



Peristaltic Pumping of a Casson Fluid in an Elastic Tube

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ABSTRACT

This paper is concerned with the peristaltic transport of an incompressible non-Newtonian fluid in an elastic tube. Here the flow is due to three different peristaltic waves and two different types of elastic tube. The constitution of blood suggests a non-Newtonian fluid model and it demands the applicability of yield stress fluid model. Among the available yield stress fluid models for blood, the non-Newtonian Casson fluid is preferred. The Casson fluid model describes the flow characteristics of blood accurately at low shear rates and when it flows through small blood vessels. Long wavelength approximation is used to linearize the governing equations. The effect of peristalsis and non-Newtonian nature of blood on velocity, plug flow velocity, wall shear stress and the flux flow rate are derived. The flux is determined as a function of inlet, outlet, external pressures, yield stress, amplitude ratio, and the elastic properties of the tube. Furthermore, it is observed that, the yield stress, peristaltic wave, and the elastic parameters have strong effects on the flux of the non-Newtonian fluid, namely, blood. One of the important observation is that the flux is more when the tension relation is an exponential curve rather than that of a fifth degree polynomial. Further, in the absence of peristalsis and when the yield stress tends to zero our results agree with the results of Rubinow and Keller (1972). This study has significance in understanding peristaltic transport of blood in small blood vessels of living organisms.

Keywords: Casson fluid; Peristaltic blood flow; Fluid flux; Amplitude ratio; Wall shear stress; Yield stress; Elastic tube.

NOMENCLATURE

a_0	radius of tube in the absence of the elasticity	T	tension
a_1	outlet radius	t	time
a_1	inlet radius	t_1, t_2, A, K	elastic parameters
a'	change in radius of the tube due to peristalsis	u	velocity of the fluid flow
a''	change in radius of the tube due to elasticity	u_p	velocity of the plug flow
c	wave speed	z	distance along the tube from the inlet end
P	pressure gradient	τ_0	yield stress
p_0	External pressure	τ_{rz}	shear stress
$p(z)$	pressure of the fluid	μ	coefficient of viscosity
p_1	inlet pressure	λ	wave length
p_2	outlet pressure	σ_1	conductivity
φ	amplitude ratio		
r_0	radius of the plug flow		

1. INTRODUCTION

The study of peristaltic flows of Newtonian and non-Newtonian fluids in symmetric and asymmetric

channel/ an elastic tube has acquired momentum interest among researchers in recent years because of its numerous practical applications in physiology and biomechanical system. In particular, a

peristaltic mechanism is involved with swallowing food through the esophagus; urine transport from the kidney to the bladder through the urethra, movement of chyme in the gastro-intestinal tract; the transport of spermatozoa in the ducts efferentus of the male reproductive tract the movement of ovum in the cervical canal of the female fallopian tubes; the transport of lymph in the lymphatic vessels; and the vasomotion of small blood vessels such as arterioles, venules, and capillaries. Also, finger and roller pumps are frequently used for peristaltic pumping of corrosive or very pure materials so as to prevent direct contact of the fluid with the pump's internal surfaces. Peristaltic transport is a form of a material transport induced by a progressive wave of area contraction or expansion along the length of a distensible tube, mixing and transporting the fluid in the direction of the wave propagation. This phenomenon is known as peristalsis. To understand the behavior of peristaltic, quite a lot of theoretical and experimental attempts have been made since the investigation of Latham (1966). Thereafter quite a good number of analytical/ numerical studies pertaining to the flow on Newtonian fluids were considered by many researchers in different physical constraints (See, Mishra *et al.* 2004, Elshehawey *et al.* 2006, Hayat *et al.* 2006, 2008, Nadeem and Akbar 2009a, 2010a, Ramana Kumari and Radhakrishnamacharya 2011).

Most of the theoretical investigations have been carried out by many researchers considering blood and other physiological fluids as Newtonian fluids. Although this approach may provide a satisfactory understanding of the peristaltic mechanism in the ureter, but it fails to provide a satisfactory model when the peristaltic mechanism is involved in small blood vessels, lymphatic vessels, and stomach. This necessitates the use of non-Newtonian models for the description of peristaltic flows in such physiological systems. It is well known that blood being suspension of cells behaves like a Newtonian fluid when it flows through tubes (arteries) with larger diameter at higher shear rate. But it exhibits Non-Newtonian characteristics when it flows through tubes with small diameter at low shear rates. Further the viscosity of the blood gets increased at low rates of shear when the red blood cells tend to aggregate and form Rouleaux. The Rouleaux is a semi-solid forming a plug flow region. In the plug flow region we have a flattened velocity profile rather than the parabolic velocity profile which is the regular behavior of a Newtonian fluid. Such a behavior can be modeled through yield stress fluid models. The yield stress for blood strongly depends on fibrinogen concentration and the hematocrit. The values of yield stress for normal human blood ranges from 0.01 and 0.06 dyn/cm². In view of this Casson model which is a yield stress model is widely used to explain the remarkable behavior of blood flow through small blood vessels at low shear rates. Several authors have reported theoretical and experimental study of the pressure flow relationship for different fluids through tapered tubes such as Newtonian, Power law and Bingham (see,

