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To Study Large Time Step High Resolution Low Dissipative Schemes for Hyperbolic Conservation Laws

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ABSTRACT

Total Variation Diminishing (TVD) schemes are low dissipative and high resolution schemes but bounded by stability criterion CFL<1 for explicit formulation. Stability criteria for explicit formulation limits time stepping and thus increase computational cost (computational time, machine cost). Research in the field of large time step (LTS) scheme is an active field for last three decades. In present work, Zhan Sen Qian's modified form of Harten LTS TVD scheme is studied and used to solve one dimensional benchmark test cases. SOD and LAX cases of shock tube problem are solved to understand the behavior of modified large time step scheme in regions of discontinuities and strong shock waves. The numerical results are found to be in good agreement with analytical results, except slight oscillations near contact discontinuity for larger values of K. Results also reveal that the discrepancy between numerical and analytical results near expansion fan, contact discontinuity and shock grows for larger values of K. Increase in discrepancy is due to the increase in truncation error. Truncation error strongly depends on step size and step size increases as CFL (or K) increases. In present work, the correction into the numerical formulation of characteristic transformation is discussed and the inverse characteristic transformations are performed using local right eigen vector in each cell interface location. This idea of extending Harten's large time step method for hyperbolic conservation laws proved to be very useful as the results shows that the modified scheme is a high resolution low dissipative and efficient scheme for 1D test cases.

Keywords: TVD scheme; Shock tube problem; Explicit scheme; Efficient scheme; 1D Euler equation.

NOMENCLATURE

A	inviscid flux Jacobi matrix
a	eigen values
c	local speed of sound
c_l	constant
E	total Energy
F	physical flux
f	numerical flux
g	flux correction
ğ	lmiter function
i	space index
K	CFL restriction parameter
n	time index
p	pressure
Q	vector of unknown conservative variable
R	right eigen vector matrix
R^{-1}	left eigen vector matrix
u	velocity in x-direction
α	characteristic variable
ε	small positive parameter
γ	ratio of specific heat
ρ	density

1. INTRODUCTION

Computational fluid dynamics (CFD) is a technology that enables to study the dynamics of things that flow (Anderson, 1995)(John C. Tannehill, 1997). CFD as a computational technology is eminently suited to develop the concept of numerical test rig (Anderson J. D., 2002)(Mukkarum Husain C.-H. L., 2012). Computation of fluid flows containing expansion fan, contact discontinuity and shock waves is a challenging task due to the presence of complex physics (B.Laney, 1998)(T. Schwartzkop, 2002).

Classical shock-capturing methods provide accurate results only for smooth or weak shock flows, and are not robust enough for strong shock wave calculations (Lax, 1970's)(A. Harten, 1976). Oscillation and stability problems are major concern for classical shock capturing methods across the discontinuities and for strong shock waves. To resolve stability problem linear numerical dissipation or artificial viscosity is added into classical shock-capturing methods (H. C. Yee, 1989) (H. C. Yee, 1987) (H. C. Yee, 1988). But in flow where discontinuities and strong shock waves are present this methodology alone will not promise to compute a physically accurate solution. Some examples of classical shock capturing methods are Mac Cormack scheme, Lax–Wendroff scheme, and Beam-Warming scheme (John C. Tannehill, 1997)(B.Laney, 1998)(Malalasekra, 2007) (Boulahia .A, 2014).In 1983, Harten (Harten A. , 1983) introduced the concept of Total Variation Diminishing (TVD) scheme. TVD schemes are monotonicity preserving schemes and therefore, it must not create local extrema and the value of an existing local minimum must be non-decreasing and that of a local maximum must be non-increasing. Numerical dissipation terms in TVD methods are nonlinear (H. C. Yee, 1987). The quantity varies from one grid point to another and usually consists of automatic feedback mechanisms to control the amount of numerical dissipation. While TVD formulations are very reliable, versatile, and quite accurate, it is bounded by stability criterion CFL<1 for explicit formulation.

Stability criteria for explicit formulation limits time stepping and thus increase computational cost. Research in the field of large time step (LTS) scheme is an active field for last three decades. In 1986, Harten proposed a large-time-step (LTS) TVD scheme (Harten, 1986) which is stable for values of CFL >1. Computation of scalar problems depicts that Harten’s LTS scheme is a high resolution and efficient scheme. However, computation of hyperbolic conservation laws show some spurious oscillations in the vicinities of discontinuities when CFL > 1 (ZhanSen Qian C. L., 2011)(ZhanSen Qian C.-H. L., 2012) (Huang Huang, C.L., 2013).

