



Electrothermal Instability in a Porous Medium Layer Saturated by a Dielectric Nanofluid

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(Received June 4, 2015; accepted November 8, 2015)

ABSTRACT

The onset of convection in a porous medium saturated by a dielectric nanofluid with vertical AC electric field is investigated. The flux of volume fraction of a nanoparticle with the effect of thermophoresis is taken to be zero on the boundaries and the eigenvalue problem is solved using the Galerkin method. The model used for nanofluid incorporates the combined effect of Brownian diffusion, thermophoresis and electrophoresis, while for porous medium Darcy model is employed. The results show that increase in the AC electric Rayleigh-Darcy number, the Lewis number, the modified diffusivity ratio and the concentration Rayleigh-Darcy number are to hasten the onset of convection. The size of convection cells does not depend on nanofluid parameters, but decreases with increasing the AC electric Rayleigh-Darcy number. The non-existence of oscillatory convection is also obtained.

Keywords: Porous medium; Nanofluids; Electrohydrodynamic instability; Brownian motion and thermophoresis.

NOMENCLATURE

a	dimensionless wave number	β	coefficient of thermal expansion
a_c	critical wave number	μ	viscosity
c	specific heat	ρ	density of the nanofluid
d	diameter of nanoparticles	ρ_0	reference density of nanofluid
D_B	Brownian diffusion coefficient	ρ_e	charge density
D_T	thermophoretic diffusion coefficient	ρ_p	density of nanoparticles
\bar{E}	root mean square value of the electric field	(ρc)	heat capacity
\bar{f}_e	force of electrical origin	$(\rho c)_m$	effective heat capacity
\bar{g}	acceleration due to gravity	φ	porosity of the porous medium
K	permeability of the porous medium	ϵ	dielectric constant
k_m	effective thermal conductivity	ϕ	volume fraction of the nanoparticles
L_e	Lewis number	ϕ_0	reference scale for the nanoparticle fraction
N_A	modified diffusivity ratio	ψ	root mean square value of the electric potential
N_B	modified specific heat increment	γ	thermal expansion coefficient of dielectric constant
p	pressure	∇_p^2	horizontal Laplacian operator
\bar{v}	Darcy velocity	∇^2	Laplacian operator
R_D	thermal Rayleigh-Darcy number		
$R_{D,c}$	critical thermal Rayleigh-Darcy number		
R_e	AC electric Rayleigh-Darcy number		
t	time	Superscripts	
T	temperature	'	perturbed quantities
(x, y, z)	space co-ordinates	Subscripts	
		p	particle
		b	basic state

0 lower boundary
 1 upper boundary

1. INTRODUCTION

The term ‘nanofluid’ first coined by Choi (1995) refers to liquid dispersions of submicron solid particles or nanoparticles, whose characteristic dimension is of order of tens or hundreds of nanometers. Nanoparticles used in nanofluid are typically made of oxide ceramics (Al₂O₃, CuO), metal carbides (SiC) or metals (Al, Cu) and base fluids are water, oil, bio-fluids, polymer solutions, other common fluids. The presence of the nanoparticles in the fluid increased the effective thermal conductivity of the fluid and consequently enhanced the heat transfer characteristics. The enhanced thermal properties of nanofluids make them potentially useful in many energetical systems where improved heat transfer or efficient heat dissipation is required such as cooling of micro-electronic components, cooling of nuclear systems, radiators and automatic transmissions etc. There are some review papers that show applications and detail characteristic feature of nanofluids (Wang, 2007; Wong and Leon, 2010; Saidur, 2011). A nanofluid modelling was made by Buongiorno (2006) by considering the effects of Brownian diffusion and thermophoresis. This model was applied to study the onset of convection in a nanofluid layer by Tzou (2008a,b) Nield and Kuznetsov (2009, 2013, 2014), Umavathi *et al.* (2015), Shivakumara *et al.* (2015), Agrawal *et al.* (2014), Rana and Agrawal (2015), Yadav *et al.* (2013a,b, 2014a, 2015a,b, 2016a,b,c,d,e) and Sheikholeslami *et al.* (2013, 2015a,b).

