Journal of Applied Fluid Mechanics, Vol. 9, No. 4, pp. 1645-1654, 2016.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.18869/acadpub.jafm.68.235.24755

Cross-Diffusion Effects on the Onset of Double Diffusive Convection in a Couple Stress Fluid Saturated Rotating Anisotropic Porous Layer

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(Received February 3, 2015; accepted August 23, 2015)

ABSTRACT

In this paper, we have investigated the onset of double diffusive convection (DDC) in a couple stress fluid saturated rotating anisotropic porous layer in the presence of Soret and Dufour effects using linear stability analyses which is based on the usual normal mode technique. The onset criteria for both stationary and oscillatory modes obtained analytically. The effects of the Taylor number, mechanical anisotropy parameter, Darcy Prandtl number, solute Rayleigh number, normalized porosity parameter, Soret and Dufour parameters on the stationary and oscillatory convections shown graphically. The effects of couple stresses are quite significant for large values of the non-dimensional parameter and delay the onset of convection. Taylor number has stabilizing effect on double diffusive convection, Dufour number has stabilizing effect in stationary mode while destabilizing in oscillatory mode. The negative Soret parameter stabilizes the system and positive Soret parameter destabilizes the system in the stationary convection, while in the oscillatory convection the negative Soret coefficient destabilize the system and positive Soret coefficient stabilizes the system.

Keywords: Couple stress fluid; Rotation; Anisotropy; Soret parameter; Dufour parameter; Double-diffusive convection (DDC).

1. INTRODUCTION

The problem of double diffusive convection in porous media has attracted considerable interest during the last few decades because of its wide range of applications, from the solidification of binary mixtures to the migration of solutes in water-saturated soils. Other examples include geophysical systems, electro-chemistry, and the migration of moisture through air contained in fibrous insulation. The problem of double diffusion convection in porous media has been extensively investigated and the growing volume of work devoted to this area is well documented by Ingham and Pop (1998, 2005).

Although the problem of double diffusive convection has been extensively investigated for Newtonian fluids, but relatively little attention has been devoted to this problem with non-Newtonian fluids. Until, such type of problems has not received much attention and the investigations of such fluids are desirable. Importance of non-Newtonian fluids with suspended particles in modern technology and industries are recently growing. The study of such fluids has applications in a number of processes that occur in industry, such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in bath, exotic lubrication and colloidal and suspension solutions. These fluids deform and produce a spin field due to the microrotation of suspended particles forming micropolar fluid developed by Eringen (1966). The micropolar fluids take care of local effects arising from microstructure and as well as the intrinsic motions of microfluidiscs. The spin field due to microrotation of freely suspended particles set up an antisymmetric stress, known as couple stress, and thus forming couple stress fluid. Thus, couple stress fluid, according to Eringen (1966), is a particular case of micropolar fluid when microrotation balances with the natural vorticity of the fluid. In the category of non –Newtonian fluids couple stress fluids have distinct features, such as polar effects. The theory of polar fluids and related theories are models for fluids whose microstructure is mechanically significant. Stokes (1966) gave the constitutive equations for couple stress fluids. The theory proposed by Stokes is the simplest one for microfluids, which allows polar effects such as the presence of couple stress, body couple and nonsymmetric tensors. There are few studies available on the Rayleigh –Benard problem for couple-stress fluids, extensions including the issue of stability/ onset [see Malashetty *et al*. (2003), and references their in].

The study of the effect of external rotation on thermal convection has attracted significant experimental and theoretical interest. Because of its general occurrence in geophysical and oceanic flows, it is important to understand how the Coriolis force influences the structure and transport properties of thermal convection. The study of thermal convection in rotating porous media is motivated both theoretically and by its practical applications in engineering; some of the important areas of applications in engineering include the food processing, chemical process, solidification and centrifugal casting of metals and rotating machinery. During the last two decades, there has been a great deal of effort lead by many researchers on the study of effect of external rotation on the Rayleigh Benard convection. In the literature there are plenty of works available on the problem of understanding how the Coriolis force influences the onset of thermal convection. The linear dynamics of rotating Rayleigh Benard convection with rigid stress-free boundaries has been thoroughly investigated by Chandrashekhar (1961). Who determined the marginal stability boundary and critical horizontal wave numbers for the onset of convection and overstability as a function of the Taylor number. An excellent review of research on thermal convection in a rotating porous medium is given by Vadasz (2000).

Anisotropy is generally a consequence of preferential orientation or asymmetric geometry of porous matrix of fibers in industry and nature. It is particularly important in a geological context, since sedimentary rocks generally have a layered structure; the permeability in the vertical direction is often much less than in the horizontal direction. Despite the practical importance of the topic, in context varying from fibrous insulating material to sedimentary rocks, only few studies have been reported on convection in an anisotropic porous medium [see Mckibbin (1992), Storesletten (2004), Nield (2007), Sidheshwar and Vanishree (2010)].