Zhan Sen Qian studied Harten LTS scheme and noticed that these spurious oscillations which are not present in the computation of scalar problems are due to the numerical formulation of the characteristic transformation used by Harten for extending the method for hyperbolic conservation laws. He proposed to perform the inverse characteristic transformations by using the local right eigenvector matrix at each cell interface location to overcome these spurious oscillations. His results for shock tube problem depict that the oscillations are eliminated without increasing the entropy fixing parameter. Shock tube is one of the few 1D problem for which analytical solution is possible and hence it is often used as a test case for validation of numerical schemes(ZhanSen Qian C. L., 2011)(ZhanSen Qian C.-H. L., 2012)(SOD, 1978)(Mukkarum Husain C.-H. L., 2009)(Mukkarum Husain C.-H. L., 2009).

In present work, Zhan Sen Qian’s modified form of Harten LTS TVD scheme is studied and applied to solve one dimensional benchmark test cases. SOD and LAX cases of shock tube problem are solved to understand the behavior of modified large time step scheme in regions of discontinuities and strong shock waves. It is observed that the inverse characteristic transformations by using the local right eigenvector matrix in each cell interface location results a high resolution low dissipation and remarkably efficient scheme.

2. SHOCK TUBE PROBLEM

The shock tube problem consists of a tube of fluid that is initially at rest. A central diaphragm in the tube separates two states of the fluid. The fluid to the left has a higher pressure and energy as compared with the fluid on the right. The analytical solution to this problem is known. It consists of a shock wave moving to the right, a contact discontinuity moving to the right with the speed of the fluid and a rarefaction moving to the left. A shock wave inside a shock tube may be generated by a small burst or through the buildup of high pressures which cause diaphragm(s) to burst and a shock wave to propagate down the shock tube (compressed-gas driven).Boundary conditions used in present computation for SOD and LAX test cases are described in Table 1.The size of computational domain is $0 \leq x \leq 2$ and number of grids are 1000.

Table 1 Description of 1D Test Cases

	SOD	LAX
P_R	0.1	0.571
ρ_R	0.125	0.5
V_R	0.0	0.0
P_L	1.0	3.528
ρ_L	1.0	0.445
V_L	0.0	0.698

3. GOVERNING EQUATIONS

In this paper 1D transient Euler equation in a conservation form is used:

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + A\frac{\partial Q}{\partial x} = 0 \tag{2}$$

where

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho E \end{bmatrix} \quad ; \quad F = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ (\rho E + p)u \end{bmatrix} \tag{3}$$

$$A = \frac{\partial F}{\partial Q}\begin{bmatrix} 0 & 1 & 0 \\ (\gamma-3)\frac{u^2}{2} & (3-\gamma) & (\gamma-1) \\ (\gamma-1)u^3-\gamma uE & -\frac{3}{2}(\gamma-1)u^2+\gamma E & \gamma u \end{bmatrix} \quad (4)$$

Equation (1) in numerical flux form can be written as:

$$u_i^{n+1} = u_i^n - \lambda\left(f^n_{i+\frac{1}{2}} - f^n_{i-\frac{1}{2}}\right) \quad (5)$$

where $\lambda = \frac{\Delta x}{\Delta t}$

Harten used the scheme of (LeVeque, 1982) (LeVeque, 1982) (M. Morales-Hernandez, 2012) and proposed large time step TVD scheme (Harten, 1986) which is second order accurate using $(2K+3)$ points explicit discretization for hyperbolic conservation laws increasing the CFL restriction uptoK.The numerical flux for LTS TVD is given by:

$$f_{i+\frac{1}{2}} = \frac{1}{2}[F_{i+1}+F_i] + \frac{1}{2\lambda}\sum_{k=1}^{m} R^k_{i+\frac{1}{2}}\left(g^k_{i+1}+g^k_i\right) - \frac{1}{\lambda}\sum_{k=1}^{m} R^k_{i+\frac{1}{2}}\left[\sum_{l=-K+1}^{K-1} C_l\left(v^k+\beta^k\right)_{i+l+\frac{1}{2}}\ \alpha^k_{i+l+\frac{1}{2}}\right] \quad (6)$$

here;

$$g_i^k = s\cdot\max\left[0,\min\left(\sigma^k_{i+\frac{1}{2}}\left|\alpha^k_{i+\frac{1}{2}}\right|, s\cdot\sigma^k_{i-\frac{1}{2}}\alpha^k_{i-\frac{1}{2}}\right)\right] \quad (7)$$

$$s = \mathrm{sgn}\left(\alpha^k_{i+\frac{1}{2}}\right) \quad (8)$$

$$\gamma^k_{i+\frac{1}{2}} = \Delta_{i+\frac{1}{2}}g^k/\alpha^k_{i+\frac{1}{2}} \quad (9)$$