Electroconvection in a dielectric fluid saturated porous medium in the presence of an electric field is of particular importance in view of its possibility of reduction of fluid viscosity in enhancing petroleum production and a control of heat and mass transfer in high voltage devices by electric field (Moreno *et al.*, 1996). Rudraiah and Gayathri (2009) have investigated the effect of thermal modulation and vertical electric field on the electroconvection in a horizontal dielectric fluid saturated densely packed porous layer and they have also discussed the importance of ETC in porous media. El-Sayed *et al.* (2011) have analyzed the nonlinear stability analysis of wave propagation in two superposed dielectric fluids saturated-porous media in the presence of vertical electric field producing surface charges. The onset of ETC in a rotating Brinkman porous layer has been investigated by Shivakumara *et al.* (2011). An extensive review on this topic has been by Nield and Bejan (2013). Very recently, Awasthi *et al.* (2014) analyzed the viscous potential flow analysis of Electro-hydrodynamic Rayleigh-Taylor instability. They observed that upper fluid fraction and electric field both have stabilizing effect on the stability of the

considered system while dielectric constant ratio plays dual role on the stability of the system.

Under the situations, the study of electric field on the onset of dielectric nanofluid convection in a porous medium seems to be significance in enhancing petroleum production and in electrical equipment such as distribution transformers, regulating transformers and shunt reactors (Asadzadeh *et al.*, 2012), and has not been given any attention in the literature. Therefore, the purpose of the research treated here is to examine theoretically the effect of a vertical AC electric field on the criterion for the onset of convection in a nanofluid saturated horizontal layer of porous medium.

2. PROBLEM FORMULATION

We consider an infinite horizontal layer of incompressible dielectric nanofluid-saturated porous layer of thickness d , heated from below. A Cartesian co-ordinate system (x, y, z) is chosen in which z axis is taken at right angle to the boundaries. The nanofluid is confined between two parallel plates $z = 0$ and $z = d$, where the temperatures at the lower and upper boundaries are taken to be T_0 and T_1 , respectively, T_0 being greater than T_1 . Nanofluid layer is subjected to a uniform vertical AC electric field applied across the layer; lower surface is grounded and upper surface is kept at an alternating potential whose root mean square is ν_1 . For simplicity, Darcy’s law is assumed to hold and the Oberbeck--Boussinesq approximation is employed. Homogeneity and local thermal equilibrium in the porous medium is assumed. According to the works of Nield and Kuznetsov (2009) and Shivakumara *et al.* (2011), the governing equations under this model are:

$$\nabla \cdot \bar{\mathbf{v}} = 0, \tag{1}$$

$$0 = -\nabla p - \frac{\mu}{K} \bar{\mathbf{v}} + \left[\phi \rho_p + \rho_0 (1 - \phi) \right] \times \{1 - \beta (T - T_1)\} \bar{\mathbf{g}} + \bar{\mathbf{f}}_e, \tag{2}$$

$$\left[(\rho c)_m \frac{\partial}{\partial t} + (\rho c)(\bar{\mathbf{v}} \cdot \nabla) \right] T = k_m \nabla^2 T + \phi (\rho c)_p \times \left[D_B \nabla \phi \cdot \nabla T + \left(\frac{D_T}{T_1} \right) \nabla T \cdot \nabla T \right], \tag{3}$$

$$\left[\frac{\partial}{\partial t} + \frac{1}{\phi} (\bar{\mathbf{v}} \cdot \nabla) \right] \phi = D_B \nabla^2 \phi + \frac{D_T}{T_1} \nabla^2 T, \tag{4}$$

where $\bar{\mathbf{v}} = (u, v, w)$ is the Darcy velocity, t is the time, ϕ is the porosity of the porous medium,

K is the permeability of the porous medium, ϕ is the nanoparticles volume fraction, p is the pressure, T is the temperature, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient, g is the gravitational acceleration, ρ_p is the density of the particle (ρc) is the heat capacity of nanofluid, $(\rho c)_m$ is the effective heat capacity, k_m is the effective thermal conductivity, ρ_p is the density of nanoparticles, ρ_0 , μ and β are the density, viscosity and thermal volumetric expansion coefficient of nanofluid, respectively and \mathbf{f}_e is the force of electrical origin which can be expressed by Landau and Lifshitz (1960) for incompressible nanofluid as

$$\mathbf{f}_e = \rho_e \bar{\mathbf{E}} - \frac{1}{2} (\bar{\mathbf{E}} \cdot \bar{\mathbf{E}}) \nabla \epsilon. \quad (5)$$