The analysis of double diffusive convection becomes complicated in case the diffusivity of one property is much greater than the other. Further, when two transport processes take place simultaneously; they interfere with each other and produce cross diffusion effects. The flux of mass caused by temperature gradient and the flux of heat caused by concentration gradient are known as Soret and Dufour coefficients, respectively. There are only few studies available on the effect of cross diffusion on double diffusive convection because of the complexity in determining these coefficients. The double diffusive convection in porous medium in the presence of Soret and Dufour coefficients has been analyzed by Rudraiah and Makashetty (1986). Linear and nonlinear DDC in a fluid saturated anisotropic porous layer with Soret effect and cross diffusion effects are investigated by Gaikwad *et al*. (2009a, b).

More Recently, Gaikwad and Kamble (2014) have studied about linear stability analysis of double diffusive convection in a horizontal sparsely packed rotating anisotropic porous layer in the presence of Soret effect. Kapil Chaudhary (2015) has studied on the onset of convection in a dusty couple stress fluid with variable gravity through a porous medium in hydromagnetics. Muthucumaraswamy *et al*. (2013) have studied the rotational effects on unsteady flow past a accelerated isothermal vertical plate with variable mass transfer in the presence of chemical reaction of first order. Rani and Reddy (2013) investigated the Soret and Dufour effects on transient double diffusive free convection of couple stress fluid past a vertical cylinder. Many authors have studied the effect of anisotropiy and/ or rotation on the onset of DDC in a horizontal porous layer [see malashetty and Swamy (2007), Malashetty and Heera (2008), Malashetty *et. al* (2011) and (2013)].

More recently, Banyal (2013) has investigated a mathematical theorem on the onset of stationary convection in couple stress fluid. P. Kumar (2012) has investigated the thermoslolutal magneto-rotating convection in a couple stress through porous medium. Concerned to our area the recent study on rotation with nanofluid has investigated by Dhanajyay *et al.* (2011, 2012, 2013a & 2013b).

Although, the problem of DDC in an isotropic porous medium has been investigated extensively very little attention has been devoted to the study of DDC in a rotating anisotropic porous layer with Soret and Dufour effects. Further, the effect of rotation on the onset of DDC in couple stress fluid – saturated anisotropic porous layer with Soret and Dufour effect is not available. The intent of the present paper is therefore to study the onset of DDC in a couple stress fluids saturated rotating anisotropic porous layer with cross diffusion effects heated and salted from below using linear stability analysis.

2. GOVERNING EQUATIONS

The physical configuration of the problem is shown in above Fig. (i). We consider an infinite horizontal couple stress fluid saturated porous layer confined between the planes $z = 0$ and $z = d$, with the vertically downward gravity force $\mathbf{g}$ acting on it. A Cartesian frame of reference is chosen with the origin in the lower boundary and the z-axis vertically upwards. The porous layer rotates uniformly about the z-axis with a constant angular velocity $\Omega = (0, 0, \Omega)$. A uniform adverse temperature difference ΔT and a stabilizing concentration difference ΔS are maintained between the lower and upper boundaries. The porous medium is assumed to possess isotropy in the horizontal plane in mechanical property. The Darcy model with time derivative is employed for the momentum equation and both the cross-diffusion terms are included in the temperature and concentration equations.

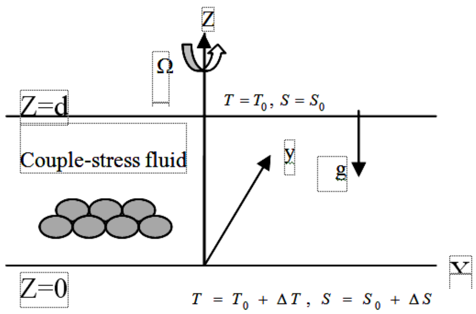

Fig. (i). Physical Configuration.

The basic state assumed to be quiescent, and we superpose an infinitesimal perturbation on the basic state. The equations for the perturbation quantities under the Boussinesq approximation are

$$\nabla \cdot \mathbf{q} = 0\,, \tag{1}$$

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} = -\nabla \mathrm{p} + \rho_0 g\left(\beta_T T - \beta_S S\right) - \frac{2\rho_0}{\varepsilon}\mathbf{\Omega}\times\mathbf{q} - (\mu - \mu_c\nabla^2)\mathbf{q}_a, \tag{2}$$

$$\gamma\frac{\partial T}{\partial t} + (\mathbf{q}\cdot\nabla)T = \kappa_{11}\nabla^2 T + \kappa_{12}\nabla^2 S\,, \tag{3}$$

$$\varepsilon\frac{\partial S}{\partial t} + (\mathbf{q}\cdot\nabla)S = \kappa_{21}\nabla^2 T + \kappa_{22}\nabla^2 S\,, \tag{4}$$

