. (10) and (11) with the boundary conditions (12) when the value of S varies in the range $S_c < S$, where S_c is the critical value of S for which Eqs. (10) and (11) with the boundary (12) have solutions. When $S = S_c$ there is only one solution, and when $S < S_c$, no solution exists. In previous studies for a viscous fluid and a power-law fluid, Merkin (1985), Weidman *et al*. (2006), Postelnicu and Pop (2011) and Rosca and Pop (2013a, b) have shown that the first solution is stable and physically realizable, while the second solution is not. Therefore, in this study, it is expected that only the first solutions are physically relevant. However, the second solutions are still of mathematical interest since they are also solutions to the differential equations. Due to space limitation, we have not described here the stability analysis, which show the physically realizable of the dual solutions of the boundary value problem (10 to 12).

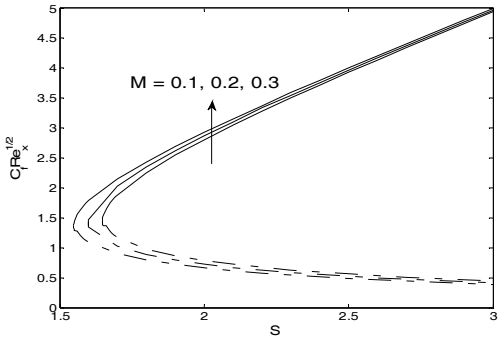

Fig. 1. Variation of the skin friction coefficient with *S* for different values of *M* when *φ* = 0.1.

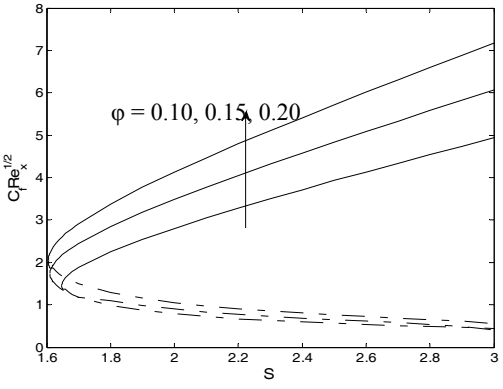

Fig. 2. Variation of the skin friction coefficient with *S* for different values of *φ* when *M* = 0.1.

It can be observed from Fig. 1 that the skin friction coefficient is consistently higher for higher values of S and M. In Fig. 2, it is seen that with increasing value of φ, the skin friction coefficient increases. From the figures, the changes observed as the nanoparticle volume fraction increases is greater than the changes occurring by increasing the magnetic parameter M, implying that nanofluid volume fraction affects the skin friction coefficient more than the magnetic parameter. Therefore, an increase in M leads to an increase of $C_f \, \mathrm{Re}_x^{1/2}$ for the first solution and a decrease in $C_f \, \mathrm{Re}_x^{1/2}$ for the second solution, respectively, as can be seen from Fig. 1.

Table 2 Values of S_c for different values of M and φ

M	φ	S_c
0.1	0.10	1.6479
0.2		1.5990
0.3		1.5478
0.1	0.15	1.6109
	0.20	1.6049

Furthermore, it is also seen that as the effect of suction increase, the skin friction coefficient for all values of the magnetic parameter seems to converge. On the other hand, the skin friction coefficient for all values of the nanoparticle volume fraction diverges as suction increases. It is also interesting to note that the range of solutions widen by increasing M and φ. The values of S_c are shown in Table 2 for different values of M and φ.

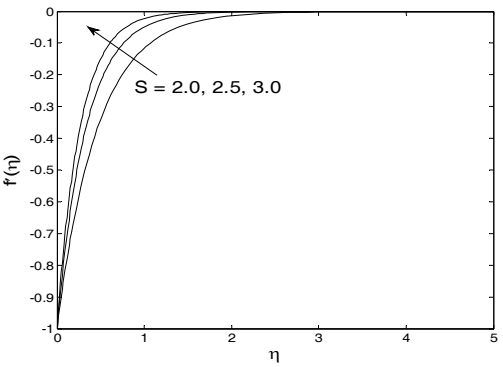

Fig. 3. Velocity profiles for different values of *S* when *M* = *φ* = 0.1.

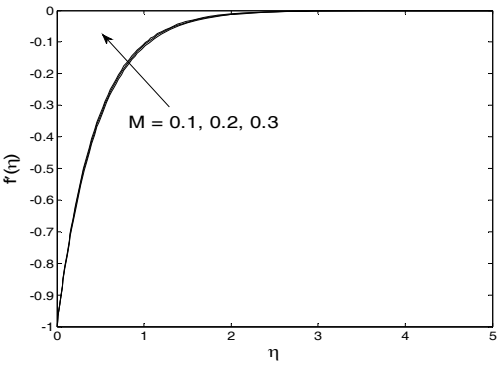

Fig. 4. Velocity profiles for different values of *M* when *S* = 2 and *φ* = 0.1.

