



# Effect of Inclination Angle and Magnetic Field on Convection Heat Transfer for Nanofluid in a Porous Cavity

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(Received June 25, 2015; accepted October 28, 2015)

## ABSTRACT

In this paper, the effect of inclination angle and magnetic field in a two-dimensional porous cavity filled with Cu-water nanofluid has been studied numerically. The equations are framed using the Darcy-Brinkman-Forchheimer model. The control volume technique is used to solve the governing equations and SIMPLE algorithm is employed for the momentum equations. Comparison test was done with previous available literatures and the results are found to be in good agreement. The results are presented for different values of inclination angle ( $0^\circ \leq \gamma \leq 180^\circ$ ), Hartmann number ( $0 \leq Ha \leq 100$ ), Darcy number ( $10^{-5} \leq Da \leq 10^{-1}$ ) and solid volume fraction ( $0\% \leq \phi \leq 5\%$ ) while the porosity  $\epsilon$ , Rayleigh number  $Ra$  and Prandtl number  $Pr$  are fixed at 0.6,  $10^6$  and 6.2, respectively. It is found that the influence of solid volume fraction is strongly affected by the presence of strong magnetic field and the inclination angle  $90^\circ$  in the porous medium.

**Keywords:** Nanofluid; Magnetic field; Porous medium; Inclination angle.

## NOMENCLATURE

$B_0$	magnetic field strength	$X, Y$	dimensionless coordinates
$C_p$	specific heat		
$Da$	Darcy number	$\alpha$	thermal diffusivity
$Fc$	Forchheimer coefficient	$\beta$	thermal expansion coefficient
$g$	gravitational acceleration vector	$\gamma$	inclination angle
$Ha$	Hartmann number	$\theta$	dimensionless temperature
$k$	thermal conductivity	$\epsilon$	porosity
$K$	permeability	$\phi$	solid volume fraction
$L$	cavity length	$\sigma$	electrical conductivity
$Nu$	local Nusselt number	$\sigma_s$	specific heat ratio
$\overline{Nu}$	average Nusselt number	$\mu$	dynamic viscosity
$p$	fluid pressure	$\nu$	kinematic viscosity
$P$	dimensionless pressure	$\rho$	density
$Pr$	Prandtl number	$\tau$	dimensionless time
$Ra$	Rayleigh number	<b>Subscripts</b>	
$t$	time	$c$	cold
$T$	temperature	$f$	fluid
$u, v$	velocity components in x, y directions	$h$	hot
$U, V$	dimensionless velocity components	$nf$	nanofluid
$x, y$	Cartesian coordinates	$p$	nanoparticle

## 1. INTRODUCTION

From the researches, natural convection in closed cavities has been attracted very much

due to its number of applications in many industries such as nuclear energy, double pan windows, heating and cooling of building, so-

lar collectors, electronic cooling and micro-electromechanical systems. In physical applications, there are systems which has been inclined with the surface which are different from normal horizontal cavity. The effect of inclination angle is also studied to understand the physical configuration of the problem.

There are several authors Soong *et al.* (1996), Vinogradov *et al.* (2011), Karimipour *et al.* (2013) and Cheng and Liu (2014) studied the influence of inclination angle on flow field and heat transfer in a tilted cavity. A numerical study carried out by Aydin *et al.* (1999) in an inclined square cavity with heating and cooling is applied on the adjacent walls. Their results indicate that for inclination angles in the range  $0^\circ$  to  $90^\circ$ , and for the high magnitude of Rayleigh number the flow field have the unsteady nature. Polat and Eilgen (2002) studied the laminar natural convection in an inclined shallow cavity. They showed that the inclination angle plays a significant role in the heat transfer and the volumetric flow rate, and also that the average Nusselt number is strongly affected by the inclination angle.

The double diffusive natural convection in an inclined rectangular enclosure in the presence of magnetic field is numerically analyzed by Teamah *et al.* (2011). They found that, the maximum heat and mass transfer was observed at the angles of  $45^\circ$  and  $135^\circ$ . Kherief *et al.* (2012) examined the influence of magnetic field and the angle of inclination on natural convection induced by a vertical temperature. Their results revealed that the dynamic and temperature fields are strongly affected by the variations of magnetic field and the inclination angle. The maximum natural convection state significantly depends on the angle of inclination which was found by Selamat *et al.* (2012).

Low thermal conductivity of conventional heat transfer fluids such as water, oil, ethylene glycol failed to produce an energy efficient heat transfer performance. To overcome this situation a high thermal conductivity of nanoparticles are added in the base fluid which results in high heat transfer rates. A nanofluid is a fluid in which the nanoparticles are suspended in the traditional fluids. There are number of works presented by Sheikholeslami<sup>1</sup> *et al.* (2013), Al-Zamily (2014), Sheikholeslami and Ganji (2014), Sheikholeslami<sup>1</sup> *et al.* (2014), Mahmoodi *et al.* (2015), Sheikholeslami and Ellahi (2015) and Sheikholeslami and Rashidi (2015a) using the nanofluid for the purpose of getting augmented heat transfer in a cavity.

Ben-Cheikh *et al.* (2013) studied the convection of water based nanofluids in a cavity where the bottom wall is heated by non-uniform distribution. Results of their study showed that the augmentation of heat transfer strongly depend on the type of nanofluid and for high Rayleigh number, the flow field assumes unsteady nature. Also they observed that, increase in the solid volume fraction of nanoparticles leads to the steady state flow field from unsteady flow. Sheikholeslami<sup>2</sup> *et al.* (2013) applied the heat-line technique to visualize the nanofluid flow and heat transfer using two phase simulation model. Moreover the authors Sheikholeslami<sup>2</sup> *et al.* (2014), Sheikholeslami *et al.* (2014a) and Sheikholeslami and Rashidi (2015b) numerically analyzed the effect of magnetic field using nanofluid and in the case of non-uniform magnetic field considered by Sheikholeslami<sup>1</sup> *et al.* (2015).