Here $\bar{\mathbf{E}}$ is the root mean square value of the electric field, ρ_e is the charge density and ϵ is the dielectric constant. The first term on the right hand side is the Coulomb force due to a free charge and the second term depends on the gradient of ϵ . If an AC electric field is applied at a frequency much higher than the reciprocal of the electrical relaxation time, the free charge does not have time to accumulate. Moreover, the electrical relaxation times of most dielectric liquids appear to be sufficiently long to prevent the buildup of free charge at standard power line frequencies. At the same time, dielectric loss at these frequencies is so low that it makes no significant contribution to the temperature field. Under the circumstances, only the force induced by non-uniformity of the dielectric constant is considered. Furthermore, since the second term in the above equation depends on $(\bar{\mathbf{E}} \cdot \bar{\mathbf{E}})$ rather than

$\bar{\mathbf{E}}$ and the variation of $\bar{\mathbf{E}}$ is very rapid, the root mean square value of $\bar{\mathbf{E}}$ can be assumed as the effective value. In other words, we can treat the AC electric field as the DC electric field whose strength is equal to the root mean square value of the AC electric field. Assuming the free charge density is negligibly small, the relevant Maxwell equations are (Roberts, 1969):

$$\nabla \times \bar{\mathbf{E}} = 0, \quad (6)$$

$$\nabla \cdot (\epsilon \bar{\mathbf{E}}) = 0. \quad (7)$$

In view of Eq. (6), $\bar{\mathbf{E}}$ can be expressed as

$$\bar{\mathbf{E}} = -\nabla \psi, \quad (8)$$

where ψ is the root mean square value of the electric potential.

The dielectric constant is assumed to be a linear function of temperature in the form

$$\epsilon = \epsilon_0 [1 - \gamma(T - T_1)] = 0, \quad (9)$$

where $\gamma (>0)$ is the thermal expansion coefficient of dielectric constant and is assumed to be small.

In previous studies of convective instability problems for nanofluids, the volumetric fraction of nanoparticles was prescribed at the boundaries. But it is observed that this type of boundary condition on volume fraction of nanoparticles is physically not realistic because in practice controlling the nanoparticle volume fraction on the boundaries may be difficult. Thus it is advisable to replace the boundary conditions by a set that are more realistic physically. In this paper, we assume that the temperature is constant and nanoparticles flux including the effect of thermophoresis is zero on the boundaries. This boundary condition on the nanoparticle volume fraction is made possible by accounting for the contributions of the effect of thermophoresis to the nanoparticle flux. In this respect this model is more realistic physically than previous. Thus the boundary conditions are:

$$\text{at } z = 0, \quad w = 0, T = T_0, D_B \frac{d\phi}{dz} + \frac{D_T}{T_1} \frac{dT}{dz} = 0 \quad (10a)$$

$$\text{at } z = d, \quad w = 0, T = T_1, D_B \frac{d\phi}{dz} + \frac{D_T}{T_1} \frac{dT}{dz} = 0 \quad (10b)$$

3. BASIC STATE

The basic state is given as:

$$\begin{aligned} \bar{\mathbf{v}} &= 0, T = T_b(z), p = p_b(z), \phi = \phi_b(z), \\ \epsilon &= \epsilon_b(z), \psi = \psi_b(z), E = E_b(z). \end{aligned} \quad (11)$$

The solution of the basic state is:

$$T_b = T_0 - \frac{\Delta T}{d} z, \phi_b = \phi_0 + \left(\frac{D_T \Delta T}{D_B T_1 d} \right) z,$$

$$E_b = \frac{E_0}{1 + \gamma \Delta T z / d} \hat{\mathbf{k}},$$

$$\psi_b(z) = -\frac{E_0 d}{\gamma \Delta T} \log \left(1 + \frac{\gamma \Delta T}{d} z \right) \hat{\mathbf{k}},$$

$$\epsilon_b = \epsilon_0 \left(1 + \frac{\gamma \Delta T}{d} z \right) \hat{\mathbf{k}},$$

where subscript b denote the steady state, $\Delta T = (T_0 - T_1)$ and $E_0 = -\frac{\psi_1 \gamma \Delta T / d}{\log(1 + \gamma \Delta T)}$ is the root mean square value of the electric field at $z = 0$.

