



# Radiation Effect on Mixed Convection Boundary Layer Flow of a Viscoelastic Fluid over a Horizontal Circular Cylinder with Constant Heat Flux

H. Ahmad<sup>†</sup>, T. Javed and A. Ghaffari

*Department of Mathematics and Statistics, FBAS, International Islamic University, Islamabad 44000, Pakistan*

<sup>†</sup>Corresponding Author Email: [hussainahmad686@yahoo.com](mailto:hussainahmad686@yahoo.com)

(Received November 8, 2014; accepted July 4, 2015)

## ABSTRACT

In the present article, radiation effect on mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder with constant heat flux has been numerically analyzed. The governing boundary layer equations are transformed to dimensionless nonlinear partial differential equations. The equations are solved numerically by using Keller-box method. The computed results are in excellent agreement with the previous studies. Skin friction coefficient and Nusselt number are emphasized specifically. These quantities are displayed against the curvature parameter. The effects of pertinent parameters involved in the problem namely effective Prandtl number and mixed convection parameter on skin friction coefficient and Nusselt number are shown through graphs and table. Boundary layer separation points are also calculated with and without radiation and a comparison is shown. The presence of radiation helps to decrease or increase the skin friction coefficient for the negative or positive values of the mixed convection parameter accordingly. The decrease in value of effective Prandtl number helps to increase the value of skin friction coefficient and Nusselt number for viscoelastic fluids.

**Keywords:** Mixed convection; Boundary layer flow; Thermal radiation; Effective Prandtl number; Numerical solution.

## 1. INTRODUCTION

Mixed convection flows over horizontal circular cylinders are very important in the circumstances often encountered in the cases of geothermal power generation and drilling operation when the free stream velocity and induced buoyancy velocity are of comparable order. It becomes one of the most important problems due to its fundamental nature as well as many engineering applications. A literature survey reveals that Merkin (1977) was the first one who gave comprehensive analysis of mixed convection boundary layer flow over a horizontal circular cylinder. He investigated boundary layer separation point along surface of the cylinder. Later on Badr (1983) studied mixed convection heat transfer from an isothermal horizontal circular cylinder and Nazar *et al.* (2004) studied mixed convection boundary layer flow from a horizontal circular cylinder with constant surface heat flux. Recently, Bhuiyan *et al.* (2014) analyzed Joule heating effects on MHD natural convection flows in presence of pressure stress work and viscous

dissipation from a horizontal circular cylinder.

In recent years, the flow of viscoelastic fluids has gained considerable interest due to its applications in engineering and several manufacturing processes e.g., petroleum drilling, manufacturing of food, paper, paints, coating, inks and jet fuels etc. The viscoelastic fluid is of second grade nature. A comprehensive discussion on second and third order fluid was done by Dunn and Rajagopal (1995). Ariel (1995), Rajagopal *et al.* (1986) and Rajagopal (1986) also studied viscoelastic fluids in different geometries. It is very necessary to mention the work done by Cortell (2006), Abel *et al.* (2002), Hayat *et al.* (2008) and Sajid *et al.* (2010) on second grade fluids. Anwar *et al.* (2008) investigated the steady mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder with constant surface temperature.

The study of convective heat transfer with thermal radiation has great importance especially in the processes involving high temperature such as gas turbines, nuclear power plants and thermal energy

storage etc. Hossain and Thakar (1996) discussed the thermal radiation effects using the Rosseland diffusion approximation on mixed convection along vertical plate with uniform free stream velocity and surface temperature. Hossain *et al.* (1999) investigated the thermal radiation of a gray fluid which is emitting and absorbing radiation in a non-scattering medium. Raptis *et al.* (2004) discussed radiative flow in the presence of a magnetic field. Hayat *et al.* (2007) studied the influence of thermal radiation on MHD flow of a second grade fluid. Sajid and Hayat (2008) investigated the influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. Mukhopadhyay (2009) discussed the effect of radiation and variable fluid viscosity on flow and heat transfer along a symmetric wedge. Prasad *et al.* (2013) considered the problem of MHD flow and heat transfer in a power law liquid film at a porous surface in presence of thermal radiation. Boundary layer flow and heat transfer over a permeable exponentially shrinking sheet in presence of thermal radiation was investigated by Sharma *et al.* (2014) by taking partial slip conditions into account. Shit and Majee (2014) considered hydromagnetic flow over an inclined non-linear stretching sheet with variable viscosity in presence of thermal radiation and chemical reaction. Thermal dispersion-radiation and melting effects on mixed convection flow from vertical plate embedded in non-Newtonian fluid saturated non-Darcy porous medium was investigated by Prasad *et al.* (2014). Very recently, Choudhury and Das (2014) studied viscoelastic MHD free convective flow through porous media in presence of radiation and chemical reaction with heat transfer. Radiation effect on natural convection laminar flow from a horizontal circular cylinder was by Molla *et al.* (2011). He investigated that due to increase in radiation velocity and thermal boundary layer thickness increases. From the available literature, it appears that radiation effects on mixed convection flow of a viscoelastic fluids over a horizontal circular cylinder with constant heat flux has not yet been considered in literature.