Chaturani and Prahalad 1985, Chakravarthy and Mandal 2000, Mandal 2005). Recent works on Casson fluid flow and heat transfer under different physical situations were carried out by several researchers (Abolbashari *et al.* 2015, Krishnamurthy *et al.* 2015, Ramesh *et al.* 2015). Various experiments performed on blood with varying hematocrit, anticoagulants, temperatures, etc. strongly suggest the behavior of blood as a non-Newtonian fluid namely, Casson fluid. In particular, the Casson fluid model describes the flow characteristics of blood more accurately at low shear rates and when it flows through small blood vessels. Keeping this in view, Oka (1971) was the first among the others who developed the generalized non-Newtonian model namely, Casson model as a special case for the study of flow characteristics in an elastic tube. Jayaraman *et al.* (1981) extended Oka's work and suggested that Casson fluids are found to be more practically applicable in developing models for blood Oxygenators. Habtu and Radhakrishnamacharya (2011) studied the effect of peristalsis on dispersion in a micropolar fluid. Peristaltic transport of some different non-Newtonian fluids in an elastic tube investigated theoretically by Nadeem and Akbar (2009b, 2010b, 2010c, 2011). The flow of non-Newtonian fluids in elastic inflatable and collapsible tubes is important to biofluid mechanics encountered in human body and other applications; for instant, transport of food and liquids in human throat (pharynx), the tube (esophagus) connecting the throat and stomach, and intestines. The knowledge on the mechanisms of pharyngeal, esophageal and intestinal transport of food and liquids is very useful for the treatment of patients with malfunctioning of these transport processes. Rubinow and Keller (1972) studied the flow of a viscous fluid through an elastic tube with applications to blood flow. Mishra *et al.* (2003) studied pulsatile flow of a viscous fluid through a porous elastic vessel of variable cross-section. Sarkar and Jayaraman (2001) concentrated on the analysis of oscillatory flow in the annulus of an elastic tube with applications to catheterized artery. Takagi and Balmforth (2011a, b) made some investigations on peristaltic transport of both rigid objects and viscous fluid in an elastic tube. Recently, Vajravelu *et al.* (2011) studied the Herschel-Bulkley fluid flow in an elastic tube. To the best of the author's knowledge no attempt is available in the literature which deals with the combined effects of different types of peristaltic transport waves of non-Newtonian fluid and the different types of elastic tubes. This particular study is useful in filling the gap in this direction.

Thus the main motivation of the present paper is to study the effects of peristaltic transport of a non-linear non-Newtonian fluid model through an elastic tube. Here the non-Newtonian model is Casson model and this model can be used for moderate shear rates in smaller diameter tubes. Here the peristaltic transport of an elastic tube is produced by choosing the peristaltic different wave train on the tube due to variation in tube radius and wave amplitudes. Long wave length approximation

is used to linearize the governing equations. The physical quantities involved in the problem are written in the non-dimensional form and the expressions for the flow quantities such as velocity, plug flow velocity, and flux are determined. The resulting equations of the fluid flow are solved analytically subjected to the appropriate boundary conditions. Finally the effect of flux for different values of the pertinent parameters, and peristalsis for a Casson fluid is presented graphically. The results and the discussion presented in this study may be helpful to further understand the peristaltic motion of non-Newtonian yield stress fluids.

2. BASIC EQUATIONS AND THE MATHEMATICAL FORMULATION

Consider the axisymmetric peristaltic flow of a steady viscous, incompressible non-Newtonian fluid namely, Casson fluid in an elastic tube of radius $a(z)$ and length L . The blood is modeled as a non-Newtonian Casson fluid and the flow is axisymmetric. The axisymmetric geometry facilitates the choice of the cylindrical co-ordinate system (r, φ, z) where r and z denote the radial and axial coordinates and φ is the azimuthal angle. The constitutive equation of a Casson fluid is

$$\sqrt{\tau} = \sqrt{\mu \gamma^* + \tau_0} \text{ for } \tau \geq \tau_0$$

$$\gamma^* = 0 \text{ for } \tau \leq \tau_0$$
(1)

where τ is the component of the shear stress and μ is the viscosity coefficient of casson fluid, γ^* is the rate of shear strain and τ_0 is the yield stress of fluid. The momentum equation governing the flow is

$$\frac{\mu}{r} \frac{\partial}{\partial r} (r \tau_{rz}) = P,$$
(2)

where

$$(\tau_{rz})^{1/2} = \begin{cases} \mu^{1/2} (-\partial u / \partial r)^{1/2} + \tau_0^{1/2}, & \tau \geq \tau_0 \\ 0, & \tau < \tau_0 \end{cases}$$
(3)