However the influence of nanofluid is studied using the Brownian and thermophoresis motion effects Sheikholeslami<sup>3</sup> *et al.* (2014), Sheikholeslami *et al.* (2014b), Sheikholeslami and Ganji (2015) and Sheikholeslami<sup>2</sup> *et al.* (2015). Recently Mansour and Bakier (2015) analyzed the influence of various thermal boundary conditions with convection heat transfer in an inclined cavity using Cu-water nanofluid in the presence of magnetic field. They found that increase in  $Ha$  caused the decreasing heat transfer rate.

Nowadays, convection heat transfer by nanofluid in the fluid saturated porous medium is the ongoing research field. There are few recent works studied by Sheikhezad and Nazari (2013), Sheikholeslami and Ganji (2014) and Nguyen *et al.* (2015) handled the influence of nanofluid in the porous medium. They found out that, the higher heat transfer rate was attained by using the nanofluid in the porous medium. At a recent time, Mahdi *et al.* (2015) studied various research works on nanofluid heat transfer and presented a review article showed that the application of nanofluids in porous cavities yields the maximum heat transfer rate.

The aim of the present work is to study the effect of inclination angle and magnetic field in the fluid saturated porous medium. The inclined cavity is filled by high thermally conducting copper nanoparticles that are stably distributed in the base fluid water. The results obtained from the numerical method are plotted in the view of streamlines, isotherms, mid-height velocity profiles and average Nusselt number

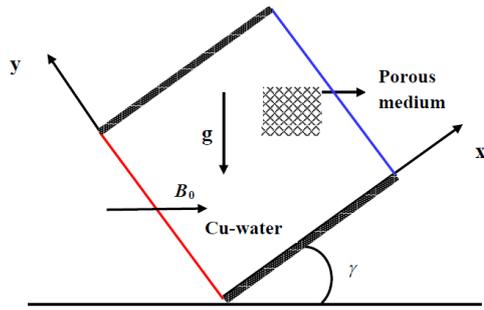


Fig. 1. Physical configuration of the problem.

graphs.

2. MATHEMATICAL FORMULATION

Figure 1 represents the physical configuration of the present study. A two-dimensional porous square cavity inclined at an angle  $\gamma$  with respect to the horizontal plane. The height and width of the cavity is commonly denoted by  $L$  and is filled by Cu-water nanofluid. The cavity is isothermally heated from the left wall with a uniform temperature  $T_h$  and the right vertical wall with temperature  $T_c$  ( $T_h > T_c$ ) while the top and bottom walls are perfectly insulated. The gravity acts downwards and the magnetic field acts normal to the hot wall and parallel to the  $x$ -axis. The generalized non-Darcy model called Darcy-Brinkman-Forchheimer is used for the present work. It is assumed that the medium is isotropic, homogeneous and in thermodynamic equilibrium with the fluid. The physical properties of the fluid are assumed to be constant except the density variation, which is approximated by Boussinesq model. According the above assumptions the non-dimensional governing equations are written as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$\frac{1}{\epsilon} \frac{\partial U}{\partial \tau} + \frac{1}{\epsilon^2} \left( U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = - \frac{\partial P}{\partial X} + \frac{v_{nf}}{\alpha_f} \frac{1}{\epsilon} \nabla^2 U - \frac{v_{nf}}{\alpha_f} \frac{U}{Da} - \frac{Fc}{\sqrt{Da}} U \sqrt{U^2 + V^2} + \frac{\beta_{nf}}{\beta_f} Ra Pr \sin \gamma \theta \tag{2}$$

$$\frac{1}{\epsilon} \frac{\partial V}{\partial \tau} + \frac{1}{\epsilon^2} \left( U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = - \frac{\partial P}{\partial Y} + \frac{v_{nf}}{\alpha_f} \frac{1}{\epsilon} \nabla^2 V - \frac{v_{nf}}{\alpha_f} \frac{V}{Da} - \frac{Fc}{\sqrt{Da}} V \sqrt{U^2 + V^2} + \frac{\beta_{nf}}{\beta_f} Ra Pr \cos \gamma \theta - Ha^2 Pr V \tag{3}$$

$$\sigma_s \frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \nabla^2 \theta \tag{4}$$

where  $Fc = \frac{1.75}{\sqrt{150} \epsilon^{3/2}}$ . The dimensionless variables in the above equations are defined as

$$(X, Y) = \frac{(x, y)}{L}, \quad (U, V) = \frac{(u, v)}{\alpha_f / L}, \quad \tau = \frac{\alpha_f t}{L^2}, \tag{5}$$

$$P = \frac{p L^2}{\rho_{nf} \alpha_f^2}, \quad \theta = \frac{T - T_c}{T_h - T_c}.$$

and the non-dimensional parameters are defined by, the Darcy number  $Da = \frac{K}{L^2}$ , the Rayleigh number  $Ra = \frac{g \beta_f L^3 (T_h - T_c)}{v_f \alpha_f}$ , the Prandtl number  $Pr = \frac{v_f}{\alpha_f}$ , the Hartmann number  $Ha = B_0 L \sqrt{\frac{\sigma_{nf}}{\rho_{nf} v_f}}$ , the specific heat ratio  $\sigma_s = \frac{\epsilon(\rho c_p)_{nf} + (1 - \epsilon)(\rho c_p)_{porous}}{(\rho c_p)_{nf}}$ ,