Figs. 3 – 8 show the velocity and concentration

profiles are influenced by the changing of the values of S, φ, K and K_s. These profiles satisfy the far field boundary conditions (13) and (17) asymptotically which validates the numerical results obtained. In Figs. 3 and 4, the effects of suction and magnetic parameters on the velocity are shown. The velocity inside the boundary layer decreases (in absolute sense) as the effect of suction at the boundary increases. Furthermore, the magnitude of velocity also decreases as the magnetic parameter rises. This is due to the retarding force that occurs as the presence of transverse magnetic field sets in the Lorenz force effect (Ibrahim and Shankar 2013). The velocity curves presented in Fig. 4 show that the rate of transport is considerably reduced with the increase of M. It clearly indicates that the transverse magnetic field opposes the transport phenomena. This is due to the fact that the variation of M leads to the variation of the Lorentz force and the Lorentz force produces more resistance to the transport phenomena (Ishak *et al*. 2008). This implies an increasing manner of the velocity gradient at the surface, thus increase the skin friction coefficient as presented in Fig. 1.

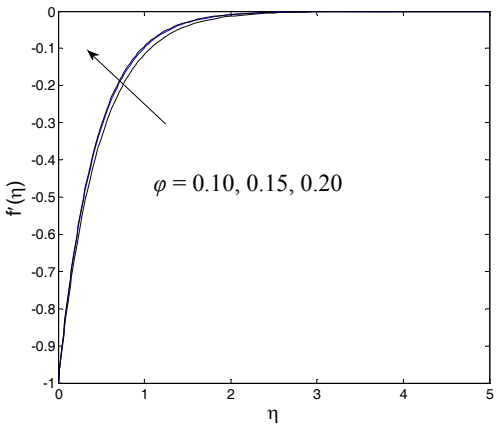

Fig. 5. Velocity profiles for different values of φ when S = 2 and M = 0.1.

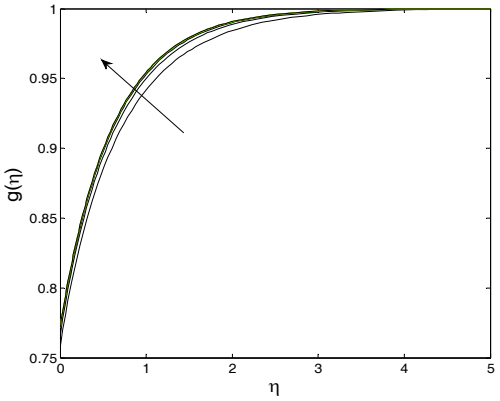

Fig. 6. Concentration profiles for different values of φ when S = 2, M = 0.1 and K = K_s = Sc =1.

Figs. 5 and 6 show the effects of φ on both velocity and concentration, respectively. As expected from the physical behavior of nanofluid, the increase in nanoparticle volume fraction φ causes the velocity $|f'(\eta)|$ to decrease and concentration to increase. The effects of heterogeneous reaction and homogeneous reaction on concentration are shown in Figs. 7 and 8, respectively. The increase in both heterogeneous reaction rate K_s and homogeneous reaction rate K lowers the concentration. This phenomenon concurs with previous observations made by Bachok *et al*. (2011) and Khan and Pop (2010). However, it is worth mentioning that the changes occurring due to difference values of K_s are greater than those of K.

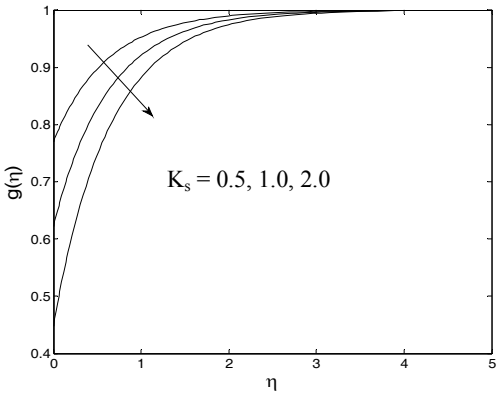

Fig. 7. Concentration profiles for different values of K_s .when S = 2, M = φ = 0.1 and K = Sc = 1.

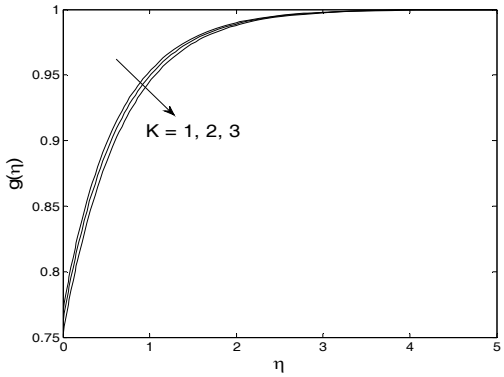

Fig. 8. Concentration profiles for different values of K when S = 2, M = φ = 0.1 and K_s = Sc = 1.

4. CONCLUSIONS

The MHD homogeneous-heterogeneous reaction in a nanofluid flow due to a permeable shrinking surface is studied. Numerical solutions were obtained using bvp4c in MATLAB. The effects of several parameters, namely suction parameter, magnetic parameter, nanoparticle volume fraction, heterogeneous reaction and homogeneous reaction, on the skin friction coefficient, flow velocity and concentration were observed. The following conclusions can be drawn from the solution of this problem:

1. Dual solutions exist and the range of the solution domain widen as magnetic parameter and nanoparticle volume fraction increase.
2. Suction, magnetic parameter and nanoparticle volume fraction cause the skin friction coefficient to increase but the magnitude of velocity to decrease.
3. The concentration increases as the nanoparticle

volume fraction increases but decrease as the homogeneous reaction rate and heterogeneous reaction rate increase.

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