$$v^k_{i+\frac{1}{2}} = \lambda a^k_{i+\frac{1}{2}} \quad (10)$$

$$\sigma(v) = \frac{K}{2}\left\{Q\left(\frac{v}{K}\right)\left[1+\frac{K-1}{2}Q\left(\frac{v}{K}\right)\right] - \frac{K+1}{2}\left(\frac{v}{K}\right)^2\right\} \quad (11)$$

where $\sigma(v) \geq 0$ for $|v| \leq K$

$$C_{\pm k}(v) = \begin{cases} c_k\left(\mu_{\mp}(v)\right), & 1 \leq k \leq K-1 \\ \frac{K}{2}Q\left(\frac{v}{K}\right), & k = 0 \end{cases} \quad (12)$$

$$\mu_{\pm}(v) = \frac{1}{2}\left[Q\left(\frac{v}{K}\right) \pm \frac{v}{K}\right] \quad (13)$$

$$Q(v) = \begin{cases} \frac{1}{2}\left(\frac{v^2}{\varepsilon}+\varepsilon\right) & |v| < \varepsilon \\ |v| & |v| \geq \varepsilon \end{cases} \quad (14)$$

$$R = \begin{bmatrix} 1 & 1 & 1 \\ u & u+c & u-c \\ \frac{u^2}{2} & \frac{u^2}{2}+uc+\frac{c^2}{(\gamma-1)} & \frac{u^2}{2}-uc+\frac{c^2}{(\gamma-1)} \end{bmatrix} \quad (15)$$

$$R^{-1} = \begin{bmatrix} 1-\frac{(\gamma-1)u^2}{2c^2} & (\gamma-1)\frac{u}{c^2} & -\frac{(\gamma-1)}{c^2} \\ -\frac{u}{2c}+\frac{(\gamma-1)u^2}{4c^2} & \frac{1}{2c}-\frac{(\gamma-1)u}{2c^2} & \frac{(\gamma-1)}{2c^2} \\ \frac{u}{2c}+\frac{(\gamma-1)u^2}{4c^2} & -\frac{1}{2c}-\frac{(\gamma-1)u}{2c^2} & \frac{(\gamma-1)}{2c^2} \end{bmatrix} \quad (16)$$

Harten large time step (LTS) TVD scheme proved to be high resolution second order accurate weak solution for hyperbolic conservation laws for larger values of CFL. Qian (ZhanSen Qian C. L., 2011) (ZhanSen Qian C.-H. L., 2012) pointed out that Harten's LTS TVD produce considerable error and oscillation especially in the vicinity of contact discontinuity and shock for large CFL (CFL>1). This deficiency was improved by Qian using inverse characteristic transformation with local right eigenvector matrix. The numerical flux is then given by

$$f_{i+\frac{1}{2}} = \frac{1}{2}[F_{i+1}+F_i] + \frac{1}{2\lambda}\sum_{k=1}^{m} R^k_{i+\frac{1}{2}}\left(g^k_{i+1}+g^k_i\right) - \frac{1}{\lambda}\sum_{l=-K+1}^{K-1}\left[\sum_{k=1}^{m} R^k_{i+l+\frac{1}{2}} C_l\left(v^k+\beta^k\right)_{i+l+\frac{1}{2}}\ \alpha^k_{i+l+\frac{1}{2}}\right] \quad (17)$$

Table 2: $C_l(x)$ at different K

K	C_1	C_2	C_3
2	x^2		
3	$x^2(3-x)$	x^3	
4	$x^2(6-4x+x^2)$	$2x^3(2-x)$	x^4

4. RESULTS AND DISCUSSION

The development of efficient high resolution low dissipative numerical schemes has been receiving more and more interest for last three decades (H. C. Yee, 1989)(H. C. Yee, 1987) (H. C. Yee, 1988) (Harten A. , 1983) (Shu). Present work is also focused on recently developed large time step high resolution low dissipative scheme. Modification of Harten large time step total variation diminishing (MHLTS-TVD) scheme proposed by Qian is studied for its merits and shortcomings. SOD and LAX shock tube problems are solved for validation. Shock tube is one of the few 1D problems for which analytical solution is possible to obtain and hence it is often used as a test case for validation of numerical schemes(SOD, 1978) (Mukkarum Husain C.-H. L., 2009). Simulation are carried out on Intel(R) Core(TM) i5-2410M CPU @ 2.30 GHz, 4 GB RAM.