And

$$P = -\frac{\partial p}{\partial z}$$
(4)

Here τ_0 represents the yield stress of the tube. The non-dimensional boundary conditions are

$$\tau_{rz} \text{ is finite at } r=0,$$

$$u=0 \text{ at } r=a'(z,t)$$
(5)

3. SOLUTION OF THE PROBLEM

To solve the equations (2) and (3) under the boundary conditions (5), we make use of the following non-dimensional quantities

$$\bar{r} = \frac{r}{a_0}, \bar{r}_0 = \frac{r_0}{a_0}, \bar{z} = \frac{z}{\lambda}, \bar{u} = \frac{u}{c},$$

$$\bar{a}' = \frac{a'}{a_0}, \bar{a}'' = \frac{a''}{a_0}, \bar{t} = \frac{ct}{\lambda},$$

$$\bar{\varphi} = \frac{b}{a_0}, \bar{P} = \frac{a_0^2}{\lambda \mu c}, \bar{Q} = \frac{Q}{\pi a_0^2 c}, \bar{\tau}_0 = \frac{\tau_0}{\mu(c/a_0)},$$

$$\bar{\tau}_{rz} = \frac{\tau_{rz}}{\mu(c/a_0)}, \bar{T} = \frac{T}{\lambda \mu(c/a_0)}$$
(6)

where a_0 is the radius of the tube in the absence of elasticity, λ is the length of the tube, c is the wave speed. Using a long-wavelength approximation along with low Reynolds number, one can find that the non-dimensional governing equations reduce to (dropping the bars)

$$\frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rz}) = P,$$
(7)

The non-dimensional boundary conditions are

$$\tau_{rz} \text{ is finite at } r=0,$$

$$u=0 \text{ at } r=a'(z,t)$$
(8)

Solving the equation (7) subjected to the conditions (8), we obtain the velocity field as

$$u = \frac{P}{2} \left(\frac{4}{3} r_0^{1/2} (r^{3/2} - a^{3/2}) - \frac{1}{2} (r^2 - a^2) - r_0 (r - a') \right)$$
(9)

Using the boundary condition $\partial u / \partial r = 0$ at $r = r_0$, the upper limit of the plug flow region is obtained as $r_0 = 2\tau_0 / P$. Also by using the condition $\tau_{rz} = \tau_{a'}$ at $r = a'$ (Vajravelu *et al.*, 2011) we obtain $P = 2\tau_{a'} / a'$. Hence

$$\frac{r_0}{a'} = \frac{\tau_0}{\tau_{a'}} = \tau \text{ for } 0 < \tau < 1.$$
(10)

Using relation (10) and taking $r = r_0$ in Eq.(9), we get the plug flow velocity as

$$u_p = \frac{P}{2} \left(a' r_0 - \frac{4}{3} r_0^{1/2} a'^{3/2} - \frac{1}{6} r_0^2 + \frac{a'^2}{2} \right)$$
(11)

for $0 \leq r \leq r_0$.

The volume flux Q through any cross-section is given by

$$Q = 2 \int_0^{r_0} u_p r dr + 2 \int_{r_0}^{a'} u r dr = \frac{FP a'^4}{8},$$

$$\text{where } F = \left(1 - \frac{16}{7} \tau^{1/2} + \frac{4}{3} \tau - \frac{1}{21} \tau^4 \right)$$
(12)

The above Equation (12) gives the volume flux for a tube due to different types of peristalsis with varying radius $a'(z,t)$ in the absence of elasticity.

The variation occurs also due to different types of elasticity of the tube wall. Therefore, in the present paper, the authors investigate the effects of different types of elasticity of the tube wall in the presence of different types of peristalsis and discuss the consequences in the next section.