where $\mathbf{q}$ is the Darcy velocity vector, p is the pressure, $\mathbf{g}$ is the acceleration due to gravity, μ is the fluid viscosity, μ_c is the couple stress viscosity, Ω denote the angular velocity of rotation, T is the temperature, S is the concentration, ε is the porosity. Further, $\gamma = \frac{(\rho c)_m}{(\rho c)_f}$, where $(\rho c)_f$ is the volumetric heat capacity of the fluid and $(\rho c)_m = (1-\varepsilon)(\rho c)_s + \varepsilon(\rho c)_f$ is the volumetric heat capacity of the saturated medium as a whole, with the subscripts f, s, and m denoting the properties of the fluid, solid, and porous matrix, respectively. κ_{11} and κ_{22} are effective thermal diffusivity and solutal diffusivity of the medium, κ_{12} and κ_{21} are the Dufour and Soret coefficients. Further, β_T and β_S are the thermal and solutal expansion coefficients in the medium. Eq. (2) is indeed the averaged equation applicable for couple stress fluid flow through rotating porous media. It is observed that the presence of rotation introduces an additional body force known as Coriolis force which has a profound effect on the flow of couple stress fluid through porous media. By operating curl twice on Eq. (2) we eliminate p from it and then render the resulting equation and the Eqs. (3) and (4) dimensionless using the following transformations

$$(x,y,z) = (x^*,y^*,z^*)d,\ t = t^*\left(\frac{\gamma d^2}{\kappa_{11}}\right),\ (u',v',w') = \frac{\kappa_{11}}{d}(u^*,v^*,w^*),$$

$$T' = (\Delta T)T^*,\ S' = (\Delta S)S^*, \tag{5}$$

to obtain non-dimensional equations as (on dropping the asterisks for simplicity),

$$\left[\Pr_D^{-1}\frac{\partial}{\partial t}\nabla^2 + \left(\nabla_1^2 + \xi^{-1}\frac{\partial^2}{\partial z^2}\right)\left(1 - C\nabla^2\right)\left\{\Pr_D^{-1}\frac{\partial}{\partial t} + \xi^{-1} - C\nabla^2\right\} + Ta\frac{\partial^2}{\partial z^2}\right]w -$$
$$\left(\Pr_D^{-1}\frac{\partial}{\partial t} + \xi^{-1} - C\nabla^2\right)\left(Ra_T\nabla_1^2 T - Ra_S\nabla_1^2 S\right) = 0, \tag{6}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)T - D_{fr}\nabla^2 S - w = 0\,, \tag{7}$$

$$\left(\varepsilon_n\frac{\partial}{\partial t} - \tau\nabla^2\right)S - S_{rt}\nabla^2 T - w = 0\,, \tag{8}$$

where,

$$Ra_T = \frac{\beta_T\, g\,\Delta T\, d\, K_z}{\nu\kappa_{11}}, \qquad Ra_S = \frac{\beta_S\, g\,\Delta S\, d\, K_z}{\nu\kappa_{11}},$$

$$C = \frac{\mu_c}{\mu d^2}, \quad Le = \frac{\kappa_{11}}{\kappa_{22}}, \quad \xi = \frac{K_x}{K_z}, \quad \varepsilon_n = \frac{\varepsilon}{\gamma},$$

$$D_{fr} = \frac{\kappa_{12}\,\Delta S}{\kappa_{11}\,\Delta T}, \quad S_{rt} = \frac{\kappa_{21}\,\Delta T}{\kappa_{11}\,\Delta S}, \quad Ta = \left(\frac{2\Omega k}{\nu\varepsilon}\right)^2,$$

$$Pr_D = \frac{\varepsilon\gamma\nu d^2}{\kappa_{11}k}, \quad \nabla_1^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 \quad \text{and}$$

$$\nabla^2 = \nabla_1^2 + \partial^2/\partial z^2.$$

The dimensionless groups, which appear, are thermal Rayleigh number Ra_T, solute Rayleigh number Ra_S, couple-stress parameter C, Lewis number Le, mechanical anisotropy parameter ξ, the normalized porosity ε_n, the Dufour parameter D_{fr}, and the Soret parameter S_{rt}, Taylor number Ta, and Darcy-Prandtl number Pr_D.

The Eqs. (6)– (8) are to be solved for stress free, isothermal, and isohaline boundaries. Hence the boundary conditions for the perturbation variables are given by

$$w = \frac{\partial^2}{\partial z^2} = T = S = 0,\ at\ z = 0,1. \tag{9}$$

3. Linear Stability Analysis

In this section, we predict the thresholds of both marginal and oscillatory convections using linear theory. The eigenvalue problem defined by Eqs. (6)- (8) subject to the boundary conditions (9) is solved using the time-dependent periodic disturbances in a horizontal plane. Assuming that the amplitudes of the perturbations are very small, we write

$$\left(W,T,S\right)=\left(W\left(z\right),\Theta\left(z\right),\Phi\left(z\right)\right)exp\left[i\left(lx+my\right)+\sigma t\right], \tag{10}$$