In present study, we investigate the thermal radiation effects on mixed convection boundary layer of a viscoelastic fluid over a horizontal circular cylinder considering the Rosseland diffusion approximation which is extension of the work by Kasim *et al.* (2013). The governing equations are transformed into convenient forms which are solved numerically by an efficient finite difference scheme named Keller-box method. The results are compared with those reported by Nazar *et al.* (2004) by taking  $K = 0$  (Newtonian case) and  $Pr_{eff} = 1$  (without radiation) and hence found excellently matched.

## 2. MATHEMATICAL FORMULATION

We consider the laminar, incompressible mixed convection flow of a viscoelastic fluid past a horizontal circular cylinder with constant heat flux

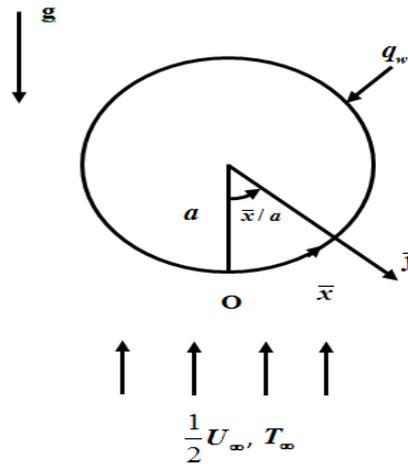


Fig. 1. The physical model of the problem.

in presence of heat radiation. The radius of circular cylinder be  $a$  and it is maintained at a constant surface heat flux  $q_w$ . The physical model is shown in Fig.1. It is also assumed that the cylinder is kept in a flow of constant free stream velocity  $\frac{1}{2}U_\infty$  (Merkin (1977) which is vertically upward so that the free stream velocity for the boundary layer is  $\bar{u}_e(\bar{x}) = U_\infty \sin\left(\frac{\bar{x}}{a}\right)$ , and a constant free stream temperature be  $T_\infty$ . Here  $q_w > 0$  and  $q_w < 0$  correspond to assisting flow and opposing flow cases respectively. The length of circular cylinder is considered long enough to neglect end point effects so that the flow field is assumed two dimensional. With these assumptions and Boussinesq and boundary layer approximations, the basic equations governing the flow are

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{1}$$

$$-\bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} = \bar{u}_e \frac{d\bar{u}_e}{dx} + \bar{v} \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{k_o}{\rho} \left[ \frac{\partial \bar{u}}{\partial x} \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial \bar{u}}{\partial x} \frac{\partial^3 \bar{u}}{\partial x \partial y^2} + \bar{v} \frac{\partial^3 \bar{u}}{\partial y^3} - \frac{\partial \bar{u}}{\partial y} \frac{\partial^2 \bar{u}}{\partial x \partial y} \right] + g \beta (T - T_\infty) \sin\left(\frac{\bar{x}}{a}\right), \tag{2}$$

$$\rho C_p \left( \bar{u} \frac{\partial T}{\partial x} + \bar{v} \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{3}$$

where  $\bar{x}$  and  $\bar{y}$  are the Cartesian coordinates measured along and normal to the surface of the cylinder respectively. Here  $\bar{x}$  is measured from the lower stagnation point of the cylinder,  $\bar{u}$  and  $\bar{v}$  are components of velocity in the  $\bar{x}$  and  $\bar{y}$  directions respectively,  $\nu$  is kinematic viscosity,  $\rho$  is density of the fluid,  $T$  is temperature,  $C_p$  is the

specific heat at constant pressure,  $k$  is thermal conductivity of the fluid and  $k_0$  is the viscoelastic material parameter. The third term on the right hand side of Eq. (2) represents the viscoelastic behavior of the fluid. The case  $k_0=0$  corresponds to Newtonian fluid,  $g$  is acceleration due to gravity,  $q_r$  is radiative heat flux,  $\beta$  is thermal expansion coefficient. The negative sign with the second term on right hand side of the Eq. (3) shows net radiative heat flux leaving the control volume. To fulfill thermodynamics requirements as suggested by Dunn and Rajagopal (1995),  $k_0$  is considered positive. The radiative heat flux  $q_r$  is simplified by the Rosseland diffusion approximation [see S. Rosseland (1936) and Magyari and Pantokratoras (2011)] as

$$q_r = -\frac{4\sigma}{3a_R} \frac{\partial T^4}{\partial y} \tag{4}$$

where  $\sigma$  is the Stefan-Boltzmann constant,  $a_R$  is the Rosseland mean absorption coefficient. For a boundary layer flow over a hot surface, Eq. (4) of the net radiation heat flux absorbed in the fluid [see Magyari and Pantokratoras (2011)] reduces to