3.2 Theoretical Determination of Flux

We now calculate theoretically the flux Q of an incompressible Casson fluid of viscosity μ in an elastic tube (see Figure 1) of radius $a(z,t) = a'(z,t) + a''(z)$ where $a'(z,t)$ is due to the peristalsis and $a''(z)$ is due to the elastic nature of the tube, and length L (an integral multiple of wave length λ). In the present analysis, the following three types of peristaltic wave forms are considered.

$$a'(z,t) = 1 + \phi \begin{cases} \sin(z-t) & \text{Sinusoidal wave} \\ \left\{ \frac{8}{\pi^2} \sum_{m=1}^{\infty} \frac{\sin(\pi(2m-1)/8)}{(2m-1)^2} \right\} \sin[(2m-1)\pi(z-t)] & \text{Trapezoidal wave} \\ \left\{ \frac{4}{\pi} \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)} \cos[(2m-1)\pi(z-t)] \right\} & \text{Square wave} \end{cases} \quad (13)$$

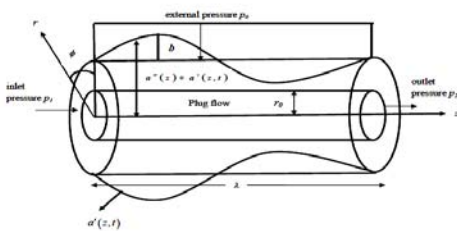


Fig. 1. Geometry of the physical model.

Further, we assume that the fluid enters in to the tube with the pressure p_1 and leaves it with pressure p_2 , while pressure outside the tube is p_0 . If z denotes the distance along the tube from the inlet end, then the pressure $p(z)$ in the fluid at z decreases from $p(0) = p_1$ to $p(\lambda) = p_2$. As a consequence of the pressure difference $p(z) - p_0$, between the inside and outside of the tube, the tube may expand or contract: Hence, the shape of its cross section may deform due to the elastic property of the wall. Therefore, the conductivity σ_1 of the tube at z will depend on the pressure difference. We consider $\sigma_1 = \sigma_1[p(z) - p_0]$ as a known function of $(p(z) - p_0)$. This conductivity is assumed to be the same as that of a uniform tube having the same cross section as that at z . We assume that the Q is related to the pressure gradient by the relation

$$Q = \sigma_1(p - p_0)(-dp/dz) \quad (14)$$

Now, from (12) and (14), we observe that

$$\sigma_1(p - p_0) = Fa'^4/8 \quad (15)$$

Taking elastic property in addition to the peristaltic movement of the tube wall into consideration, we can take $\sigma_1 = (F(a' + a''))^4/8$ where a' is the change in radius of the tube due to peristalsis and a'' is the change in the radius of the tube due to elasticity. As the flow is of Poiseuille type, at each cross section, the radius a'' is a function of pressure $p - p_0$, $a''(p - p_0)$ and the wall deformation due to the infinite train of peristaltic waves represented by equation (13) which is a function of z and t . Integrating (14) with respect to z from $z = 0$ and using the inlet condition $p(0) = p_1$, we obtain

$$Qz = \int_{p(z)-p_0}^{p_1-p_0} (\sigma_1(p')) dp' \quad (16)$$

where $p' = p(z) - p_0$. This equation determines $p(z)$ implicitly in terms of Q and z . To find Q , we set $z = 1$ and $p(1) = p_2$ in equation (16) to obtain

$$Q = \int_{p(1)-p_0}^{p_1-p_0} (\sigma_1(p')) dp' \quad (17)$$

Now, using (15) in (17), we have

$$Q = \int_{p_2-p_0}^{p_1-p_0} \frac{F(a'+a'')^4}{8} dp' \quad (18)$$

Eq. (18) can be solved if we know the form of the function $a''(p - p_0)$. If the stress or tension $T(a'')$ in the tube wall is known as a function of a'' , then $a''(p')$ can be found using the equilibrium condition (Rubinow and Keller 1972)

$$T(a'')/a'' = p - p_0 \quad (19)$$

3.3 Application to Flow Through an Artery

We now find flow through an artery by two different methods.

3.3.1 Method of Rubinow and Keller

Roach and Burton (1959) determined the static pressure-volume relation of a 4 cm long piece of the human external iliac artery, and converted it into a tension versus length curve. Using least squares method, (Rubinow and Keller, 1972) we have

$$T(a'') = t_1(a'' - 1) + t_2(a'' - 1)^5 \quad (20)$$

where $t_1 = 13$ and $t_2 = 300$. When we substitute (19) in (20) we get

$$p' = p - p_0 = \frac{1}{a''} [t_1(a'' - 1) + t_2(a'' - 1)^5] \quad (21)$$

$$dp' = \left[\frac{t_1}{a'^2} + t_2 \left(4a'^3 - 15a'^2 + 20a' - 10 + \frac{1}{a'^2} \right) \right] da'' \tag{22}$$