The appropriate initial and boundary conditions are:

$$\begin{aligned} \tau = 0 : U = V = 0, \quad \theta = 0, \quad 0 \leq X \leq 1, \quad 0 \leq Y \leq 1, \\ \tau > 0 : U = V = 0, \quad \theta = 1, \quad X = 0, \quad 0 \leq Y \leq 1, \\ U = V = 0, \quad \theta = 0, \quad X = 1, \quad 0 \leq Y \leq 1, \\ U = V = 0, \quad \frac{\partial \theta}{\partial Y} = 0, \quad Y = 0 \text{ \& \ } 1, \quad 0 \leq X \leq 1. \end{aligned}$$

The effective density of the nanofluid ( $\rho_{nf}$ ) is given as

$$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_p,$$

where  $\phi$  is the solid volume fraction of nanoparticles and the subscripts  $f, p$  are represents the base fluid and the nanoparticles, respectively.

Thermal diffusivity of the nanofluid is

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}.$$

Here  $k_{nf}$  is the effective thermal conductivity of the nanofluid and can be found in the following expression

$$k_{nf} = k_f \left[ \frac{k_p + 2k_f - 2\phi(k_f - k_p)}{k_p + 2k_f + \phi(k_f - k_p)} \right],$$

where  $k_f$  and  $k_p$  are the thermal conductivities of the base fluid and dispersed particles.

In addition, the heat capacity of the nanofluid  $(\rho c_p)_{nf}$  can be determined by

$$(\rho c_p)_{nf} = (1 - \phi) (\rho c_p)_f + \phi (\rho c_p)_p.$$

In a similar way, the thermal expansion of the nanofluid obtained in the following form

$$(\rho \beta)_{nf} = (1 - \phi) (\rho \beta)_f + \phi (\rho \beta)_p.$$

**Table 1 Thermo-physical properties of base fluid and nanoparticle**

Physical properties	Fluid phase	Cu
$c_p(J/kgK)$	4179	385
$k(W/mk)$	0.613	401
$\rho(kg/m^3)$	997.1	8933
$\beta(1/K)$	$21 \times 10^{-5}$	$1.67 \times 10^{-5}$

The Brinkman (1952) model is used to calculate the dynamic viscosity of the nanofluid and is given by

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}},$$

where  $\mu_f$  is the dynamic viscosity of the base fluid. The thermophysical properties of the base fluid and the nanofluid are listed in Table 1. The heat transfer coefficients in terms of local Nusselt number is defined by

$$Nu = -\frac{k_{nf}}{k_f} \frac{\partial T}{\partial Y}. \quad (6)$$

The average Nusselt number at the hot wall is computed as

$$\overline{Nu} = \int_0^1 Nu \, dX. \quad (7)$$

### 3. NUMERICAL ANALYSIS

The Darcy-Brinkman-Forchheimer equations combined with the boundary conditions are solved by the control volume technique. The pressure velocity coupling equations are solved by the well known SIMPLE algorithm presented by Patankar (1980). The convective and diffusion terms are approximated using the power law and central difference schemes, respectively. The time step is chosen to be  $10^{-5}$  for the present work. The resulting set of algebraic equations are obtained by Thomas algorithm with line by line iterative process. This algorithm is iterated until a convergent solution is arrived. The convergence criteria chosen in the present problem is given by

$$\frac{\sum |\phi_{i,j}^{n+1} - \phi_{i,j}^n|}{\sum |\phi_{i,j}^{n+1}|} < \eta,$$

upto the temperature ( $\theta$ ) and the velocities ( $U$  and  $V$ ) have been met simultaneously. In the above expression,  $\eta$  is chosen in the order  $10^{-5}$ ,  $n$  is any time level and  $\phi$  represents  $\theta, U$  or  $V$ .

**Table 2 Grid independent study on the average Nusselt number at  $\gamma = 45^\circ, Ha = 50, Da = 10^{-3}$  and  $\phi = 0.05$**

Grid size	$\overline{Nu}$	Error %
$41 \times 41$	5.722	1.99
$61 \times 61$	5.608	0.80
$81 \times 81$	5.563	0.37
$101 \times 101$	5.542	0.16
$121 \times 121$	5.533	0.03
$141 \times 141$	5.531	

### 3.1 Grid Independent Test and Code Validation

To select a better grid size for the present study, grid test was performed for different mesh sizes on the average Nusselt number for the hot wall at  $\gamma = 45^\circ, Ha = 50, Da = 10^{-3}$  and  $\phi = 0.05$  which is shown in Table 2. From the table, we observed that the increased grid size from  $121 \times 121$  to  $141 \times 141$  renders comparatively the lowest error percentage. Henceforth the size  $121 \times 121$  gives a grid independent solution of the present problem.

In order to check the accuracy of the present problem, validation is carried out by comparing the calculated average Nusselt number at the heated wall with the numerical results of Ghasemi *et al.* (2011) and Nguyen *et al.* (2015) which is displayed in Table 3 and 4. By data from the table, we notice that there is good agreement between the present and previous published results.

## 4. RESULTS AND DISCUSSION

Natural convection in a nanofluid filled inclined porous cavity with the presence of magnetic field is studied numerically. In this work, some of the non-dimensional parameters such as porosity, Rayleigh number and the specific heat ratio are fixed at 0.4,  $10^6$  and 1 respectively. With the above fixed parameters, the main goal of the present study is to analyze the soul effect of inclination angle varying between  $0^\circ$  and  $180^\circ$ , Hartmann number varying between 0 and 100, and the Darcy number between  $10^{-5}$  to  $10^{-1}$ .