Where, l & m are horizontal wavenumbers and σ is the growth rate. Infinitesimal perturbations of the rest state may either damp or grow depending on the value of the parameter σ. Substituting Eq. (10) into the linearized version of Eqs. (6)- (8) we obtain

$$\left[\begin{array}{l}\left\{\begin{array}{l}\sigma Pr_D^{-1}\left(D^2-a^2\right)+\left(\xi^{-1}D^2-a^2\right)\\-\left(1-C\left(D^2-a^2\right)\right)\end{array}\right\}\times\\ \left\{\begin{array}{l}Pr_D^{-1}\sigma+\xi^{-1}\\-C\left(D^2-a^2\right)\end{array}\right\}+Ta\,D^2\end{array}\right]w-$$

$$[\sigma\,\mathrm{Pr}_D^{-1}+\xi^{-1}-C(D^2-a^2)]\times$$

$$[-Ra_T a^2\Theta+Ra_S a^2\Phi]=0=0 \tag{11}$$

$$W+(D^2-a^2-\sigma)\Theta+D_{fr}(D^2-a^2)\Phi=0, \tag{12}$$

$$W+S_{rt}(D^2-a^2)\Theta+(\tau(D^2-a^2)-\varepsilon_n\sigma)\Phi=0, \tag{13}$$

where, $D\equiv d/dz$ and $a^2=l^2+m^2$. The boundary conditions (9) now read

$$W=D^2W=\Theta=\Phi=0\ \ at\ z=0,1. \tag{14}$$

We assume the solutions of Eqs. (11)- (13) satisfying the boundary conditions (14) in the form

$$\left(W\left(z\right),\Theta\left(z\right),\Phi\left(z\right)\right)=\left(W_0,\Theta_0,\Phi_0\right)\sin n\pi z,$$

$$(n=1,2,3,......)\,. \tag{15}$$

The most unstable mode corresponds to $n=1$ (fundamental mode).Therefore, substituting Eq. (15) with $n=1$ into Eqs. (11)- (13), we obtain a matrix equation

$$Ra_T=\left[\frac{-LeRa_S\delta^2(1+\sigma-S_{rt})}{\delta^2(-1+LeD_{fr})-Le\sigma\varepsilon_n}\right]+$$

$$\left[\frac{\pi^2Ta\xi Pr_D(Le\delta^4D_{fr}S_{rt}-(\delta^2+\sigma)(\delta^2+Le\sigma\varepsilon_n))}{a^2(\sigma+(\xi+2C\delta^2\xi)Pr_D)(\delta^2(-1+LeD_{fr})-Le\sigma\varepsilon_n))}\right]$$

$$-\left[\frac{(\eta+\dfrac{\delta^2\sigma}{Pr_D}+\delta_1^2)(-Le\delta^4D_{fr}S_{rt}+(\delta^2+\sigma)(\delta^2+Le\sigma\varepsilon_n))}{a^2(\delta^2(-1+LeD_{fr})-Le\sigma\varepsilon_n)}\right], \tag{16}$$

where, $\delta^2=\pi^2+a^2$, $\delta_1^2=\pi^2\xi^{-1}+a^2$, and $\eta=1+C\delta^2$. We note that η is a representative of the viscosity of the fluid, and it is evident that the suspended particles add to the viscosity. The growth rate σ is in general a complex quantity such that $\sigma=\sigma_r+i\sigma_i$. The system with $\sigma_r<0$ is always stable, while for $\sigma_r>0$ it will become unstable.

3.1 Stationary State

For the validity of principle of exchange of stabilities (i.e., steady case), we have $\sigma=0$ (i.e., $\sigma_r=\sigma_i=0$) at the margin of stability. Then the Rayleigh number at which marginally stable steady mode exists becomes

$$Ra_T^{St}=\frac{\delta^2(-1+LeD_{fr}S_{rt})(\eta+\delta_1^2)}{a^2(-1+LeD_{fr})}+\frac{\pi^2Ta\delta^2(-1+LeD_{fr}S_{rt})}{a^2(1+2C\delta^2)(-1+LeD_{fr})}+\frac{LeRa_S(-1+S_{rt})}{(-1+LeD_{fr})}. \tag{17}$$

It is important to note that the critical wave number a_c^{st} depends on the couple stress parameter and Taylor number. In the absence of Taylor number, i.e. when $Ta=0$, Eq. (17) gives

$$Ra_T^{St}=\frac{\delta^2(-1+LeD_{fr}S_{rt})(\eta+\delta_1^2)}{a^2(-1+LeD_{fr})}+\frac{LeRa_S(-1+S_{rt})}{(-1+LeD_{fr})}. \tag{18}$$

In the absence of Soret and Dufour effects (i.e., $S_{rt}=D_{fr}=0$) the stationary Rayleigh number given by Eq. (18)) reduces to

$$Ra_T^{St}=\frac{(\pi^2+a^2)(\pi^2\xi^{-1}+a^2)[1+C(\pi^2+a^2)]}{a^2}+Ra_S Le. \tag{19}$$

This result coincides with the results of Malashetty and Premila (2011).