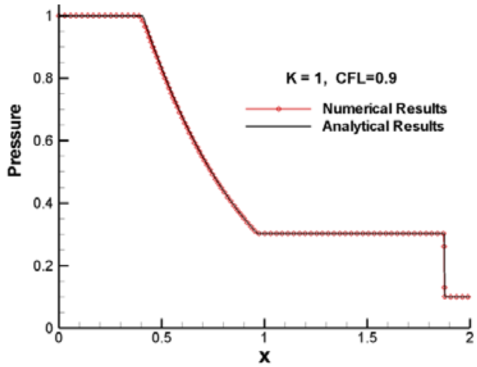

Fig. 1. Comparison of pressure profile for SOD case, K=1, CFL=0.9.

Fig 1-12 describes the numerical results computed for pressure, density and Mach number profiles along with analytical results for SOD case taking $K = 1, 2, 3$ and 4 for 0.9, 1.8, 2.8, and 3.8 values of CFL, respectively. Numerical results depicts that shock wave is resolved better than contact discontinuity. Reason behind this result is the basic difference between these two waves. For shock wave characteristic lines are convergent while for contact discontinuity they are parallel to each other, and therefore the dissipation near to the shock wave is controlled up to a small extent in the time marching steps irrespective of the numerical dissipation as compared to contact discontinuity. The numerical results are in good agreement with analytical results, except slight oscillations near contact discontinuity for large values of K.

Similarly, Fig 13-24 shows the numerical results computed for pressure, density and Mach number along with analytical results for LAX case taking $K = 1, 2, 3$ and 4 for 0.9, 1.8, 2.8, and 3.8 values of CFL, respectively. Same behavior is found as discussed above.

Table 3 K, No. of Iteration, and Simulation Time for SOD

K	No. of Iteration	Simulation Time (sec)	Simulation Time per Iteration(sec)
1	609	0.3432022	0.0005635504
2	305	0.1872012	0.0006137744
3	197	0.156001	0.0007918832
4	146	0.1248008	0.0008548

Table 3 and 4 shows the time required, number of iterations, and simulation time per iteration of SOD and LAX test cases. Significance of using large time step scheme is evident in both cases. Although, time for one step is increased as K increases due to the addition of more terms in the calculation of numerical flux but total number of iterations is correspondingly decreased with increase in K due to the increase in CFL, which play dominant role to decrease total simulation time. Therefore, scheme becomes more and more efficient as K increases.

Table 4 K, No. of Iteration, and Simulation Time for LAX

K	No. of Iteration	Simulation Time (sec)	Simulation Time per Iteration(sec)
1	578	0.3276021	0.0005667856
2	290	0.1872012	0.0006455214
3	187	0.1404009	0.000750807
4	138	0.1248008	0.0009043536

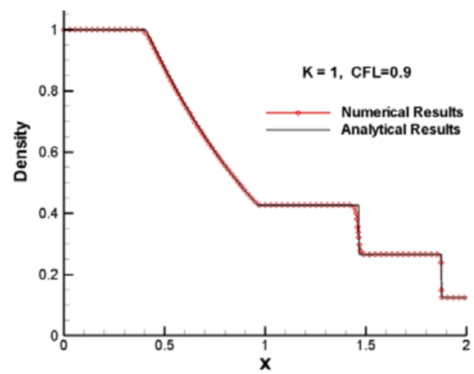

Fig. 2. Comparison of density profile for SOD case, K=1, CFL=0.9.

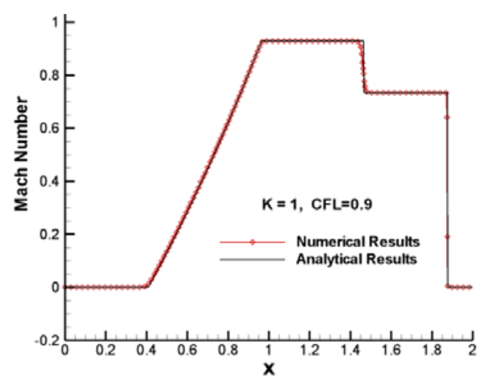

Fig. 3. Comparison of Mach number profile for SOD case, K=1, CFL=0.9.

Fig 25, 26 and 27 depicts the comparison of numerical and analytical results of density profile near expansion fan, contact discontinuity and shock respectively for the test case of SOD. Results reveal that the discrepancy between numerical and analytical results near expansion fan, contact discontinuity and shock grows for larger values of K. Increase in discrepancy is due to the increase in truncation error. Truncation error strongly depends on step size. Since step size increase as CFL (or K) increase therefore truncation error also increases. Similar behavior is observed in comparison of numerical and analytical results of pressure profile and Mach number profile.