$$q_r = -\frac{16\sigma T^3}{3a_R} \frac{\partial T}{\partial y} \tag{5}$$

Using Eq. (5) in Eq. (3) we get

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{eff} \frac{\partial^2 T}{\partial y^2} \tag{6}$$

where  $k_{eff}$  is effective thermal conductivity and is defined as

$$k_{eff} = k + \frac{16\sigma T^3}{3a_R} = k_{cond} + k_{rad} \tag{7}$$

Let us assumed that the fluid-phase temperature differences within the flow is sufficiently small as reported by Raptis *et al.* (2004) so that the linearization about the ambient temperature  $T_\infty$  reduces Eq. (5) as

$$q_r = -\frac{16\sigma T_\infty^3}{3a_R} \frac{\partial T}{\partial y} \tag{8}$$

It is worth mentioning here that the use of the Rosseland diffusion approximation is valid in the interior of a medium but it is not employed near the boundaries. It is good only for an optically thick boundary layer. Since the expression in Eq. (8) does not contain any term for the radiation from the boundary surface, therefore, is not valid to predict a complete description of this physical situation near the surface. In other words, the boundary surface effects are negligible in the interior of an optically thick boundary layer region, which is due to the fact that the

radiation from the boundaries becomes very weak before reaching the interior [see Molla *et al.* (2011)].

Since the governing equations for viscoelastic fluid are one order higher than those of Newtonian fluids, therefore we need an extra boundary condition  $\frac{\partial \bar{u}}{\partial \bar{y}} \rightarrow 0$  as  $\bar{y} \rightarrow \infty$ , as suggested by Garg and Rajagopal (1990) to solve partial differential Eqs. (1,2) and (6) numerically. The boundary conditions for the considered problem are given by

$$\left. \begin{aligned} \bar{u} = 0, \bar{v} = 0, \frac{\partial T}{\partial \bar{y}} = -\frac{q_w}{k_{eff}} \\ \text{at } \bar{y} = 0, \bar{x} \geq 0, \\ \bar{u} \rightarrow \bar{u}_e(\bar{x}), \frac{\partial \bar{u}}{\partial \bar{y}} \rightarrow 0, T \rightarrow T_\infty \\ \text{as } \bar{y} \rightarrow \infty, \bar{x} \geq 0. \end{aligned} \right\} \tag{9}$$

The non-dimensional variables are introduced as follows:

$$\left. \begin{aligned} \bar{x} = \frac{x}{a}, \bar{y} = \text{Re}^{\frac{1}{2}} (y/a), \bar{u} = u/U_\infty, \\ \bar{v} = \text{Re}^{\frac{1}{2}} (v/U_\infty), \bar{u}_e(\bar{x}) = \frac{u_e(\bar{x})}{U_\infty}, \\ \theta = \frac{\text{Re}^{\frac{1}{2}} k_{eff}}{a q_w} (T - T_\infty) \end{aligned} \right\} \tag{10}$$

where  $\text{Re} = aU_\infty/\nu$  is the Reynolds number. After substituting Eq. (10), Eqs. (1,2) and (6) go over in

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{11}$$

$$\begin{aligned} u \frac{\partial \bar{u}}{\partial \bar{x}} + v \frac{\partial \bar{u}}{\partial \bar{y}} = u_e \frac{d\bar{u}_e}{d\bar{x}} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + K \left[ \frac{\partial \bar{u}}{\partial \bar{x}} \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \right. \\ \left. u \frac{\partial^3 \bar{u}}{\partial \bar{x} \partial \bar{y}^2} + v \frac{\partial^3 \bar{u}}{\partial \bar{y}^3} - \frac{\partial \bar{u}}{\partial \bar{y}} \frac{\partial^2 \bar{u}}{\partial \bar{x} \partial \bar{y}} \right] + \lambda \theta \sin(\bar{x}) \end{aligned} \tag{12}$$

$$u \frac{\partial \theta}{\partial \bar{x}} + v \frac{\partial \theta}{\partial \bar{y}} = \frac{1}{\text{Pr}_{eff}} \frac{\partial^2 \theta}{\partial \bar{y}^2} \tag{13}$$

where  $K$  is the dimensionless viscoelastic parameter,  $\lambda$  is the constant mixed convection parameter,  $\text{Pr}_{eff} = \frac{\text{Pr}}{1+N_r}$  is the effective Prandtl number,  $N_r$  being radiation parameter [see Magyari and Pantokratoras (2011)] and  $\text{Pr}$  is Prandtl number which are defined as

$$\left. \begin{aligned} K = \frac{k_0 U_\infty}{\rho a \nu}, \lambda = \frac{Gr}{\text{Re}^{\frac{5}{2}}}, \\ N_r = \frac{16\sigma T_\infty^3}{3k a_R}, \text{Pr} = \frac{\mu C_p}{k} \end{aligned} \right\} \tag{14}$$