For an isotropic porous medium, that is when $\xi=1$, Eq. (19) gives

$$Ra_T^{St}=\frac{1}{a^2}(\pi^2+a^2)^2[1+C(\pi^2+a^2)]+Ra_S Le, \tag{20}$$

this result is given by Malashetty et al. (2010) for the onset of double diffusive convection for stationary mode in a couple stress fluid saturated isotropic porous medium. Further, in the absence of couple stresses, that is $C = 0$, the Eq. (20) reduces to

$$Ra_T^{St} = \frac{1}{a^2}(\pi^2 + a^2)^2 + Ra_S Le \,. \tag{21}$$

This is the classical result for double diffusive convection (DDC) in a porous medium for the stationary mode (Nield and Bejan (2006)). For single component couple stress fluid saturated porous medium, $Ra_S = 0$, the stationary Rayleigh number given by Eq. (21) reduces to

$$Ra_T^{St} = \frac{(\pi^2 + a^2)^2}{a^2} \,, \tag{22}$$

which has the critical value $Ra_{T.c}^{St} = 4\pi^2$ for $a_c^{St} = \pi^2$ obtained by Horton and Rogers (1945), and Lapwood (1948).

3.2 Oscillatory State

We now set $\sigma = i\sigma_i$ in Eq. (16) and clear the complex quantities from the denominator, to obtain

$$Ra_T = \Delta_1 + i\sigma_i \Delta_2 \,, \tag{23}$$

where, Δ_1 and Δ_2 are not mentioned here for brevity

Since Ra_T is a physical quantity, it must be real. Hence, from Eq. (23) it follows that either $\sigma_i = 0$ (steady onset) or $\Delta_2 = 0$ ($\sigma_i \neq 0$, oscillatory onset). For oscillatory onset $\Delta_2 = 0$ ($\sigma_i \neq 0$), we obtain

$$Ra_T^{Osc} = \frac{\delta^4 w^2}{\Pr_D} + \delta^4 \begin{pmatrix} (-1+D_{fr}LeS_{rt})\eta + \\ (1-D_{fr}LeS_{rt})\delta_1^2 \end{pmatrix} + Le(\eta+\delta^4) w^2 \varepsilon_n + \frac{\pi^2 Ta\xi \Pr_D \begin{pmatrix} (-1+D_{fr}S_{rt})\delta^4 \\ +Lew^2\varepsilon_n \end{pmatrix}}{a^2\delta^2 \begin{pmatrix} (-1+D_{fr}Le + \\ 2C(-1+D_{fr}Le)\delta^2) \end{pmatrix} \xi \Pr_D} + \frac{\delta^2(-1+S_{rt})LeRa_S}{(-1+LeD_{fr})\delta^2} \tag{24}$$

The analytical expression for oscillatory Rayleigh number given by Eq. (24) is minimized with respect to the wavenumber numerically for various values of physical parameters in order to know their effects on the onset of oscillatory convection.

4. Result And Discussion

The neutral stability curves in the $Ra_T - a$ plane for various parameter values are as shown in Figs.1-7. We fixed the values for the parameters except the varying parameter ($Ta = 100, S_{rt} = 0.05, D_{fr} = 0.005, C = 0.3, \xi = 0.5, \eta = 0.3, \Pr_D = 10, Le = 10$). From these figures, it is clear that the neutral curves are topologically connected. This connectedness allows the linear stability criteria to be expressed in terms of the critical Rayleigh number, below which the system is stable and unstable above. It is to be noted that 'isotropic case' means mechanically and thermally isotropic. Here we have not plotted the graph of thermal anisotropy parameter η. Because the limitation of the length of the paper, we have not incorporated the different graphs with different parameters.

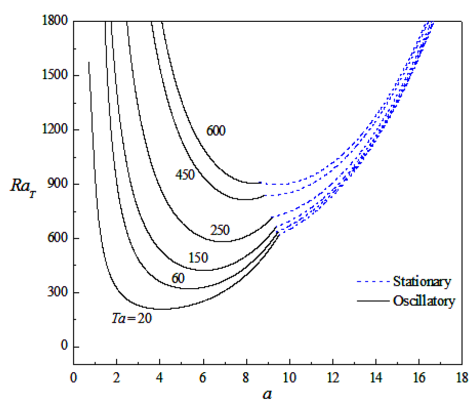

Fig. 1. Neutral stability curves for different values of Ta.

Fig. 1 depicts the effect of Taylor number Ta on the neutral stability curves. From this figure, we find that the effect of increasing the Taylor number Ta is to increase the value of the Rayleigh number for stationary and oscillatory modes and the corresponding wavenumber. Thus, the Taylor number Ta has stabilizing effect on the double diffusive convection in a couple stress fluids saturated rotating anisotropic porous layer with cross diffusions. That is for high rotation rates were necessary to significantly increase the Rayleigh number.

Fig. 2 shows the neutral stability curves for different values of the couple stress parameter C. We observed from this figure that the minimum value of the Rayleigh number for both stationary and oscillatory modes increases with an increase in the value of the couple stress parameter C, indicating that the effect of the couple stress parameter is to stabilize the system. That is the presence of couple stresses, it is shown that their effect is to delay the onset of convection and oscillatory convection always occurs at a lower value of the Rayleigh number at which steady convection sets in. the effect of couple stresses are quite large for large values of the non-dimensional parameter C. We also find that the bigger value of C, the larger the value of the Rayleigh number indicating that the effect of large couple stresses is to delay the onset of convection.