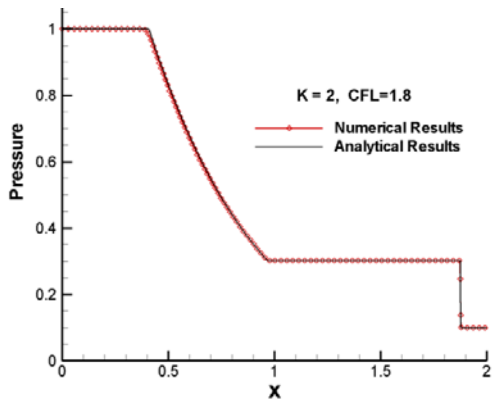

Fig. 4. Comparison of pressure profile for SOD case, K=2, CFL=1.8.

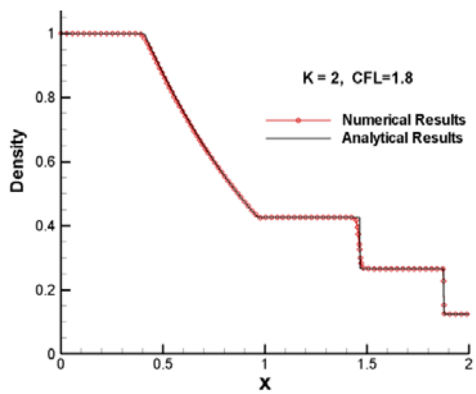

Fig. 5. Comparison of density profile for SOD case, K=2, CFL=1.8.

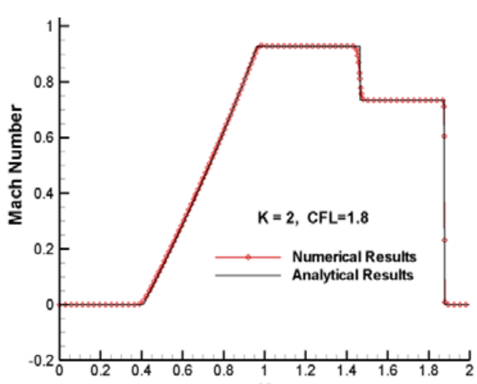

Fig. 6. Comparison of Mach number profile for SOD case, K=2, CFL=1.8.

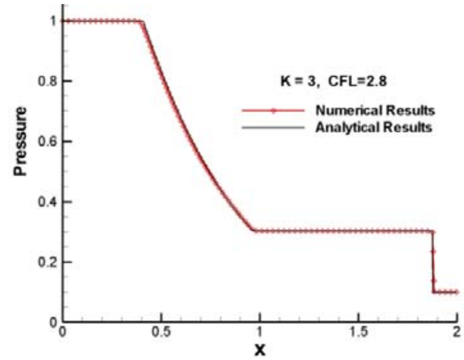

Fig. 7. Comparison of pressure profile for SOD case, K=3, CFL=2.8.

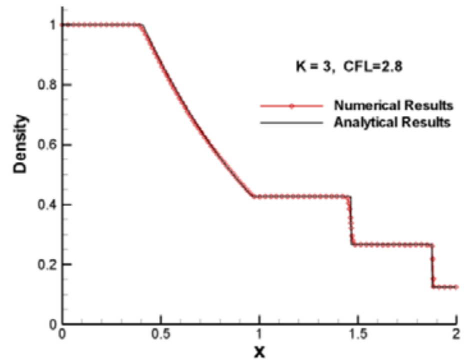

Fig. 8. Comparison of density profile for SOD case, K=3, CFL=2.8.

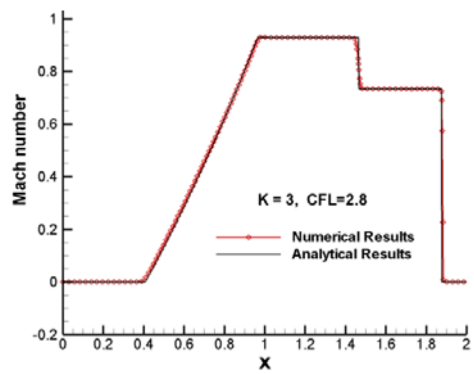

Fig. 9. Comparison of Mach number profile for SOD case, K=3, CFL=2.8.

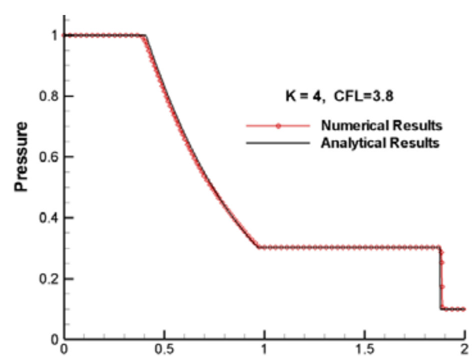

Fig. 10. Comparison of pressure profile for SOD case, K=4, CFL=3.8.