4. PERTURBATION EQUATIONS

Let the initial basic state as described by equation be slightly perturbed so that the perturbed state is

given by:

$$\begin{aligned} \mathbf{v} &= \mathbf{v}', \quad p = p_b(z) + p', \quad T = T_b(z) + T', \\ \phi &= \phi_b(z) + \phi', \quad \epsilon = \epsilon_b + \epsilon', \quad \vec{\mathbf{E}} = \vec{\mathbf{E}}_b + \vec{\mathbf{E}}', \\ \psi &= \psi_b + \psi', \end{aligned} \quad (12)$$

where the prime denote the perturbed quantities. On substituting the Eq. (12) into the Eqs. (1)–(10), linearizing by neglecting the products of primed quantities, eliminating the pressure term from the momentum equation by operating curl twice and retaining the vertical component and converting the resulting equations to non-dimensional form by introducing the following variables:

$$\begin{aligned} (x'', y'', z'') &= (x', y', z')/d, \\ (u'', v'', w'') &= (u', v', w')d/\alpha_m, \\ t'' &= (\alpha_m t')/(\sigma d^2), \quad T'' = (T' - T_1)/\Delta T, \\ \phi'' &= (\phi' - \phi_0)/\phi_0, \quad \psi'' = \psi'/(\gamma E_0 \Delta T d), \end{aligned}$$

where ϕ_0 is a reference scale for the nanoparticle fraction and $\alpha_m = k_m/(\rho c)$, $\sigma = (\rho c)_m/(\rho c)$, we obtain the linear stability equations (dropping the dashes (") for simplicity) in non dimensional form as:

$$\nabla^2 w = R_D \nabla_H^2 T - R_n \nabla_H^2 \phi + R_e \nabla_H^2 \left(T - \frac{\partial \psi}{\partial z} \right), \quad (13)$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T - \frac{N_B}{L_e} \left(N_A \frac{\partial T}{\partial z} + \frac{\partial \phi}{\partial z} \right) \quad (14)$$

$$\frac{1}{\sigma} \frac{\partial \phi}{\partial t} + \frac{N_A}{\phi} w = \frac{1}{L_e} \nabla^2 \phi + \frac{N_A}{L_e} \nabla^2 T \quad (15)$$

$$\nabla^2 \psi = \frac{\partial T}{\partial z}. \quad (16)$$

In the above equations the following non-dimensional parameters are given as:

$L_e = \frac{\alpha_m}{D_B}$ is the Lewis number,

$R_D = \frac{\rho_0 g \beta \Delta T K d}{\mu \alpha_m}$ is the thermal Rayleigh-

Darcy number, $R_n = \frac{(\rho_p - \rho_0) \phi_0 g K d}{\mu \alpha_m}$ is the nanoparticle Rayleigh-Darcy number,

$R_e = \frac{\gamma^2 \epsilon E_0^2 (\Delta T)^2 K}{\mu \alpha_m}$ is the AC electric

Rayleigh-Darcy number, $N_A = \frac{D_T \Delta T}{D_B T_1 \phi_0}$ is the

modified diffusivity ratio, $N_B = \phi \frac{(\rho c)_p \phi_0}{(\rho c)}$ is the

modified particle-density increment.

In non-dimensional form, the boundary conditions become:

$$w = \frac{\partial \psi}{\partial z} = T = 0, \quad \frac{\partial \phi}{\partial z} + N_A \frac{\partial T}{\partial z} \quad \text{at } z=0 \text{ and } z=1. \quad (17)$$

5. NORMAL MODES ANALYSIS

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form:

$$\begin{aligned} [w, T, \phi, \psi] &= [W(z), \Theta(z), \Phi(z), \Psi(z)] \\ &\times \exp(i k_x x + i k_y y + n t) \end{aligned} \quad (18)$$

where k_x and k_y are wave numbers in x and y directions, respectively, while n is the growth rate of disturbances.

On using Eq. (18), into Eqs. (13)–(16), we have:

$$(D^2 - a^2)W + R_D a^2 \Theta - R_n a^2 \Phi + R_e a^2 \left(\Theta - \frac{\partial \Psi}{\partial z} \right) = 0, \quad (19)$$

$$W + \left[D^2 - a^2 - n - \frac{N_A N_B}{L_e} D \right] \Theta - \frac{N_B}{L_e} D \Phi = 0 \quad (20)$$

$$-\frac{N_A}{\phi} W + \left[\frac{1}{L_e} (D^2 - a^2) - \frac{n}{\sigma} \right] \Phi + \frac{N_A}{L_e} (D^2 - a^2) \Theta = 0, \quad (21)$$

$$(D^2 - a^2) \Psi - D \Theta = 0, \quad (22)$$

where $D \equiv \frac{d}{dz}$ and $a = \sqrt{k_x^2 + k_y^2}$ is the resultant dimensionless wave number. The boundary conditions in view of normal mode analysis are:

$$W = \Theta = D \Psi = 0, \quad D \Phi + N_A D \Theta = 0 \quad \text{at } z = 0, 1. \quad (23)$$

The growth rate n is in general a complex quantity such that $n = \omega_r + i \omega_i$, the system with $\omega_r < 0$ is always stable, while for $\omega_r > 0$ it will become unstable. For neutral stability, the real part of ω is zero. Hence, we now write $n = i \omega_i$, where ω_i is real and is a dimensionless frequency.