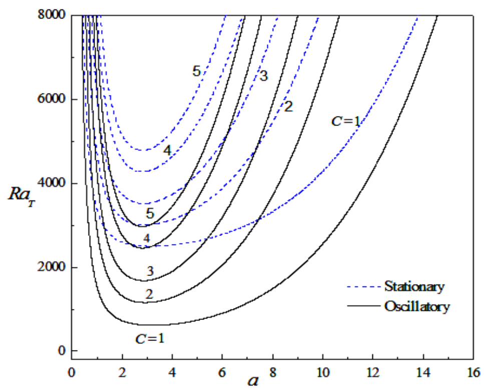

Fig. 2. Neutral stability curves for different values of C .

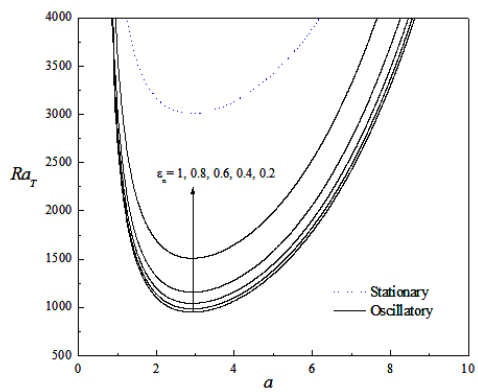

Fig. 3. Neutral stability curves for different values of ε_n .

Fig.3 shows the marginal stability curves for different values of normalized porosity parameter ε_n. We observe that the effect of increasing the normalized porosity parameter decreases the minimum of the Rayleigh number, indicating that the effect of normalized porosity parameter is to advance the onset of double diffusive convection. Further, it is important to note that the effect of normalized porosity is significant for small ε_n.

Fig. 4 shows the neutral stability curves for different values of mechanical anisotropy parameter ξ. We observe from these figures that the convection sets in as oscillatory mode prior to the stationary convection. It can be observed that an increase in ξ decreases the minimum of the Rayleigh number for both stationary and oscillatory state, indicating that, the effect of increasing mechanical anisotropy parameter ξ is to advance the onset of stationary and oscillatory convection. Further, we find that the minimum of Rayleigh number shift towards the smaller values of the wavenumber with increasing mechanical anisotropy parameter. This indicates that the cell width increases with increasing mechanical anisotropy parameter.

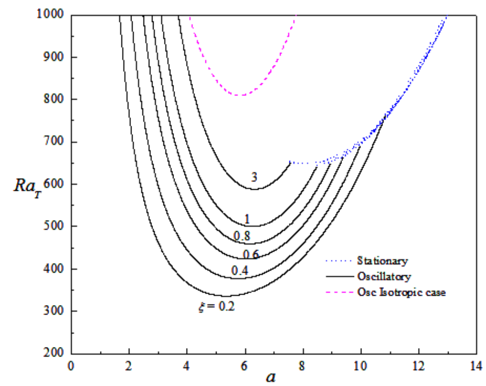

Fig. 4. Neutral stability curves for different values of ξ .

Fig. 5 depicts the effect of solute Rayleigh number Ra_S on the neutral stability curves for stationary and oscillatory convections. We find that the effect of increasing Ra_S is to increase the value of the Rayleigh number for stationary and oscillatory modes and the corresponding wavenumber. Thus, the solute Rayleigh number Ra_S has a stabilizing effect on the double diffusive convection rotating fluid saturated anisotropic porous medium.

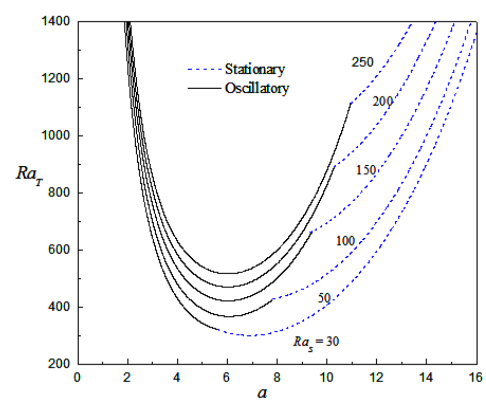

Fig. 5. Neutral stability curves for different values of Ra_S .

Fig. 6 shows the marginal stability curves for different values of Lewis number Le. It is observed that with the increase of Le the values of Rayleigh number and the corresponding wavenumber for oscillatory mode decrease while those for stationary mode increase. Therefore, the effect of Le is to advance the onset of oscillatory convection while its effect is to inhibit the stationary convection.

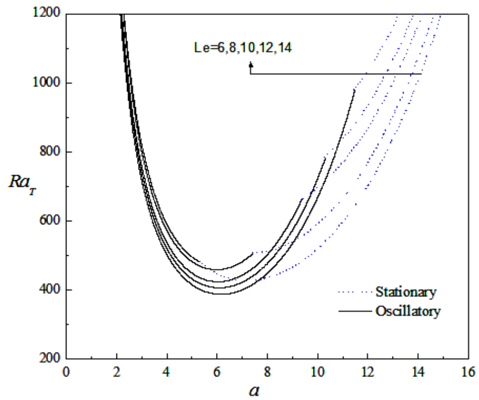

Fig. 6. Neutral stability curves for different values of Le.