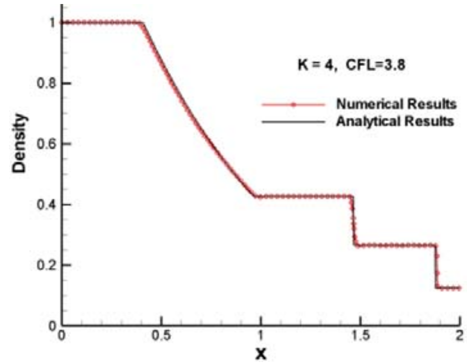

Fig. 11. Comparison of density profile for SOD case, K=4, CFL=3.8.

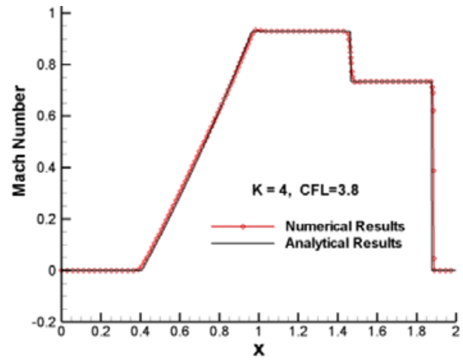

Fig. 12. Comparison of Mach number profile for SOD case, K=4, CFL=3.8.

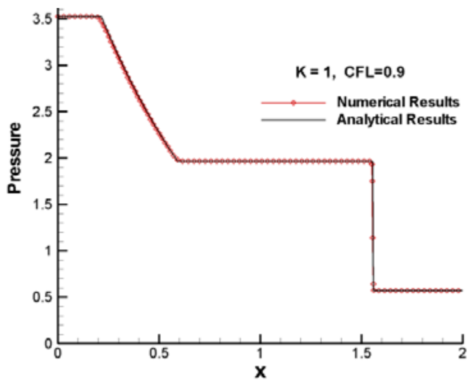

Fig. 13. Comparison of pressure profile for LAX case, K=1, CFL=0.9.

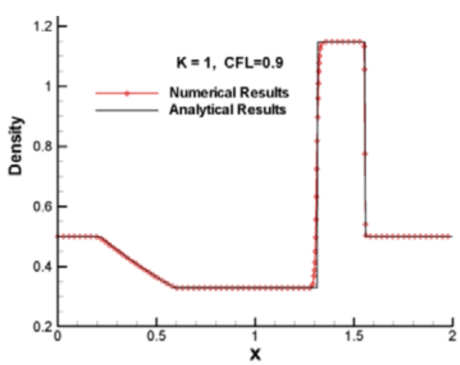

Fig. 14. Comparison of density profile for LAX case, K=1, CFL=0.9.

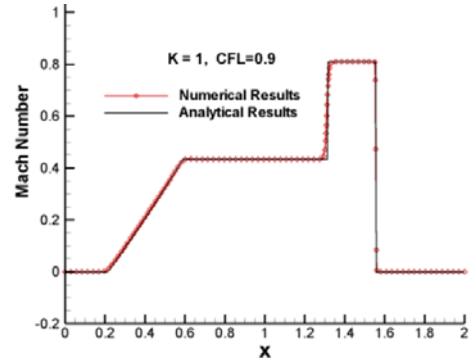

Fig. 15. Comparison of Mach number profile for LAX case, K=1, CFL=0.9.

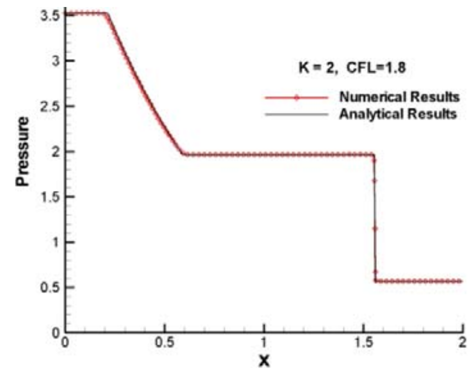

Fig. 16. Comparison of pressure profile for LAX case, K=2, CFL=1.8.