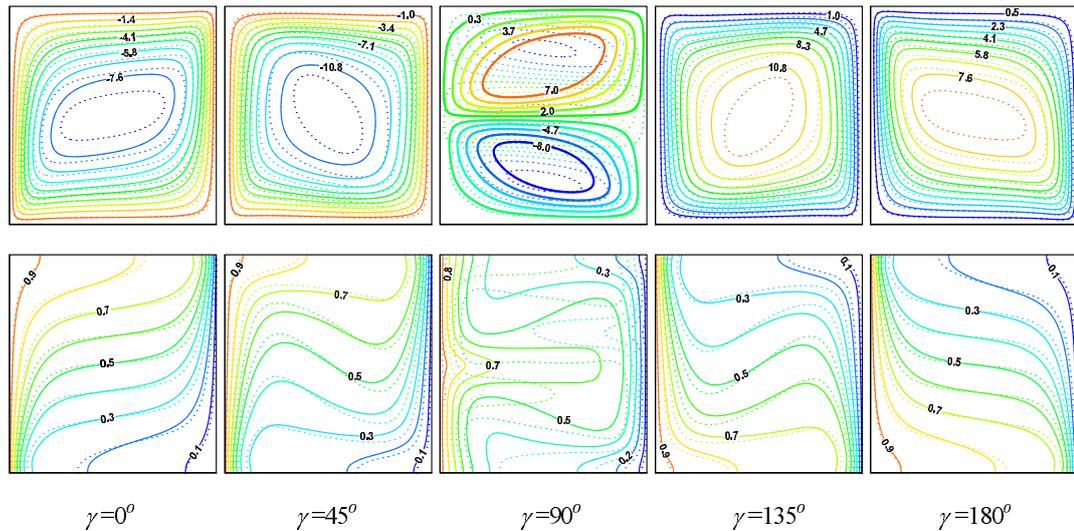
Figure 2 represents streamlines and isotherms of nanofluid ( $\phi = 0.05$ ) and pure water ( $\phi = 0.0$ ) for various inclination angles with fixed  $Ha = 50$  and  $Da = 10^{-3}$ . For  $\gamma = 0^\circ$ , the cavity is a normal horizontal cavity where the hot and cold walls are placed at the left and right vertical walls, respectively. In this situation, the fluid

**Table 3 Validation of average nusselt number for different Hartmann number and solid volume fraction at  $Ra = 10^5$**

$Ha$	$\phi$	$Nu$		$ \psi _{max}$	
		Ghasemi <i>et al.</i> (2011)	Present	Ghasemi <i>et al.</i> (2011)	Present
0	0.0	4.738	4.754	11.053	11.003
	0.02	4.820	4.833	11.313	11.264
	0.04	4.896	4.909	11.561	11.512
15	0.0	4.143	4.162	8.484	8.456
	0.02	4.179	4.198	8.615	8.585
	0.04	4.211	4.230	8.734	8.704
30	0.0	3.150	3.171	5.710	5.696
	0.02	3.138	3.159	5.682	5.668
	0.04	3.124	3.145	5.642	5.629

**Table 4 Validation of average nusselt number for different solid volume fraction at  $\gamma = 0^\circ, Ha = 0$  and  $\epsilon = 0.4$**

$Ra$	$Da$	$\phi$	$ \psi _{max}$		$Nu$	
			Nguyen <i>et al.</i> (2015)	Present	Nguyen <i>et al.</i> (2015)	Present
$10^3$	$10^{-2}$	0.0	0.283	0.282	1.007	1.020
		0.025	0.264	0.263	1.081	1.103
		0.5	0.246	0.245	1.160	1.171

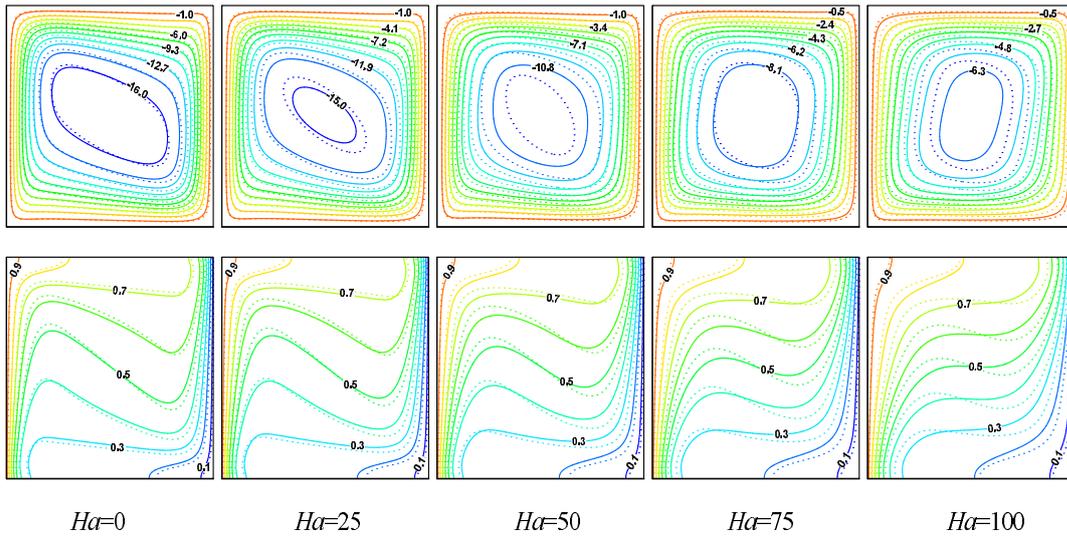


**Fig. 2. Steady state streamlines, isotherms for different inclination angle for nanofluid with  $\phi = 0.05$  (solid line) and pure water (dotted line) at  $Ha = 0$  and  $Da = 10^{-3}$ .**