The neutral stability curves for different values of Darcy-Prandtl number Pr_D are presented in Fig.7, from this figure it is evident that for small and moderate values of Pr_D the value of oscillatory Rayleigh number decreases with the increase of Pr_D, however this trend is reversed for large values of Pr_D.

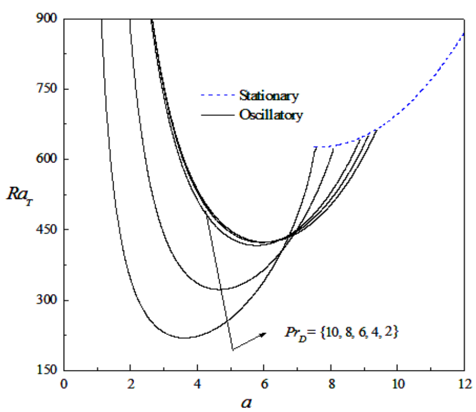

Fig. 7. Neutral stability curves for different values of Pr_D.

Fig. 8 depicts the stationary stability curves for different values of Soret parameter S_{rt}. We observe that with an increase in the negative Soret parameter S_{rt}, the stationary Rayleigh number increases indicating that the negative Soret parameter stabilizes the system. On the other hand, for positive Soret parameter, the minimum of the stationary Rayleigh number decreases with an increase of the Soret parameter, indicating that the positive Soret parameter S_{rt} destabilizes the system.

Fig. 9 shows oscillatory mode for different values of Soret parameter S_{rt}. We find that with an increase in the negative Soret parameter, the oscillatory Rayleigh number decreases, indicating that the negative Soret parameter destabilizes the system. On the other hand, for positive Soret parameter, the minimum of the oscillatory Rayleigh number increases with an increase of the Soret parameter, indicating that the positive Soret parameter S_{rt} has a stabilizing effect.

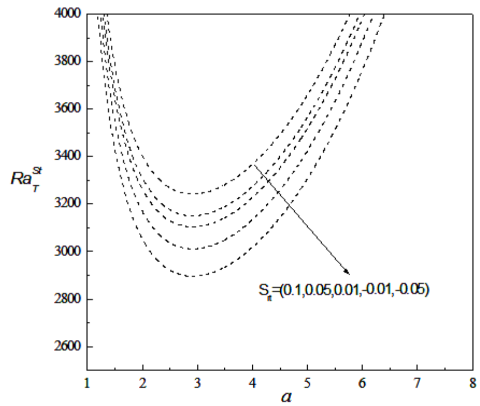

Fig. 8. Stationary neutral stability curves for different values of S_{rt}.

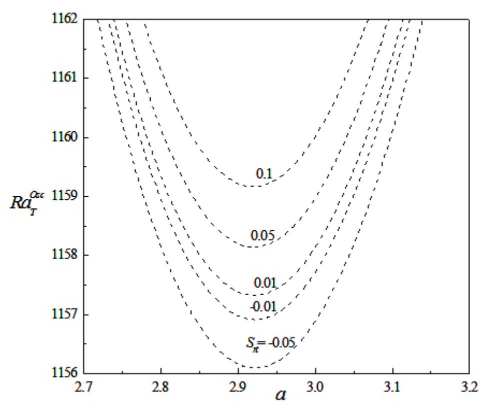

Fig. 9. Oscillatory neutral stability curves for different values of S_{rt}.

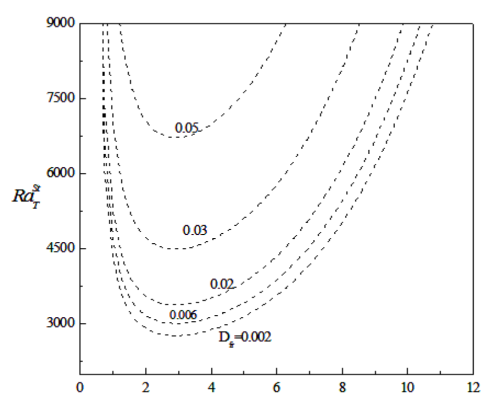

Fig. 10. Stationary neutral stability curves for different values of D_{fr}.

Fig. 10 shows the neutral stability curves for stationary mode for different values of Dufour parameter D_{fr}. We find that the stationary Rayleigh number increases with an increase in the value of the Dufour parameter, indicating that the effect of Dufour parameter is to enhance the stability of the system. Fig. 11 shows the neutral stability curves for oscillatory mode for different values of Dufour parameter D_{fr}. We observe that the oscillatory Rayleigh number decreases with increasing the value of the Dufour parameter, indicating that the Dufour parameter advances the oscillatory convection.