Using (21) in (18), we get the flux as given below:

$$Q = \frac{F}{8} \left(g(a_1'') - g(a_2'') \right) \tag{23}$$

Where

$$g(a) = t_1 \left[\frac{a'^3}{3} + 2a'd' + 6a'^2a'' + 4d'^3 \log \log a' - \frac{a'^4}{a'} \right] + t_2 \left[\frac{a'^8}{2} + \frac{a'^7}{7} (16a' - 15) + \frac{a'^6}{6} (24a'^2 - 60a' + 20) + \frac{a'^5}{5} (16a'^3 - 90a'^2 + 80a' - 10) + \frac{a'^4}{4} (4a'^4 - 60a'^3 + 120a'^2 - 40a') + \frac{a'^3}{3} (-15a'^4 + 80a'^3 - 60a'^2 + 1) + \frac{a'^2}{2} (20a'^4 - 40a'^3 + 4a') + a' (-10a'^4 + 6a'^2) + 4a'^3 \log \log a' - \frac{a'^4}{a''} \right]$$

We observe that equation (23) reduce to the corresponding results of Rubinow and Keller (1972) for the flow of Newtonian fluid ($a' = 0$, and $\tau = 0$) in an elastic tube.

3.2.2 Method of Mazumdar

Following Mazumdar (1992), the tension relation can be written as

$$T(a'') = A \left(e^{Ka''} - e^K \right) \tag{24}$$

where $A=0.007435$ and $K=5.2625$. When we substitute (24) in (19) we get

$$p' = p - p_0 = \frac{1}{a'} \left[A \left(e^{Ka''} - e^K \right) \right] \tag{25}$$

$$dp' = A \left(e^{Ka''} \left(\frac{K}{a''} - \frac{1}{a'^2} \right) + \frac{e^K}{a'^2} \right) da'' \tag{26}$$

Using (26) in (18), we get the flux as follows:

$$Q = \frac{FA}{8} \int_{a_2''}^{a_1''} (a' + a'')^4 \left(e^{Ka''} \left(\frac{K}{a''} - \frac{1}{a'^2} \right) + \frac{e^K}{a'^2} \right) da'' \tag{27}$$

Where $a'(z)$ is given by equation (13) and

$$a'' = a''(p - p_0) \tag{28}$$

We note that a_1'' and a_2'' are determined by solving (19) with $p = p_1$ and $p = p_2$, respectively. The above integral (27) after numerical evaluation gives the value of flux for Casson's model in an elastic tube. We observe that equation (27) reduce to the corresponding results of Mazumdar (1992) for the flow of Casson fluid in an elastic tube without peristalsis (i.e. $a' = 0$). Using the above two cases we find the flux for a Casson fluid in an elastic tube.

4. RESULTS AND DISCUSSION

The objective of the present investigation is to understand the change in flow pattern, and estimate the increase in the flow resistance in a small artery due to the presence of a catheter (by modeling the Cow's blood as Casson fluid, which flows due to a peristaltic wave). Here, Blood is modeled as a Casson fluid and this model considers the yield stress parameter along with the elasticity of wall and peristalsis. The main advantage of this model is that it includes the expressions for tension as a fifth degree polynomial and also as an exponential function. Further, the flow is assumed to be steady, laminar, fully developed and axially symmetric. The expressions for velocity, plug flow velocity, wall shear stress and the flux flow rate are derived analytically. The flux is determined as a function of inlet, outlet, yield stress and the elastic property of the tube. In order to assess the quantitative effects of catheterization; the non-Newtonian fluid; Peristaltic wave motion; the elastic property of the tube; flux; and inlet and outlet radius involved in the problem, the numerical computations are carried with the Mathematica software for the analytical expressions. The numerical results are plotted graphically in Figs. 2-5.

Figs. 2(a-c) respectively, represent the profiles for the flux with the axis for different values of amplitude ratios and different peristaltic waves, namely, when the wave is sinusoidal, trapezoidal and square, by Rubinow and Keller method for the flow through an artery. From the graphical representations it is clearly noticed that as the amplitude ratio increases the flux also increases. This observation is more consistent when the peristaltic wave is sinusoidal as compared to other two ways, namely, square wave and trapezoidal wave. In Figs. 3(a)-Fig.3(c) flux with the axis profiles are presented for the same set of physical parameters except the flow through an artery by one more method namely, Mazumdar method. Comparison of the Figures reveals that the flux increases as the amplitude ratio increases. Furthermore, it is clearly notify that the flux is more when the peristaltic wave is trapezoidal as compared to other two waves.