with  $Gr = g \beta q_w a^4 / k_{eff} \nu^2$  being the Grashof number. The mixed convection parameter  $\lambda$  in terms of  $Gr$  indicates that  $\lambda > 0$  corresponds to aiding flow ( $q_w > 0$ ),  $\lambda < 0$  corresponds to opposing flow ( $q_w < 0$ ) and  $\lambda = 0$  corresponds to forced convection case of the problem. For  $K = 0$ , we get the case for viscous (Newtonian) fluids. The boundary conditions (9) becomes

$$u = 0, v = 0, \frac{\partial \theta}{\partial y} = -1 \text{ at } y = 0, x \geq 0,$$

$$u \rightarrow U_e(x), \frac{\partial u}{\partial y} \rightarrow 0, T \rightarrow 0 \text{ as } y \rightarrow \infty, x \geq 0. \tag{15}$$

To solve Eqs. (11–13) subject to the boundary conditions (15), we assume that  $u_e(x) = \sin x$  as given by Merkin (1977) and introduce the following variables:

$$\psi = xF(x, y), \theta = \theta(x, y) \tag{16}$$

where  $\psi$  and  $F$  are the dimensional and dimensionless stream functions respectively such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{17}$$

Use of Eqs. (16,17) in Eqs. (12,13) gives

$$F''' + FF'' - (F')^2 + \frac{\sin x \cos x}{x} + \lambda \frac{\sin x}{x} \theta - K \left[ FF'''' - 2FF''' + (F'')^2 + x \left\{ \frac{\partial F}{\partial x} F'''' - \frac{\partial F'}{\partial x} F''' + F'' \frac{\partial F''}{\partial x} - F' \frac{\partial F'''}{\partial x} \right\} \right] = x \left( F' \frac{\partial F'}{\partial x} - \frac{\partial F}{\partial x} F'' \right) \tag{18}$$

$$\frac{1}{Pr_{eff}} \theta'' + F \theta' = x \left( F' \frac{\partial \theta}{\partial x} - \frac{\partial F}{\partial x} \theta' \right) \tag{19}$$

subject to the boundary conditions

$$F = 0, F' = 0, \theta' = -1 \text{ at } y = 0, x \geq 0,$$

$$F' \rightarrow \frac{\sin x}{x}, F'' \rightarrow 0, \theta \rightarrow 0 \text{ as } y \rightarrow \infty, x \geq 0, \tag{20}$$

where prime denotes differentiation with respect to  $y$ .

The physical quantities of principle interest are the shearing stress and the rate of heat transfer in terms of the skin-friction coefficient  $C_f$  and the Nusselt number  $Nu$  respectively. For the present problem, these are given as

$$C_f = Re^{\frac{1}{2}} \frac{\tau_w}{\rho U_\infty^2}, Nu = Re^{\frac{1}{2}} \frac{aq_w}{k_{eff} (T_w - T_\infty)} \tag{21}$$

where  $\tau_w$  and  $q_w$  are the wall shear stress and surface heat flux respectively which are defined by

$$\tau_w = \mu \left( \frac{\partial \bar{u}}{\partial y} \right)_{y=0} + k_0 \left( \bar{u} \frac{\partial^2 \bar{u}}{\partial x \partial y} + \bar{v} \frac{\partial^2 \bar{u}}{\partial y^2} + 2 \frac{\partial \bar{u}}{\partial x} \frac{\partial \bar{u}}{\partial y} \right)_{y=0}, q_w = -k_{eff} \left( \frac{\partial T}{\partial y} \right)_{y=0} \tag{22}$$

Use of Eqs. (9,16) reduces Eq. (21) to

$$C_f = x \left( \frac{\partial^2 F}{\partial y^2} \right)_{y=0}, Nu = \frac{1}{\theta(x, 0)} \tag{23}$$

At the lower stagnation point of the cylinder i.e. at  $x \approx 0$ , the partial differential Eqs. (18,19) reduce to the form

$$f'''' + ff'' - (f')^2 + 1 + \lambda \theta - K (ff'''' - 2ff''' + f''^2) = 0 \tag{24}$$

$$\frac{1}{Pr_{eff}} \theta'' + f \theta' = 0 \tag{25}$$

with the boundary conditions

$$f(0) = 0, f'(0) = 0, \theta'(0) = -1, f'(\infty) = 1, f''(\infty) = 0, \theta(\infty) = 0. \tag{26}$$

where prime denotes differentiation with respect to  $y$ . The skin friction coefficient  $C_f$  and the Nusselt number  $Nu$  reduce to

$$C_f = xf''(0), Nu = \frac{1}{\theta(0)}. \tag{27}$$

### 3. NUMERICAL METHOD

A very useful and an accurate implicit finite difference method (Keller-box method) is employed to solve the nonlinear system of partial differential equations (18, 19) subject to the boundary conditions (20) and ordinary differential equations (24, 25) subject to the boundary conditions (26) which is very well explained by Cebeci and Bradshaw (1984) and Salleh *et al.* (2011). It is described briefly in the following steps:

- the partial differential equations as well as ordinary differential equations are expressed in form of first order equations in  $y$ , which are then written in finite difference form by approximating the functions and their derivatives in form of mean value and central differences respectively in both coordinate directions,
- the resulting non-linear finite difference equations are linearized by Newton's method,
- these linearized equations are solved by using block-tridiagonal method, for a given value of the iterative procedure is stopped when the difference in values in consecutive iteration is

less than  $10^{-15}$ . The step size  $\Delta y$  in  $y$  as well as the edge of the boundary layer  $y_\infty$  is adjusted for different values of the parameters like  $\lambda$ ,  $K$  and  $Pr_{eff}$  to maintain accuracy in the results. Therefore, the step size  $\Delta y = 0.02$  and  $\Delta x = 0.01$  has been taken in present numerical study.

#### 4. RESULT AND DISCUSSION

To analyze completely the radiation effect on the mixed convection boundary layer flow of a viscoelastic fluid with heat flux over a circular cylinder results for the skin friction coefficient  $C_f$  and Nusselt number  $Nu$  are obtained for some values of the mixed convection parameter  $\lambda$ , viscoelastic parameter  $K$  and effective Prandtl number  $Pr_{eff}$ . The comparison of the skin friction coefficient  $C_f$  and the Nusselt number  $Nu$  for a Newtonian fluid ( $K = 0$ ) as a limiting case with those reported by Nazar *et al.* (2004) is illustrated in Figs. 2 and 3. Open circles represent solution reported by Nazar *et al.* (2004) and dashed curves are our reproduced results. The

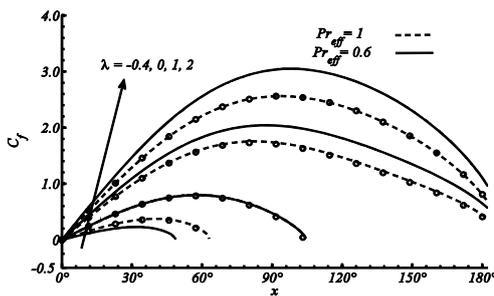


Fig. 2. Comparison of skin friction coefficient  $C_f$  against the curvature parameter  $x$  for  $K = 0$  (Newtonian fluid) and for various values of  $\lambda$ .

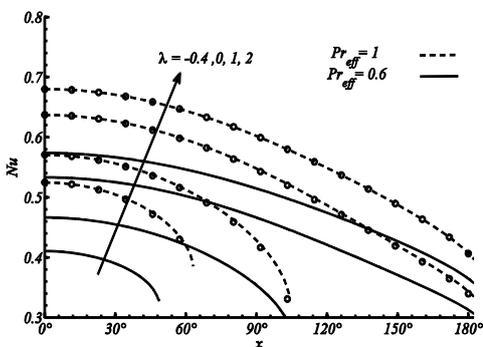


Fig. 3. Comparison of Nusselt number  $Nu$  against the curvature parameter  $x$  for  $K = 0$  (Newtonian fluid) and for various values of  $\lambda$ .

comparison shows an excellent agreement of our results with those reported by Nazar *et al.* Also the Figs. show the effect of radiation on the skin friction coefficient  $C_f$  and the Nusselt number  $Nu$ . In this article,  $Pr_{eff} = 1$  shows results without radiation and  $Pr_{eff} = 0.6$  gives results in presence of radiation.

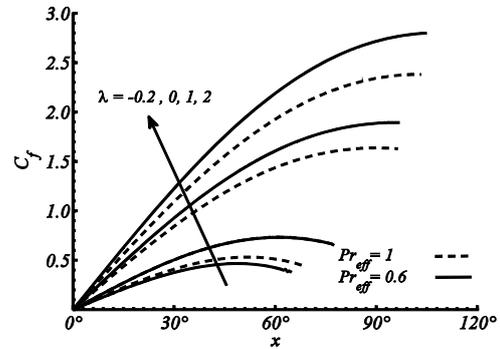


Fig. 4. Variation of skin friction coefficient  $C_f$  against the curvature parameter  $x$  for  $K = 0.2$  (viscoelastic fluid) and various values of  $\lambda$ .