The Galerkin weighted residuals method is used to obtain an analytical solution to the system of Eqs. (19)–(22). Accordingly, the base functions. W, Θ, Φ and Ψ are taken in the following way:

$$W = \sum_{p=1}^N A_p W_p, \quad \Theta = \sum_{p=1}^N B_p \Theta_p,$$

$$\Phi = \sum_{p=1}^N C_p \Phi_p, \quad \Psi = \sum_{p=1}^N D_p \Psi_p, \quad (24)$$

where

$W_p = \Theta_p = \sin p\pi z$, $\Phi_p = -N_A \sin p\pi z$, $\Psi_p = \cos p\pi z$, (satisfying the boundary conditions), A_p , B_p , C_p and D_p are unknown coefficients, and $p = 1, 2, 3, \dots, N$. On using above expression for W , Θ , Φ and Ψ into Eqs. (19)-(22) and multiplying the resulting first equation by W_p second equation by Θ_p , third equations by Φ_p and fourth equation by Ψ_p and integrating in the limits from zero to unity, we obtained a system of $4N$ linear algebraic equations in the $4N$ unknowns A_p , B_p , C_p and D_p ,

$p = 1, 2, 3, \dots, N$. For the existence of non trivial solution, the determinant of coefficients matrix must vanish, which gives the characteristic equation for the system, with the thermal Rayleigh- Darcy number R_D as the eigenvalue of the characteristic equation. For a first approximation, we take $N = 1$; this produces the result

$$\begin{vmatrix} \frac{-J}{2}, & \frac{a^2(R_D+R_e)}{2}, & \frac{a^2 N_A R_n}{2}, & \frac{a^2 \pi R_e}{2} \\ \frac{1}{2}, & \frac{(-J-i\alpha)}{2}, & 0, & 0 \\ \frac{N_A^2}{2\varphi}, & \frac{N_A^2 J}{2L_e}, & \frac{N_A^2(-i\alpha L_e - J\sigma)}{2L_e \sigma}, & 0 \\ 0, & \frac{-\pi}{2}, & 0, & \frac{J}{2} \end{vmatrix} = 0. \quad (25)$$

Here $J = (a^2 + \pi^2)$. Generally when we employ a single-term Galerkin approximation in this situation we get a value overestimate by about 3%. But in this case, the single-term Galerkin approximation gives the exact result. We note that the parameter N_B does not appear to first order of approximation because of an orthogonal property of the first-order trial functions and their first derivatives. This approximation is valid because the terms containing N_B involves as a function of N_B/L_e and the value of N_B/L_e is too small of order $10^{-2} \sim 10^{-5}$, pointing to the zero contribution of the nanoparticle flux in the thermal energy conversation.

6. RESULTS AND DISCUSSION

6.1 Stationary Convection

First, consider the case of stationary convection, i.e. $\omega_i = 0$. Then, Eq. (25) gives the following expression for the thermal Rayleigh- Darcy number R_D :

$$R_D = \frac{(a^2 + \pi^2)^2}{a^2} - \frac{a^2 R_e}{(a^2 + \pi^2)} - N_A R_n \left(1 + \frac{L_e}{\varphi}\right) \quad (26)$$

It is clear from Eq. (26) that the thermal Rayleigh-Darcy number R_D decreases with increasing the AC electric Rayleigh-Darcy number and nanofluid parameters while increases with porosity parameter φ .

To find the critical value of R_D , Eq. (26) is differentiated with respect to a^2 and equated to zero a polynomial in a_c^2 , whose coefficients are functions of the physical parameters influencing the instability is obtained in the form

$$(a_c^2)^4 + 2\pi^2(a_c^2)^3 - \pi^2 R_e(a_c^2)^2 - 2\pi^6(a_c^2) - \pi^8 = 0 \quad (27)$$

The above equation is solved numerically for various values of R_e and the minimum value of a_c^2 is obtained each time, hence the critical wave number is obtained. Using this in Eq. (26), the critical thermal Rayleigh-Darcy number $R_{D,C}$ above which the convection sets in is determined.