Fig. 4(a)-Fig. 4(b) respectively, describe the method of Rubinow and Keller for the variations of flux with the axis for different values of yield stress parameter and the elastic parameter. It is observed that the flux in non-Newtonian case is greater than the flux in the Newtonian case. This is consistent with the physical situation of the yield stress and the non-linear nature of non-Newtonian fluid namely, Casson fluid. The variation of flux with the axis for Casson fluid is calculated for different values of yield stress and is depicted in Fig.4 (a). It can be clearly seen from Fig. 4 (a) that for a given axis, the flux depends on yield stress and it decreases with increasing yield stress. This trend is even true when the flow is through a small artery by the

method of Mazumdar shown graphically in Fig. 4(c). A comparison of Fig. 4(a) and Fig. 4(c) reveals that the flux is much more enhanced in Rubinow and Keller method as compared to Mazumdar method. The effect of an increase in the elastic parameter is to increase the flux for the non-Newtonian model when the other elastic parameter is fixed shown graphically in Fig. 4(b).

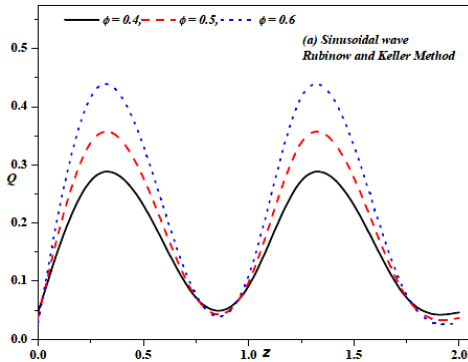


Fig. 2(a). Variation of Q Vs. z for different values of amplitude ratio with $t = 0.1$, $\tau = 0.5, t_1 = 13, t_2 = 300, a_1'' = 0.2, a_2'' = 0.3$.

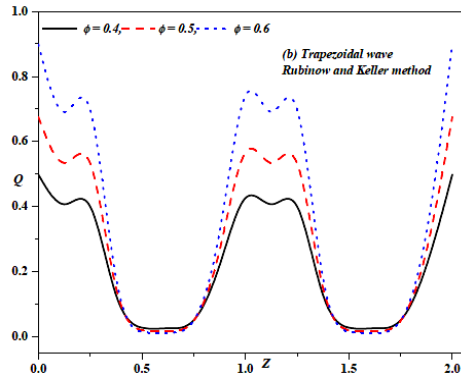


Fig. 2(b). Variation of Q Vs. z for different values of amplitude ratio with $t = 0.1, \tau = 0.5, t_1 = 13, t_2 = 300, a_1'' = 0.2, a_2'' = 0.3$.

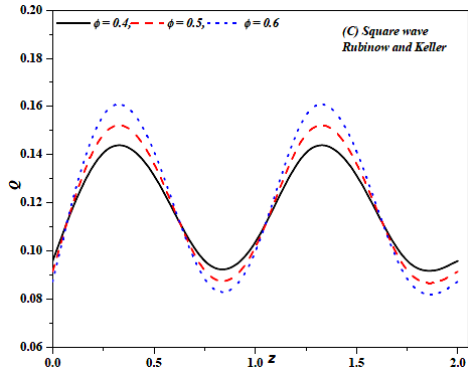


Fig. 2(c). Variation of Q Vs. z for different values of amplitude ratio with $t = 0.1, \tau = 0.5, t_1 = 13, t_2 = 300, a_1'' = 0.2, a_2'' = 0.3$.

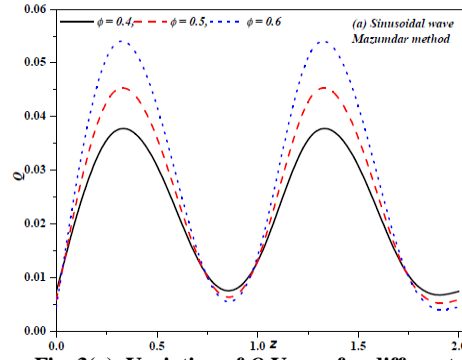


Fig. 3(a). Variation of Q Vs. z for different values of amplitude ratio with $t = 0.1, \tau = 0.5, A = 0.007435, K = 5.2625, a_1'' = 0.2, a_2'' = 0.3$.

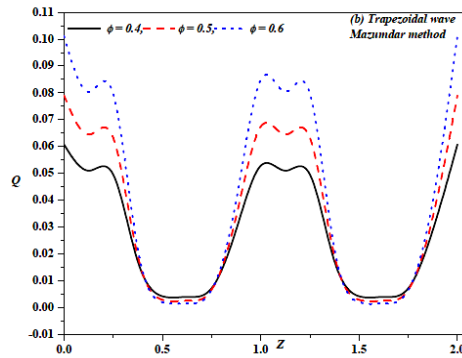


Fig. 3(b). Variation of Q Vs. z for different values of amplitude ratio with $t = 0.1, \tau = 0.5, A = 0.007435, K = 5.2625, a_1'' = 0.2, a_2'' = 0.3$.