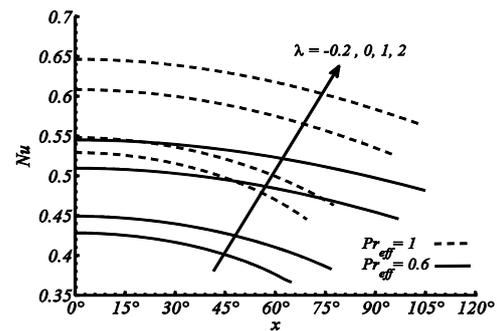
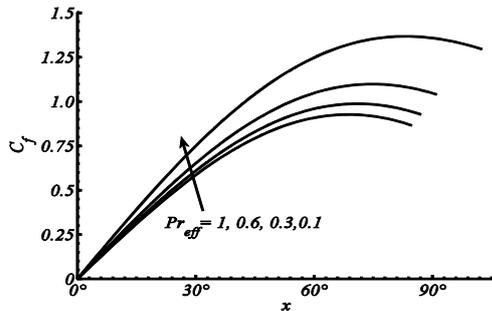


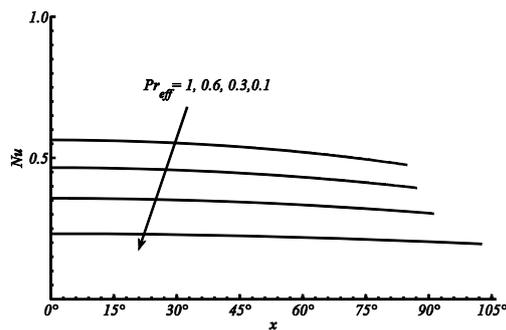
Fig. 5. Variation of Nusselt Number  $Nu$  against the curvature parameter  $x$  for  $K = 0.2$  (viscoelastic fluid) and various values of  $\lambda$ .

It is further seen that for  $\lambda < 0$ , decrease in skin friction is observed but for  $\lambda > 0$ , increase in it has been noticed due to the radiation while increase in Nusselt number has been observed due to the radiation effects. The curves in Figs. 2 and 3 show that the positive value of mixed convection parameter  $\lambda$  induces a supporting pressure gradient which results into an increase in skin friction coefficient  $C_f$  and Nusselt number  $Nu$ . For the case,  $K = 0$ , increasing  $\lambda$  delays boundary layer separation from the cylinder and can be suppressed entirely in the range  $0 \leq x \leq \pi$  by increasing  $\lambda (> 0)$  sufficiently. Figures 4 and 5 are graphs of the skin friction coefficient  $C_f$  and the Nusselt number  $Nu$  against viscoelastic parameter  $K = 0.2$ . The curves are drawn for different values of mixed convection parameter  $\lambda = -0.2, 0, 1, 2$  in

absence ( $Pr_{eff} = 1$ ) as well as in presence ( $Pr_{eff} = 0.6$ ) of the radiation. It is importantly noticed that presence of radiation ( $Pr_{eff} = 0.6$ ) reduces the skin friction for  $\lambda < 0$  but for  $\lambda > 0$  the radiation effect increases the skin friction. The figures also show that there is a critical value of  $\lambda = \lambda_0$  depending upon the viscoelastic parameter  $K$  below which a boundary layer solution is not possible. The similar effects were reported



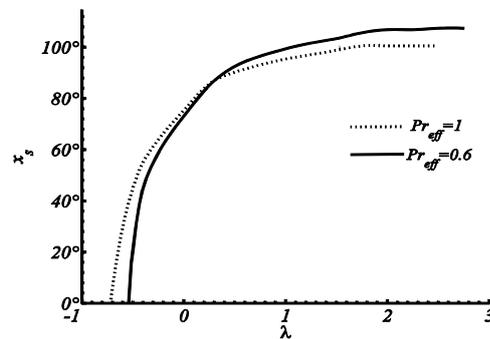
**Fig. 6. Effect of radiation on skin friction coefficient  $C_f$  for different values of  $Pr_{eff}$  when  $K = 0.2$  and  $\lambda = 0.2$ .**



**Fig. 7. Effect of radiation on Nusselt Number  $Nu$  for different values of  $Pr_{eff}$  when  $K = 0.2$  and  $\lambda = 0.2$ .**

by Merkin (1977) for the Newtonian case, the reason is that for sufficiently cooled cylinder  $\lambda < 0$ , the natural convection would start at upper stagnation point ( $x = \pi$ ) and the flow of stream upwards cannot overcome the motion of fluid next to cylinder in downwards direction under the action of buoyancy forces that oppose the development boundary layer. Figures 6 and 7 show the effect of radiation for  $K = 0.2$  and  $\lambda = 0.2$ . It is noticed that the decrease in value of effective Prandtl number  $Pr_{eff}$  leads to increase in values of skin friction coefficient  $C_f$  and decrease in the value of Nusselt number  $Nu$ . It is due to radiation, surface temperature increases which results in increase of the flow rate. Table 1 shows the numerical value of the skin friction coefficient  $C_f$  and the Nusselt number  $Nu$  for  $\lambda = 0.5$  and

$K = 0.5$  for the different values of  $Pr_{eff}$ . The Table illustrates that the values of skin friction coefficient and the Nusselt number increase with the decrease in  $Pr_{eff}$  (1,0.6,0.3,0.1). In Fig. 8, the variation of boundary layer separation point  $x_s$  with mixed convection parameter  $\lambda$  is shown for  $Pr_{eff} = 1, 0.6$  and  $K = 0.2$ . The dotted curve shows the behavior of separation point for  $Pr_{eff} = 1$  (without radiation), and solid curve gives its behavior for  $Pr_{eff} = 0.6$  (with radiation). It is pointed out that in both cases; there exists a specific value of  $\lambda = \lambda_0 (< 0)$  below which boundary layer does not exist. It also appears that presence of radiation increases the critical value  $\lambda_0$ .