It is interesting to check Eqs. (26) and (27) for existing results in the literature under some limiting cases. In the absence of nanoparticle (i.e., $R_n = 0$), the Eq. (26) reduces to

$$R_D = \frac{(a^2 + \pi^2)^2}{a^2} - \frac{a^2 R_e}{(a^2 + \pi^2)} \quad (28)$$

and coincides with Roberts (1969). In the absence of electrical field ($R_e = 0$), Eqs. (26) and (27) reduce to:

$$R_D = \frac{(a^2 + \pi^2)^2}{a^2} - N_A R_n L_e \left(\frac{1}{\varphi} + \frac{1}{L_e}\right) \quad (29)$$

and

$$a_c = \pi. \quad (30)$$

Equations (29) and (30) coincide with that of Yadav and Lee (2015b) for a thermal equilibrium case with $D_a = T_D = 0$.

6.2 Oscillatory Convection

With oscillatory onset $\omega_i \neq 0$, the real and imaginary parts of Eq. (25) yield:

$$\begin{aligned} -a^2 J L_e N_A R_n \sigma + \varphi \{ J^3 \sigma + a^2 \pi^2 R_e \sigma - J \{ L_e \omega_i^2 \\ + a^2 (R_{D,Osc} + R_e + N_A R_n) \sigma \} \} = 0, \end{aligned} \quad (31)$$

Table 1 Critical thermal Rayleigh-Darcy number $R_{D,c}$ and critical wave number a_c for different values of R_e and $S (= N_A R_n (1 + L_e / \varphi))$

R_e	S	$R_{D,c}$	a_c	R_e	S	$R_{D,c}$	a_c
0	0	39.4784	3.14	60	0	4.3756	4.33
	10	29.4784	3.14		10	-5.6244	4.33
	20	19.4784	3.14		20	-15.6244	4.33
	30	9.4784	3.14		30	-25.6244	4.33
	40	-0.5216	3.14		40	-35.6244	4.33
	50	-10.5216	3.14		50	-45.6244	4.33
20	0	28.8549	3.55	80	0	-9.0615	4.66
	10	18.8549	3.55		10	-19.0615	4.66
	20	8.8549	3.55		20	-29.0615	4.66
	30	-1.1451	3.55		30	-39.0615	4.66
	40	-11.1451	3.55		40	-49.0615	.66
	50	-21.1451	3.55		50	-59.0615	4.66
40	0	17.0832	3.96	100	0	-23.0688	4.95
	10	7.0832	3.96		10	-33.0688	4.95
	20	-2.9168	3.96		20	-43.0688	4.95
	30	-12.9168	3.96		30	-53.0688	4.95
	40	-22.9168	3.96		40	-63.0688	4.95
	50	-32.9168	3.96		50	-73.0688	4.95

$$-a^2 J L_e N_A R_n \sigma + \varphi \left(a^2 L_e \pi^2 R_e - a^2 J L_e (R_{D,osc} + R_e) + J^3 (L_e + \sigma) \right) = 0 \tag{32}$$

Equations (31) and (32) give the following expressions for the thermal Rayleigh-Darcy number $R_{D,osc}$ and the frequency of oscillation ω_i :

$$R_{D,osc} = \frac{(a^2 + \pi^2)^2}{a^2} - \frac{a^2 R_e}{(a^2 + \pi^2)} - \frac{N_A R_n \sigma}{\varphi} + \frac{(a^2 + \pi^2)^2 \sigma}{a^2 L_e} \tag{33}$$

$$\omega_i^2 = - \left[\frac{a^2 N_A R_n (L_e + \varphi - \sigma) \sigma}{L_e \varphi} + \frac{(a^2 + \pi^2)^2 \sigma^2}{L_e^2} \right] \tag{34}$$

From Eq. (34), it is interesting to note that the vertical AC electric field does not influence the existence of oscillatory convection. Following Buongiorno (2006), Nield and Kuznetsov (2013) and Yadav *et al.* (2014b) the Lewis number L_e is on the order of $10^1 \sim 10^3$, N_A is on the order of $1 \sim 10$, the nanoparticle Rayleigh-Darcy number R_n and σ are on the order of $1 \sim 10$, and Hence from Eq. (34), the value of ω_i^2 will be always negative. Since ω_i is real for oscillatory convection, therefore oscillatory convection cannot occur and the principle of the exchange of stability is valid for the case of nanofluid.