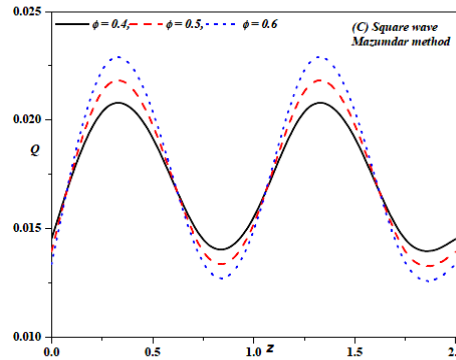


Fig. 3(c). Variation of Q Vs. z for different values of amplitude ratio with $t = 0.1, \tau = 0.5, A = 0.007435, K = 5.2625, a_1'' = 0.2, a_2'' = 0.3$.

The effects of the other remaining parameters on the flux with the axis profiles are drawn graphically in Figs. 5(a)-5(d) for the flow through an artery by Mazumdar method. From the Figs. 5(a) and 5(b) it is observed that as the elastic parameter A is to increase the flux in the presence of a Casson fluid when the other elastic parameter K is fixed. This trend is even true with the other elastic parameter, namely, K . This is shown graphically in Fig. 5(b).

The flux profiles with inlet and outlet elastic radius variations are shown graphically in Figs. 5(c)-5(d). For a fixed value of outlet radius the effect of increasing values of inlet elastic radius makes the flux to decrease and hence Q decreases as the inlet elastic radius increases. However this is the opposite of the behavior when we fix inlet elastic radius and vary the outlet elastic radius (Fig. 5(d)). This is consistent with the physical situation.

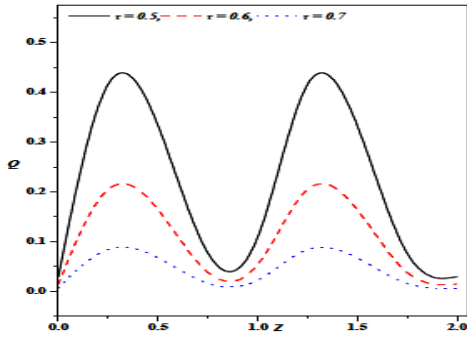


Fig. 4(a). Variation of Q Vs. z for different values of yield stress with (by Rubinow and Keller method for Sinusoidal wave), $t = 0.1, \varphi = 0.6,$

$$t_1 = 13, t_2 = 300, a_1'' = 0.2, a_2'' = 0.3.$$

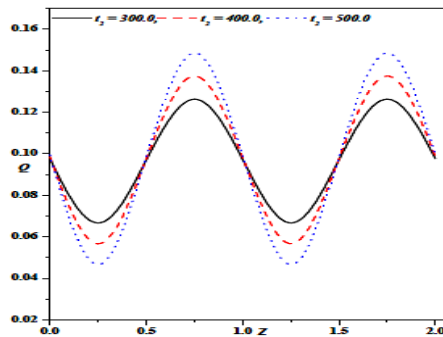


Fig. 4(b). Variation of Q Vs. z for different values of elastic parameter t_2 with (by Rubinow and Keller method for Sinusoidal wave), $t = 0.1,$

$$\varphi = 0.6, t_1 = 13, t_2 = 300, a_1'' = 0.2, a_2'' = 0.3..$$

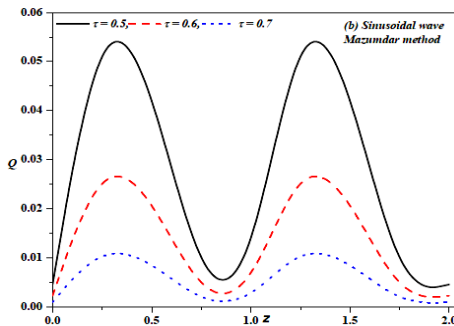


Fig. 4(c). Variation of Q Vs. z for different values of yield stress with (by Mazumdar method for Sinusoidal wave), $t = 0.1, \tau = 0.5, .$