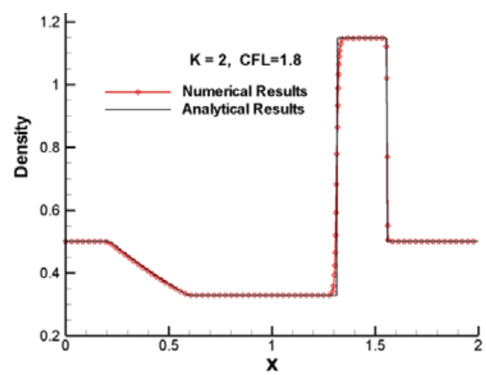

Fig. 17. Comparison of density profile for LAX case, K=2, CFL=1.8.

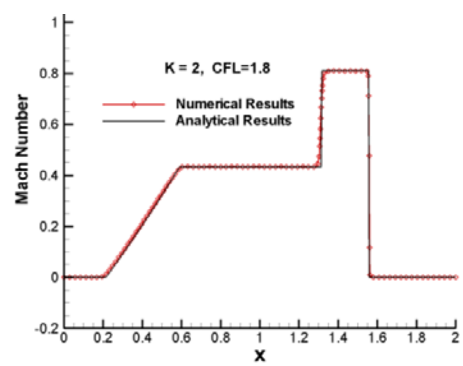

Fig. 18. Comparison of Mach number profile for LAX case, K=2, CFL=1.8.

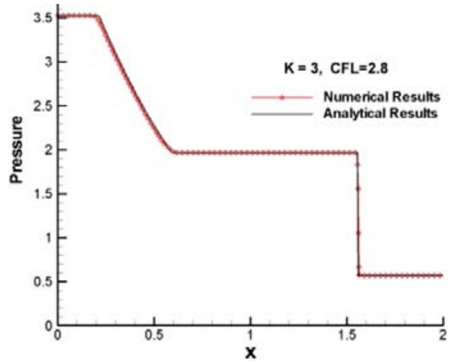

Fig. 19. Comparison of pressure profile for LAX case, K=3, CFL=2.8.

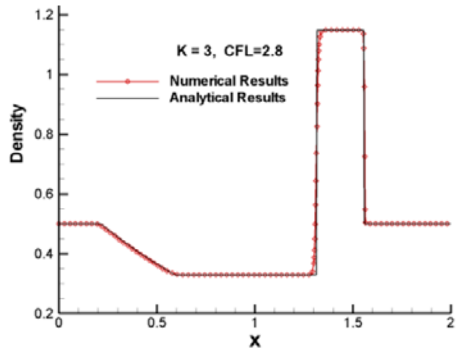

Fig. 20. Comparison of density profile for LAX case, K=3, CFL=2.8.

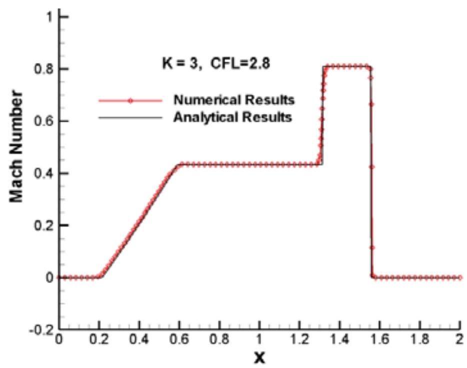

Fig. 21. Comparison of Mach number profile for LAX case, K=3, CFL=2.8.

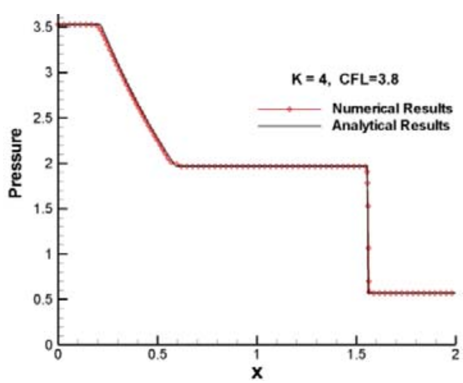

Fig. 22. Comparison of pressure profile for LAX case, K=4, CFL=3.8.

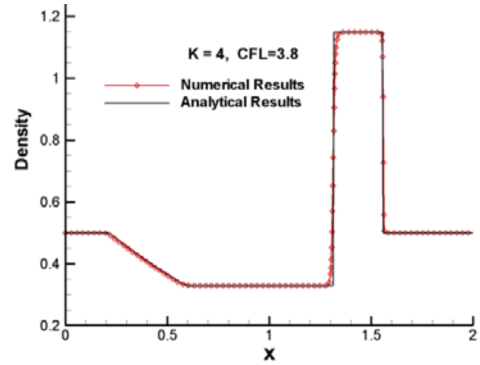

Fig. 23. Comparison of density profile for LAX case, K=4, CFL=3.8.