heated from the hot wall moves towards the top wall and the cold fluid falls downwards which produces a single clockwise cell occupied in the whole cavity. The isotherms are well distributed in the entire cavity. The cavity is tilted with an angle of  $45^\circ$ , the core lines are rotated at an angle of  $45^\circ$  where the streams travel along a tilted parallelogram.

Furthermore, the flow rate increases with increase in angle from  $0^\circ$  to  $45^\circ$ . The addition

of nanoparticles clearly affects the flow pattern due to the presence of magnetic field strength. The cavity is again rotated by an angle  $90^\circ$ , the hot and cold walls are moved to the bottom and top sides. In this case, the cavity is filled by multiple cells for pure water ( $\phi = 0.0$ ), and double circulation pattern for nanofluid ( $\phi = 0.05$ ). Here the clockwise cell is placed near the hot wall due to the bottom wall heating, and the anticlockwise cell is located close to the top wall. The isotherms binds together forming a thin



**Fig. 3. Steady state streamlines, isotherms for different  $Ha$  for nanofluid with  $\phi = 0.05$  (solid line) and pure water (dotted line) at  $\gamma = 45^\circ$  and  $Da = 10^{-3}$ .**

layer at the isothermal walls. Also from the figure, we observe that the structures of  $\gamma = 135^\circ$  and  $\gamma = 180^\circ$  are the mirror images of  $\gamma = 45^\circ$  and  $\gamma = 0^\circ$ , respectively.

The influence of magnetic field is displayed in Fig. 3. In the absence of magnetic field, the nanofluid and pure water have the same structures and the interior streamlines move diagonally due to the fixed inclination angle. On applying a small amount of magnetic field strength ( $Ha = 25$ ), the flow and thermal fields are significantly affected whereas the increase in Hartmann number leads to decrease in flow rate which is expected; since the presence of magnetic field normally retards the fluid velocity. The flow rate is very low at high  $Ha$  compared with the absence of magnetic field. For increase in Hartmann number from 0 to 100, the thermal lines placed at the vicinity of the isothermal walls slowly walks into the cavity and also that, the bend on the isotherms slightly decreases. This is because of magnetic field, which generally resists the fluid flow.

Mostly the effect of porous medium is calculated via Darcy number. Fig. 4. submits flow and thermal fields for different  $Da$ . For  $Da = 10^{-5}$ , a single clockwise cell is noticed whose rotation is about the core of the cavity. Also the presence of magnetic field obviously affects the flow pattern. The isotherms are formed parallel to the hot wall which indicates that, the conduction is dominant when the value of  $Da$  is very low. Further, the increase in  $Da$  leads to increase in flow rate and the central streamlines rotate about an angle of  $45^\circ$  owing to the inclination angle. For rising values of  $Da$ , the

isotherms moves towards the cold wall, which means that the convection is superior and thermal boundary layer is formed at the vicinity of the cold wall.

Figure 5 exhibits mid-height horizontal and vertical velocity profiles of various inclination angles for pure water. From the figure, we observe that the velocity increases due to the increase in inclination angle from  $0^\circ$  to  $45^\circ$  and the line of  $90^\circ$  assumes a pattern of sine wave. Also, the velocity lines of  $135^\circ$  and  $180^\circ$  are the mirror images of  $45^\circ$  and  $0^\circ$ , respectively. This explanation is confirmed from Fig. 2.

The effect of magnetic field on the mid-height velocity profiles are shown in Fig. 6. The velocity is decreased completely owing to the increase in  $Ha$  and resembles almost same for all Hartmann number at the middle of the cavity. The increase in  $Ha$  stimulates a decrease in the buoyancy forces, which leads to reduction of fluid motion in the cavity, whereby a decrement in magnitude of velocity profile is observed.

Figure 7 communicates the influence of Darcy number on velocity profiles at  $\gamma = 45^\circ$  and  $Ha = 50$ . The velocity is same at the mid plane of the cavity when the value of  $Da$  is very low. By definition, the Darcy number is directly proportional to the permeability of the porous medium, and so that increase in  $Da$  produces an enhanced flow circulation in the cavity which is confirmed in Fig. 4. Henceforth, the velocity increases with increase in the value of Darcy number.

Generally, increasing amount of nanoparticle volume fraction causes an enhancing heat transfer rate. But the presence of magnetic field