The stationary convection curves

in $(R_{D,c}, R_e)$ -plane for various parameter values are shown in Figs. 1-5. The values of the parameters $N_A = 2$, $L_e = 10$, $\varphi = 0.7$ and $R_n = 0.5$ are fixed except the varying parameters. The range of parameters fall in these figures is taken from the available literature (Buongiorno, 2006; Nield and Kuznetsov, 2013; Yadav *et al.*, 2011, 2012, 2014b, c, d, 2015c,d,e). From these figures, it is clear that the linear stability criterion is expressed in terms of critical thermal Rayleigh-Darcy number $R_{D,c}$, below which the system is stable and unstable above.

To validate the numerical procedure used to find the critical stability parameters, first the test computations are obtained under the limiting case of nanoparticle and electric field i.e. $R_n = R_e = 0$ and tabled in Table 1. From the Table 1, we recognize that in the absence of nanoparticles and electric field we recover the exactly well-known result that the critical Rayleigh-Darcy number $R_{D,c}$ is equal to $4\pi^2$ and the corresponding wave number a_c is π . This verifies the accuracy of the numerical method used.

The critical thermal Rayleigh-Darcy number $R_{D,c}$ and the corresponding wave number a_c as a function of AC electric Rayleigh-Darcy number R_e are obtained for different values of nanoparticles Rayleigh-Darcy number R_n are shown in Figs. 1 and 2, respectively. From Fig. 1, it is found that the critical thermal Rayleigh-Darcy number $R_{D,c}$ decreases with an increase

in the value of the AC electric Rayleigh-Darcy number R_e . That is, higher the electric field strength the less stable the system due to an increase in the destabilizing electrostatic energy to the system. From Fig. 1, it is also observed that the critical thermal Rayleigh-Darcy number decreases as nanoparticles Rayleigh-Darcy number R_n increases.

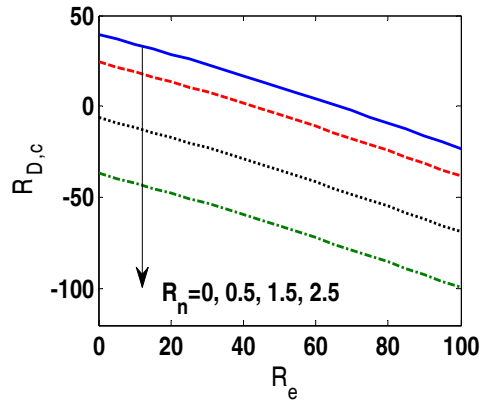


Fig. 1. Effect of AC electric Rayleigh-Darcy number R_e on the critical thermal Rayleigh-Darcy number $R_{D,c}$ for different values of concentration Rayleigh-Darcy number R_n with $N_A = 2$, $L_e = 10$, $\varphi = 0.7$.

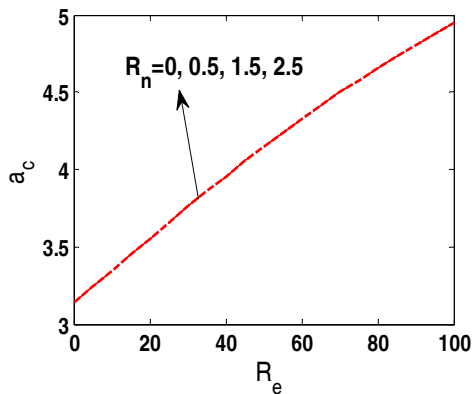


Fig. 2. Effect of AC electric Rayleigh-Darcy number R_e on the critical wave number a_c for different values of concentration Rayleigh-Darcy number R_n with $N_A = 2$, $L_e = 10$, $\varphi = 0.7$.

This is because as an increase in volumetric fraction of nanoparticles, increases the Brownian motion of the nanoparticles which cause destabilizing effect on the stability of the system. The corresponding critical wave number a_c has been plotted in Fig. 2 and it indicated that increase in the values of AC electric Rayleigh-Darcy number R_e tends to increase a_c and thus its effect is to decrease the size of convection cells. The critical wave number a_c has no change for the different value of nanoparticles Rayleigh-

Darcy number R_n . This is because nanoparticles diffuse in the base fluid so they are not proficient to change the size of convection cell. Therefore, nanoparticle parameters (such as nanoparticle Rayleigh-Darcy R_n , Lewis number L_e and modified diffusivity ratio N_A) have no significant effect on the critical wave number observed.