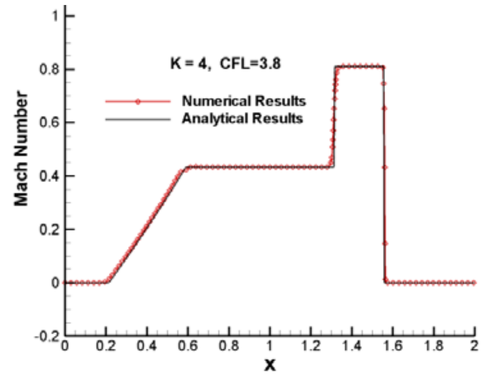

Fig. 24. Comparison of Mach number profile for LAX case, K=4, CFL=3.8.

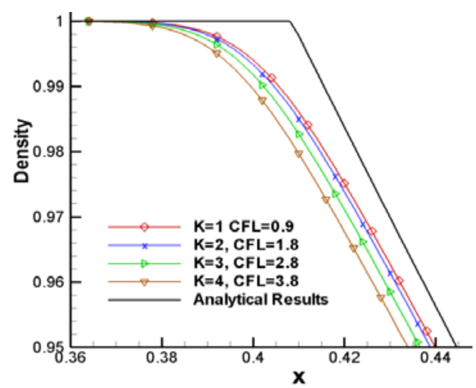

Fig. 25. Density profile near expansion fan, SOD.

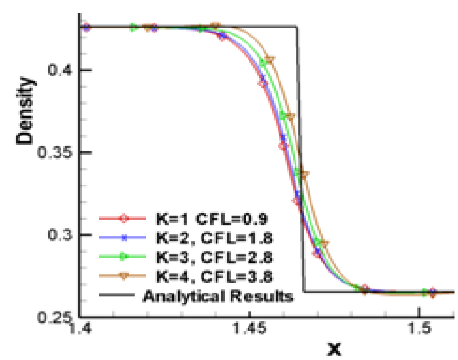

Fig. 26. Density profile near contact discontinuity, SOD.

In case of SOD, as shown in figures 1-12 the region of expansion fan is$0.4 \leq x \leq 1$, expansion wave exponentially decreased the pressure from 1 to 0.3 and the density from 1 to 0.4 while Mach number is exponentially increased from 0 to 0.95 for increasing values of x-location.The pressure across the contact wave is constant while density and Mach number is changed and contact discontinuity is occurred at $x = 1.5$. Shock is captured at $1.8 < x < 1.9$ where Mach number is reduced to zero again.

In case of LAX, as shown in figures 13-24, the region of expansion fan is$0.2 \leq x \leq 0.6$, expansion wave exponentially decreased the pressure from 3.5 to 1.95 and the density from 0.5 to 0.325 while Mach number is increased from 0 to 0.425for increasing values of x-location.The pressure across the contact wave is constant while density and Mach number is changed and contact discontinuity is occurred at $x = 1.3$. Shock is captured at $1.5 < x < 1.6$ where Mach number is reduced to zero.

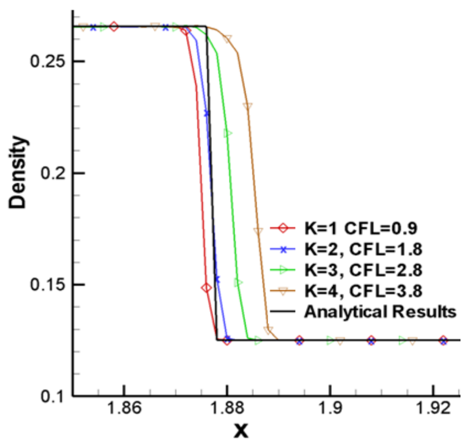

Fig. 27. Density profile near shock, SOD.

3. CONCLUSION

Computational Fluid Dynamics (CFD) as a computational technology is appropriate to develop the concept of numerical wind tunnel. But, it is still recommended to study and understand the behavior of a scheme before using for engineering design and analysis or as a research tool.

Stability criteria (CFL <=1) for explicit formulation limits time stepping and thus increase computational cost. Large time step schemes are efficient and considerably reduce computational time. It gives researcher a chance to try more possibilities during design and optimization face within stipulated time frame.

In present work, Zhan Sen Qian's modified form of Harten LTS scheme is studied and used to solve one dimensional benchmark test cases. The numerical results for LAX and SOD test cases are computed and found in good agreement with analytical result, except slight oscillations for larger values of K.

It is concluded that the correction into the numerical formulation of the characteristic transformation, for extending the Harten large time step method for hyperbolic conservation laws, by the inverse characteristic transformations using the local right eigenvector matrix in each cell interface location results a high resolution low dissipation and efficient scheme for one dimensional cases.

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