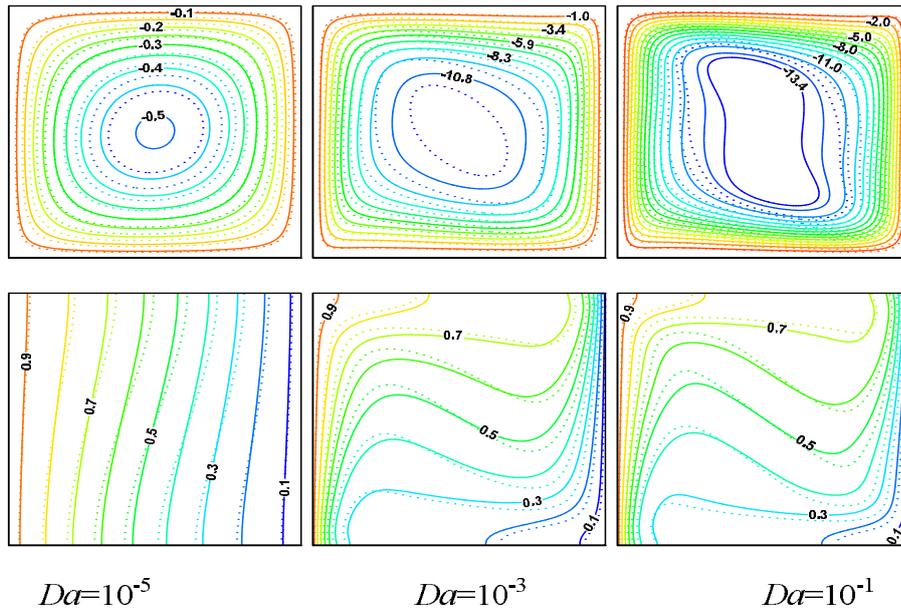


Fig. 4. Steady state streamlines and isotherms for different  $Da$  for nanofluid with  $\phi = 0.05$  (solid line) and pure water (dotted line) at  $\gamma = 45^\circ$  and  $Ha = 50$ .

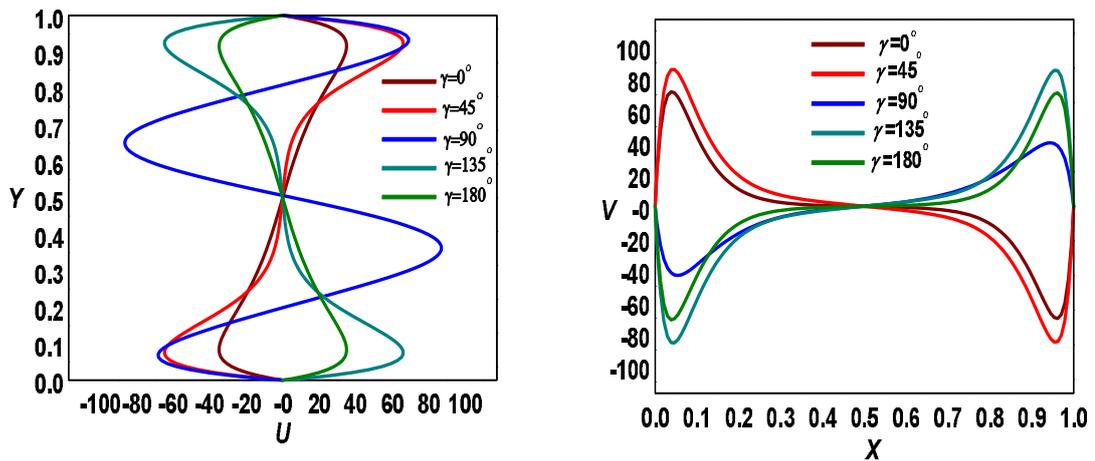


Fig. 5. The horizontal and vertical velocity profiles for pure water with different inclination angle at  $Ha = 50$  and  $Da = 10^{-3}$ .

significantly affects the effect of solid volume fraction. Ghasemi *et al.* (2011) found that the heat transfer rate increases with solid volume fraction for the value of  $Ha = 0$  and 15. But for the case of strong magnetic field ( $Ha = 30, 45$  and  $60$ ), receive a opposite behavior. Similar result is obtained in the present work. In this study, the heat transfer rate is monotonically increased with  $\phi$  for  $Ha = 0$  and 25 and decreases for  $Ha = 50, 75$  and 100. This explanation is derived in Fig. 8. To verify the above result, the comparison of the average Nusselt number between the presence and absence of  $Ha$  is made. Fig. 9. reveals the average Nusselt number at the heated wall for different inclination angles without magnetic field (left)

and with magnetic field (right). In the case of  $Ha = 0$ , the average Nusselt number increases for increasing values of  $\phi$ , but for the case of  $Ha = 50$ , the increasing rate of heat transfer is transmitted into decreasing rate for rising values of  $\phi$  except for the inclination angle  $\gamma = 90^\circ$ . Also from the figure, for all solid volume fractions, the maximum and minimum heat transfer was found at angles of  $\gamma = 135^\circ$  and  $\gamma = 90^\circ$ , respectively, in both presence and absence of magnetic field effect. In addition to that, the inclination angle  $\gamma = 90^\circ$  also affects the augmented heat transfer performance. Hence, the figures convey that the presence of magnetic field and the tilt angle  $90^\circ$  affects the influence of  $\phi$  on the heat transfer achievement.