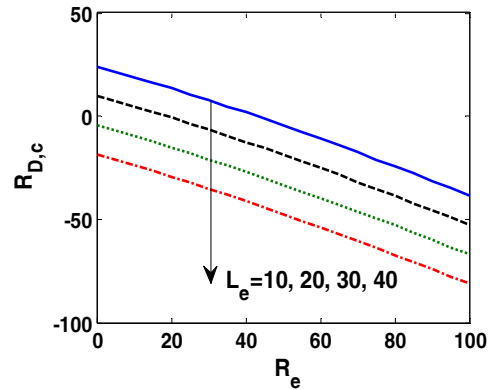


Fig. 3. Effect of AC electric Rayleigh-Darcy number R_e on the critical thermal Rayleigh-Darcy number $R_{D,c}$ for different values of Lewis number L_e with $N_A = 2$, $\varphi = 0.7$, $R_n = 0.5$.

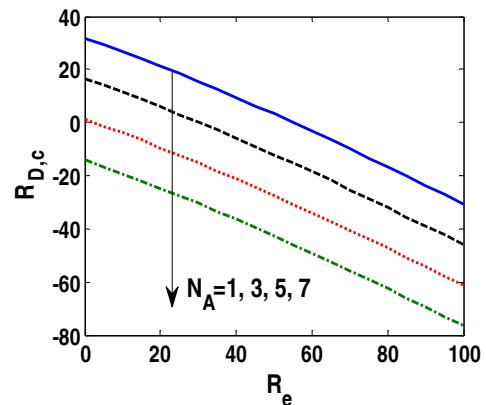


Fig. 4. Effect of AC electric Rayleigh-Darcy number R_e on the critical thermal Rayleigh-Darcy number $R_{D,c}$ for different values of modified diffusivity ratio N_A with $L_e = 10$, $\varphi = 0.7$, $R_n = 0.5$.

Figs. 3-5 show the effect of Lewis number L_e , the modified diffusivity ratio N_A and porosity parameter φ on the stability of the system. From Figs. 3-5, we found that the Lewis number L_e and the modified diffusivity ratio N_A accelerate the onset of convection, while porosity parameter φ delays the convection in a nanofluid layer. It may be happened because the thermophoresis at a higher value of thermophoretic diffusivity is more supportable

to the disturbance in nanofluids, while both thermophoresis and Brownian motion are driving forces in favour of the motion of nanoparticles.

Based on the Eq. (26), we can also conclude that the critical thermal Rayleigh-Darcy number $R_{D,c}$ depends on R_e and $S(=N_A R_n(1+L_e/\phi))$. Therefore, for simplification of our results, Tables 1 is also made which show the effect of these parameters on the stability characteristic.

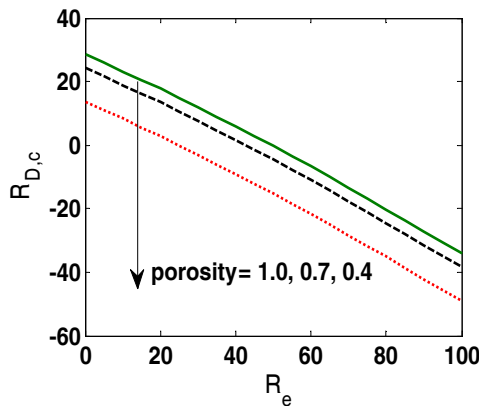


Fig. 5. Effect of AC electric Rayleigh-Darcy number R_e on the critical thermal Rayleigh-Darcy number $R_{D,c}$ for different values of porosity ϕ with $N_A = 2$, $L_e = 10$, $R_n = 0.5$.

7. CONCLUSIONS

The effect of vertical AC electric field on the onset of convection in a nanofluid-saturated porous layer is studied. The flux of volume fraction of nanoparticles with the effect of thermophoresis is taken to be zero on the isothermal boundaries and the eigenvalue problem is solved theoretically using the Galerkin method. It is observed that the instability of the fluid is reinforced with an increase in the value of AC electric Rayleigh-Darcy number R_e , the Lewis number L_e , the modified diffusivity ratio N_A and the concentration Rayleigh-Darcy number R_n . The size of convection cells depends only on AC electric Rayleigh-Darcy number R_e and decreases with increasing the AC electric Rayleigh-Darcy number R_e . It is also found that the vertical AC electric field does not influence the existence of oscillatory convection and the principle of exchange of stability is valid for nanofluid.

ACKNOWLEDGEMENT

This work was supported by human resources development program (No. 20144030200560) of the Korean Institute of Energy Technology Evaluation and Planning (KETEP) under a grant funded by the Korean government Ministry of

Trade, Industry and Energy.

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