**Fig. 8. Variation of boundary layer separation point  $x_s$  with  $\lambda$  for  $K = 0.2$ .**

## 5. CONCLUSION

The influence of radiation on mixed convection boundary layer flow of a viscoelastic fluid past a horizontal circular cylinder with constant heat flux has been studied. The boundary layer equations governing the flow and heat transfer are transformed to non-dimensional, nonlinear system of partial differential equations which are then solved numerically by a highly accurate implicit finite difference scheme (Keller-box method). We observed the effect of effective Prandtl number  $Pr_{eff}$  on flow and heat transfer rate as well as the boundary layer separation point  $x_s$  from the surface of the cylinder. The present investigation helps to conclude that:

- the presence of radiation increases the skin friction  $C_f$  for heated cylinder ( $\lambda > 0$ ) case, but it decreases the skin friction for the cooled cylinder ( $\lambda < 0$ ) case
- the radiation increases the Nusselt number  $Nu$  in both cases ( $\lambda > 0, \lambda < 0$ )
- a decrease in the value of effective Prandtl number  $Pr_{eff}$ , leads to increase in the value of both skin friction  $C_f$  and Nusselt number  $Nu$
- the decrease in value of effective Prandtl

**Table 1** Values of skin friction coefficient  $C_f$  and Nusselt number  $Nu$  for various values of  $Pr_{eff} = 1$  (without radiation), 0.6, 0.3 and 0.1 (with radiation)  $\lambda = 0.5, K = 0.5$

$x$	$Pr_{eff}=1$		$Pr_{eff}=0.6$		$Pr_{eff}=0.3$		$Pr_{eff}=0.1$	
	$C_f$	$Nu$	$C_f$	$Nu$	$C_f$	$Nu$	$C_f$	$Nu$
0	0	0.55793	0	0.46499	0	0.36160	0	0.24026
0.2	0.23878	0.55674	0.25419	0.46406	0.28022	0.36095	0.33580	0.23989
0.4	0.46582	0.55316	0.49682	0.46128	0.54907	0.35900	0.66040	0.23879
0.6	0.67007	0.54720	0.71700	0.45666	0.79585	0.35576	0.96322	0.23698
0.8	0.97318	0.52816	0.90513	0.45022	1.0111	0.35126	1.2349	0.23447
1.0	1.0589	0.51508	1.0536	0.44198	1.1874	0.34554	1.4677	0.23129
1.2	1.0814	0.55930	1.1571	0.43199	1.3193	0.33865	1.6561	0.22749
1.4	1.0964	0.49967	1.2132	0.42029	1.4044	0.33065	1.7969	0.22311
1.6	-	-	1.2226	0.40697	1.4428	0.32163	1.8893	0.21821

number  $Pr_{eff}$  results in delay of boundary layer separation point  $x_s$

**REFERENCES**

Abel, M. S., S. K. Khan and K. V. Prasad (2002). Study of visco-elastic fluid flow and heat transfer over a stretching sheet with variable viscosity. *Int. J. Non-Linear Mech.* 37, 81–88.

Anwar, I., N. Amin and I. Pop (2008). Mixed convection boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder. *Int. J. Non-Linear Mech.* 43, 814–821.

Ariel, P. D. (1995). Stagnation point flow of a viscoelastic fluid towards a moving plate. *Int. J. Eng. Sci.* 33, 1679–1687.

Badr, H. M. (1983). A theoretical study of laminar mixed convection from a horizontal cylinder in a cross stream. *Int. J. Heat Mass Transfer.* 26, 639–653.

Bhuiyan A. S., N. H. M. A. Azim and M. K. Chowdhury (2014). Joule heating effects on MHD natural convection flows in presence of pressure stress work and viscous dissipation from a horizontal circular cylinder. *Journal of Applied Fluid Mechanics.* 7(1), 7–13.

Cebeci, T. and P. Bradshaw (1984). *Physical and Computational Aspects of Convective Heat Transfer.* Springer New York.

Choudhury R. and S. K. Das (2014). Visco-Elastic MHD free convective flow through porous media in presence of radiation and chemical reaction with heat and transfer. *Journal of*

*Applied Fluid Mechanics.* 7(4), 603–609.