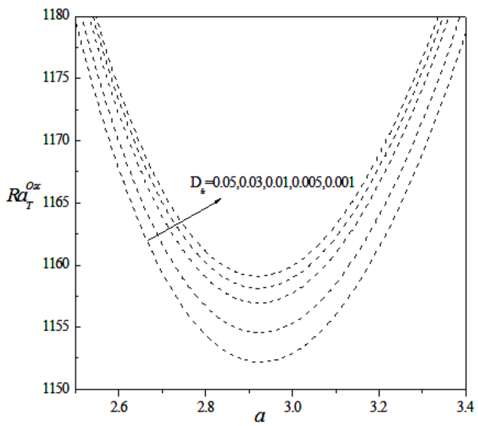

Fig. 11. Oscillatory neutral stability curves for different values of D_{fr}.

Fig. 12 shows the variation of the critical Rayleigh number for stationary and oscillatory convection with couple stress parameter C for different values of Dufour parameter D_{fr}. From this figure, we find that the critical Rayleigh number for stationary convection with increase in the value of couple stress parameter C indicating that the effect of couple stress parameter C is to stabilize the system, also we observe that the critical thermal Rayleigh number for stationary mode increases with an increase in the value of Dufour parameter D_{fr}. Thus the effect of D_{fr} is to stabilize the system in case of stationary convection. For oscillatory convection we find that greater the value of C, the larger the value of the oscillatory critical Rayleigh number, indicating that the effect of C is to delay the onset of oscillation convection. It is important to note that the effect of Dufour parameter D_{fr} is significant to the oscillatory convection for large values of C while it has marginal effect when C is small.

Fig. 13 depicts the variation of the critical and oscillatory convection with couple stress parameter C for different values of Soret parameter S_{rt}. We observe from this figure that the critical thermal Rayleigh number for stationary convection decreases with an increase in the value of Soret parameter S_{rt} indicating the destabilizing the effect of positive Soret parameter. On the other hand an increase in the magnitude of the negative Soret parameter increases the critical Rayleigh number for the stationary mode indicating the stabilizing effect of negative Soret parameter S_{rt}, also enhances the stability and the effect is very marginal. The effect of Soret parameter S_{rt} becomes insignificant for large values of the couple stress parameter C.

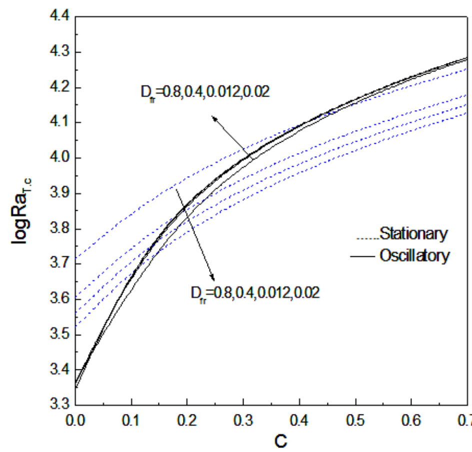

Fig. 12. Neutral stability curves for different values of D_{fr}.

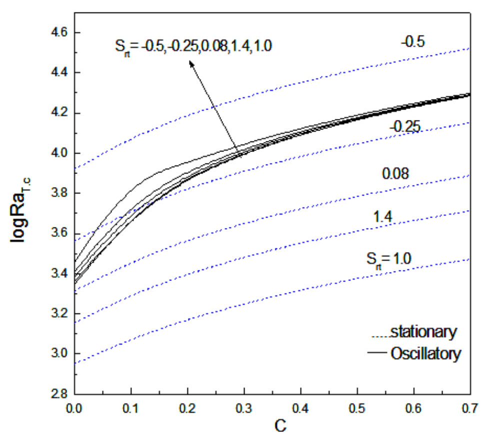

Fig. 13. Neutral stability curves for different values of S_{rt}.

5. CONCLUSIONS

Cross-diffusion effects on the onset of double diffusive convection in a couple stress fluid saturated rotating anisotropic porous layer which is heated and salted from below, is investigated analytically using linear stability analysis. The usual normal mode technique is used to solve the linear problem. The following conclusions are drawn:

1. The Taylor number Ta has a stabilizing effect on the double diffusive convection in an anisotropic porous medium.
2. The mechanical anisotropy parameter ξ has stabilizing effect on stationary and oscillatory modes. However, the convection sets in an oscillatory mode prior to the stationary mode.
3. The effect of solute Rayleigh number Ra_S is to delay both stationary and oscillatory convection.
4. The effect of Lewis number Le is to delay the onset of stationary convection while it advances the oscillatory convection.
5. The effect of normalized porosity is to advance the onset of oscillatory convection. In addition, the Darcy Prandtl Pr_D has a dual effect on the oscillatory mode.
6. The effect of couple stress parameter C is to delay, both stationary and oscillatory convection.
7. The normalized porosity parameter ε_n has a destabilizing effect in the case of oscillatory mode.
8. The Dufour parameter D_{fr} stabilizes the system in the stationary mode, while it destabilizes the system in the oscillatory mode.
9. The negative Soret parameter stabilizes the system and positive Soret parameter destabilizes the system in the stationary convection, while in the oscillatory convection the negative Soret coefficient destabilize the system and positive Soret coefficient stabilizes the system.

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