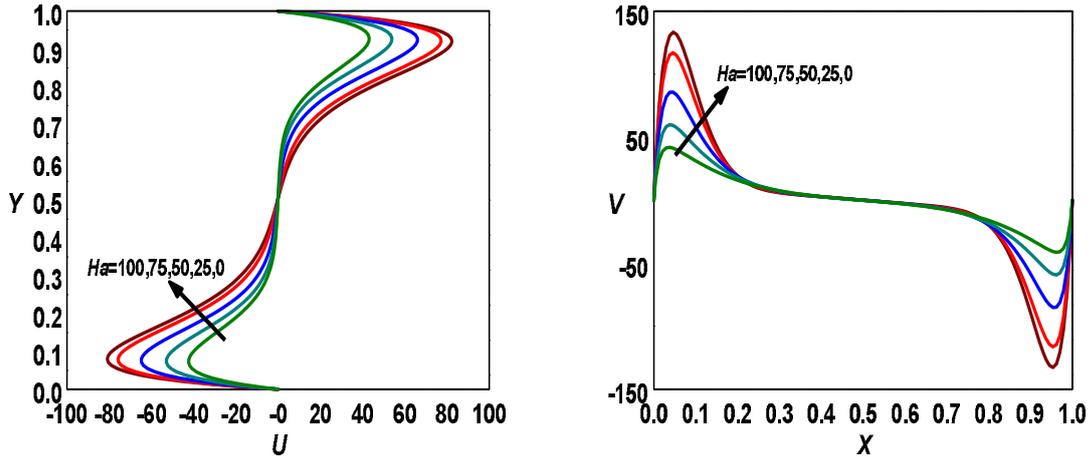


Fig. 6. The horizontal and vertical velocity profiles for pure water with different  $Ha$  at  $\gamma = 45^\circ$  and  $Da = 10^{-3}$ .

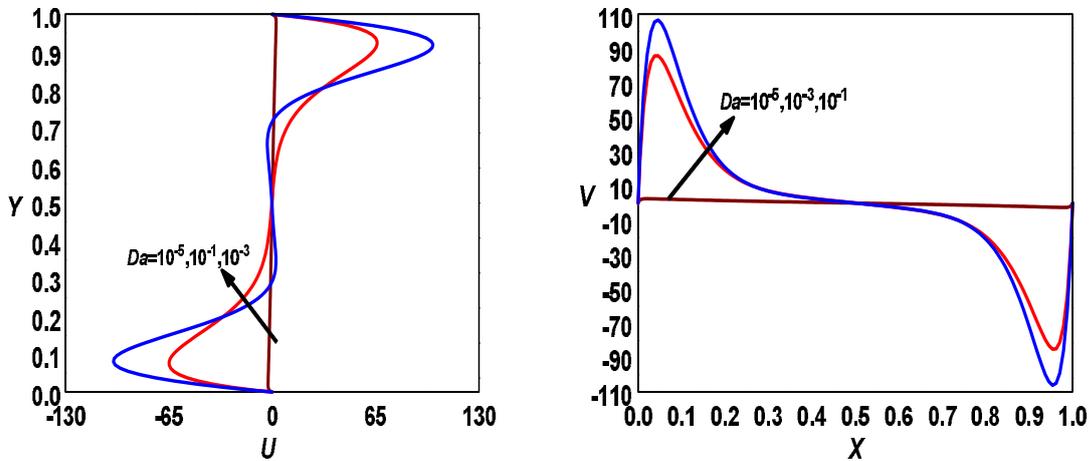


Fig. 7. The horizontal and vertical velocity profiles for pure water with different  $Da$  at  $\gamma = 45^\circ$  and  $Ha = 50$ .

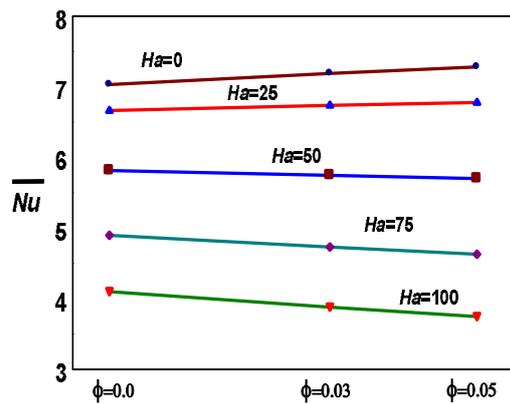


Fig. 8. Variation of average Nusselt number at the hot wall for different Hartmann number and solid volume fraction with  $\gamma = 45^\circ$  and  $Da = 10^{-3}$ .

The same study is applied on the Darcy number variation which is presented in Fig. 10. When  $Ha = 0$  and  $Da = 10^{-5}$ , the heat trans-

fer rate is very low and further increase in both the values of  $Da$  and  $\phi$  caused the increasing heat transfer rate which declare that the conduction is transition into a convection state for increase in the value of  $Da$ . Furthermore, at  $Da = 10^{-3}$  and  $10^{-1}$ , the heat transfer rate is slightly decreased as the value of  $\phi$  is increased in the presence of strong magnetic field.

Figure 11 displays the time history of average Nusselt number versus time at different values of Hartmann number and Darcy number. At initial time, the average Nusselt number decreases sharply and as time progresses, the Nusselt number becomes slightly oscillated and finally reaches the steady state.

### 5. CONCLUSION

Numerical analysis of the unsteady natural convection flow in an inclined porous cavity filled with Cu-water nanofluid has been studied in the

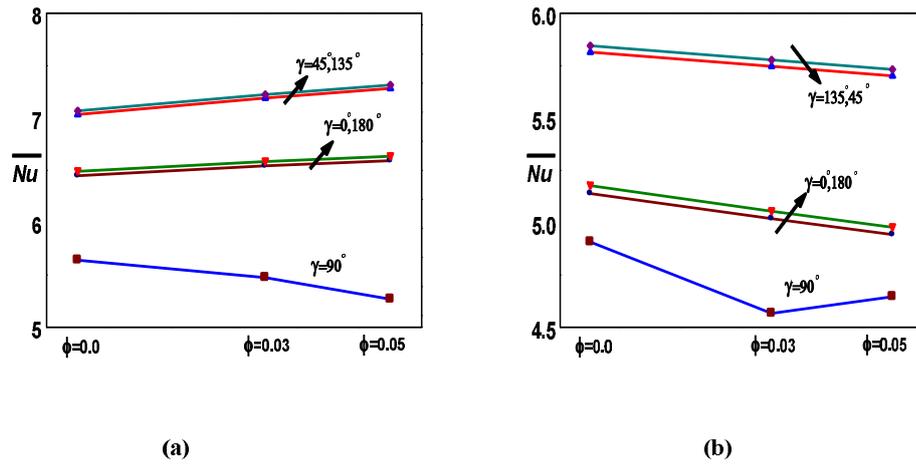


Fig. 9. Variation of average Nusselt number at the hot wall with  $\gamma$  (a)  $Ha = 0$  and (b)  $Ha = 50$  at  $Da = 10^{-3}$ .