Cortell, R. A. (2006). Note on flow and heat transfer of a viscoelastic fluid over a stretching sheet. *Int. J. Non-Linear Mech.* 41, 78–85.

Dunn, J. E. and K. R. Rajagopal (1995). Fluids of differential type: critical review and thermodynamic analysis. *Int. J. Eng. Sci.* 33, 689–729.

Garg, V. K. and K. R. Rajagopal (1990). Stagnation point flow of a non-Newtonian fluid. *Mech. Res. Commun.* 17, 415–421.

Hayat, T., Z. Abbas and T. Javed (2008). Mixed convection flow of a micropolar fluid over a nonlinear stretching sheet. *Physics Letters A.* 372, 637–647.

Hayat, T., Z. Abbas, M. Sajid and S. Asghar (2007). The influence of thermal radiation on MHD flow of a second-grade fluid. *Int. J. Heat Mass Transfer.* 50, 931–941.

Hossain, M. A. and H. S. Takhar (1996). Radiation effects on mixed convection along a vertical plate with uniform surface temperature. *Int. J. Heat Mass Transfer.* 31, 243–248.

Hossain, M. A., M. A. Alim and D. Rees (1999). The effect of radiation on free convection from a porous vertical plate. *Int. J. Heat Mass Transfer.* 42, 181–191.

Kasim, A. R. M., N. F. Mohammad, S. Shafie and I. Pop (2013). Constant heat flux solution for mixed convection boundary layer viscoelastic fluid. *Heat Mass Transfer.* 49, 163–171.

Magyari, E. and A. Pantokratoras (2011). Note on

- the effect of thermal radiation in the linearized Rosseland approximation on the heat transfer characteristics of various boundary layer flows. *Int. Commun. Heat and Mass Transfer.* 38, 554–558.
- Merkin, J. H. (1977). Mixed convection from a horizontal circular cylinder. *Int. J. Heat Mass Transfer.* 20, 73–77.
- Molla, M. M., S. C. Saha, M. A. I. Khan and M. A. Hossain (2011). Radiation effects on natural convection laminar flow from a horizontal circular cylinder. *Desalin Water Treat.* 30, 89–97.
- Mukhopadhyay, S. (2009). Effect of radiation and variable fluid viscosity on flow and heat transfer along a symmetric wedge. *Journal of Applied Fluid Mechanics* 2(2), 29–34.
- Nazar, R., N. Amin and I. Pop (2004). Mixed convection boundary layer flow from a horizontal circular cylinder with a constant surface heat flux. *Heat Mass Transfer.* 40, 219–227.
- Prasad, J. S. R., K. Hemalatha and B. D. C. N. Prasad (2014). Mixed convection flow from vertical plate embedded in the Non-Newtonian fluid saturated Non-Darcy porous medium with thermal dispersion-radiation and melting effects. *Journal of Applied Fluid Mechanics* 7(3), 385–394.
- Prasad, K. V., K. Vajravelu, P. S. Datti and B. T. Raju (2013). MHD flow and heat transfer in a power law liquid film at a porous surface in the presence of thermal radiation. *Journal of Applied Fluid Mechanics* 6(3), 385–395.
- Rajagopal, K. R. (1992). Flow of viscoelastic fluids between rotating disks. *Theor. Comput. Fluid Dyn.* 3, 185–206.
- Rajagopal, K. R., M. Renardy, Y. Renardy and A. S. Wineman (1986). Flow of viscoelastic fluids between plates rotating about distinct axes. *Rheol. Acta.* 25, 459–467.
- Raptis, A., C. Perdakis and H. S. Takhar (2004). Effect of thermal radiation on MHD flow. *Appl. Math. Comp.* 153, 645–649.
- Rosseland, S. (1936). *Theoretical Astrophysics.* Oxford University Press London.
- Sajid, M. and T. Hayat (2008). Influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. *Int. Commun. Heat and Mass Transfer.* 35, 347–356.
- Sajid, M., Z. Abbas, T. Javed and N. Ali (2010). Boundary layer flow of an Oldroyd B fluid in the Region of stagnation point over a stretching sheet. *Canadian J. of Physics.* 88, 635–640.
- Salleh, M. Z., R. Nazar, N. M. Arifin and I. Pop (2011). Numerical solutions of forced convection boundary layer flow on a horizontal circular cylinder with Newtonian heating. *Malaysian J. Math. Sci.* 5(2), 161–184.
- Sharma, R., A. Ishak, R. Nazar and I. Pop (2014). Boundary layer flow and heat transfer over a permeable exponentially shrinking sheet in the presence of thermal radiation and partial slip. *Journal of Applied and Fluid Mechanics* 7(1), 125–134.
- Shit, G. C. and S. Majee (2014). Hydro-magnetic flow over an inclined non-linear stretching sheet with variable viscosity in presence of thermal radiation and chemical reaction. *Journal of Applied and Fluid Mechanics.* 7(2), 239–247.