$$A = 0.007435, K = 5.2625, a_1'' = 0.2, a_2'' = 0.3.$$

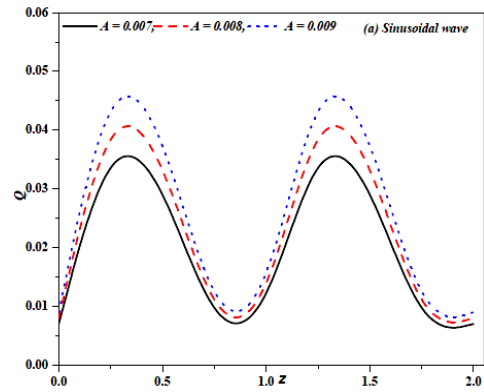


Fig. 5(a). Variation of Q Vs. z for different values of elastic parameter A with $t = 0.1, \tau = 0.5, .$

$$\varphi = 0.6, K = 5.2625, a_1'' = 0.2, a_2'' = 0.3.$$

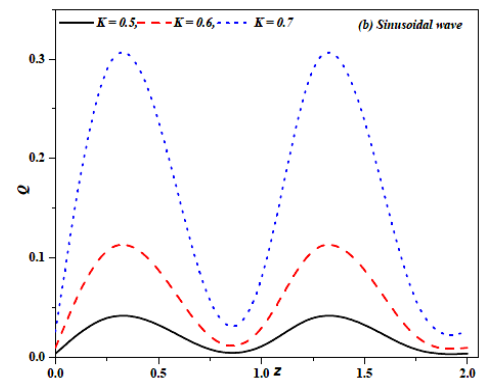


Fig. 5(b). Variation of Q Vs. z for different values of elastic parameter K with $t = 0.1, \tau = 0.5, .$

$$\varphi = 0.6, A = 0.007435, a_1'' = 0.2, a_2'' = 0.3.$$

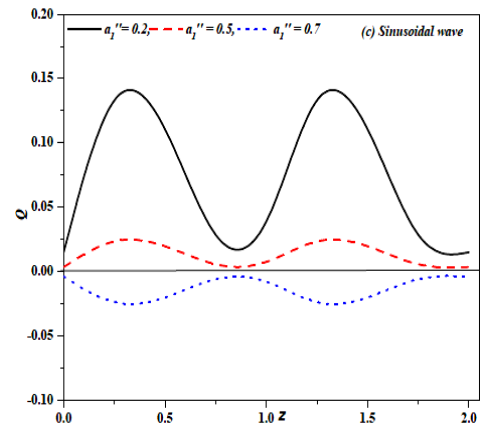


Fig. 5(c). Variation of Q Vs. z for different values of inlet elastic radius a_1'' with $t = 0.1, .$

$$\tau = 0.5, \varphi = 0.4, A = 0.007435, K = 5.2625,$$

$$a_2'' = 0.6.$$

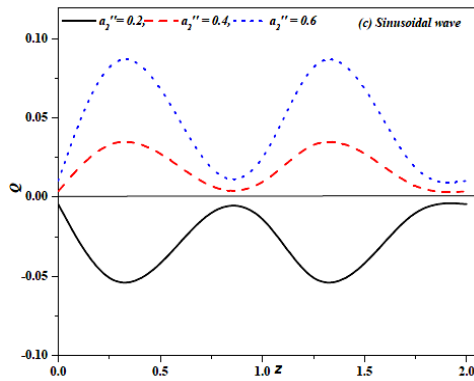


Fig. 5(d). Variation of Q Vs. z for different values of inlet elastic radius a_2'' with $t = 0.1$,

$\tau = 0.5, \varphi = 0.4, A = 0.007435, K = 5.2625, a_1'' = 0.3$.

5. CONCLUSIONS

The present study deals with the Poiseuille flow of a non-Newtonian fluid with non-zero yield stress, namely Casson fluid, to study the changes in the blood flow pattern when a catheter is inserted into an elastic tube. Blood is modeled as a Casson fluid. This fluid model considers the yield stress parameter along with the elasticity of wall and peristalsis. The results are analyzed for different values of the pertinent parameters namely, yield stress, radius, different wave forms, amplitude ratio and the elasticity of the tube wall. Some of the interesting findings are:

- the flux increases with an increase in the radius of the tube;
- the flux decreases with increasing values of the yield stress;
- the flux increases with an increase in the values of amplitude ratio;
- the flux increases with increasing values of the elastic parameters;
- the flux is more for a trapezoidal wave case than the square wave case or the sinusoidal wave case; and the back flow occurs if the inlet radius is more than the outlet radius. Rubinow and Keller method is better than the Mazumdar method with respect to the effect of yield stress and the amplitude ratio.

The newly derived Casson formulae will facilitate modeling non-Newtonian rheology in single distensible tubes which can be attributed to elasticity and peristalsis. This is a major addition to the existing modeling capabilities.

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