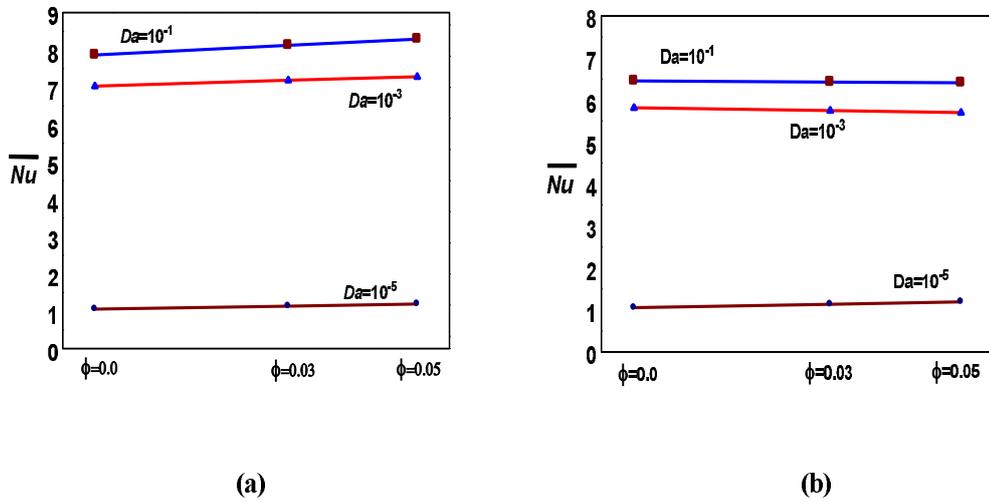


Fig. 10. Variation of average Nusselt number at the hot wall with  $Da$  (a)  $Ha = 0$  and (b)  $Ha = 50$  at  $\gamma = 45^\circ$ .

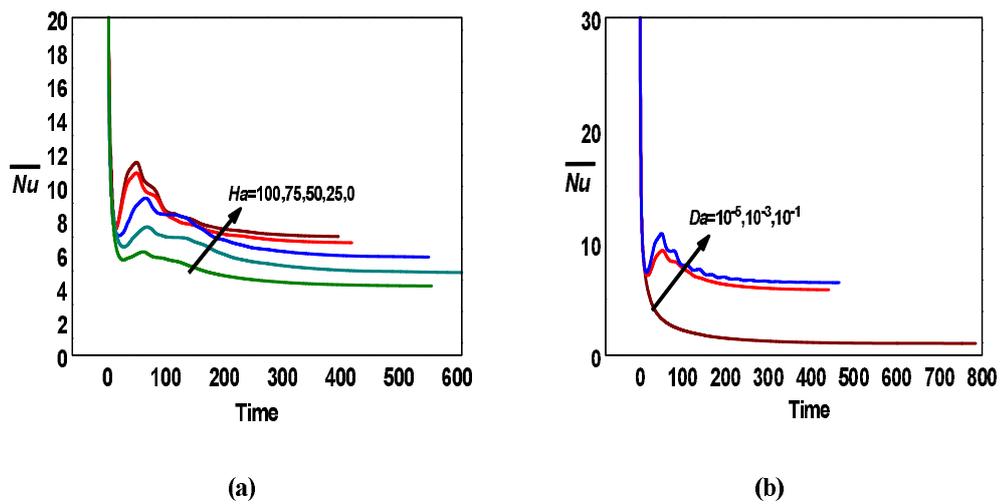


Fig. 11. Time history of average Nusselt number versus time for (a) different  $Ha$  at  $Da = 10^{-3}$ , and  $\gamma = 45^\circ$  and (b) different  $Da$  at  $\gamma = 45^\circ$  and  $Ha = 50$ .

presence of magnetic field. The results are presented to understand the effects of various non-dimensional parameters such as inclination angle  $\gamma$ , Hartmann number  $Ha$ , Darcy number  $Da$  and solid volume fraction  $\phi$ . The main conclusions obtained from the numerical results are

- The flow and thermal fields are significantly affected by adding nanoparticles in the base fluid due to the presence of magnetic field.
- The heat transfer rate decreases with increase in Hartmann number. Furthermore, the increasing rate of heat transfer was obtained with  $\phi$  for low values of Hartman numbers ( $Ha = 0$  &  $25$ ), and the rising rate transits into a decreasing rate when the magnitude of Hartmann number is large ( $Ha = 50, 75$  &  $100$ ).
- For all solid volume fractions, the best inclination angles are  $135^\circ$  &  $45^\circ$ , in both presence and absence of magnetic field respectively, because they yield a higher heat transfer rate.
- The heat transfer rate increases monotonically with  $Da$ , and also the presence of magnetic field clearly affects the influence of solid volume fraction.

#### ACKNOWLEDGMENT

The author Rajarathinam would like to thank RGNF (Rajiv Gandhi National Fellowship, India) for their financial support.

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