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Interaction of an Acceleration Wave with a Strong Shock in Transient Pinched Plasma

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ABSTRACT

In this paper, the evolution of an acceleration wave for the system of partial differential equations describing one dimensional, unsteady, axisymmetric motion of transient pinched plasma has been considered. The amplitude of the acceleration wave propagating along the characteristic associated with the largest eigenvalue has been evaluated. The interaction of the strong shock with the acceleration wave has been investigated. Effects of ambient density exponent and magnetic induction has been investigated.

Keywords: Shock wave; Acceleration Wave; Interaction of waves; Plasma.

1. INTRODUCTION

For nonlinear hyperbolic system of equations, discontinuity waves, which are also known as acceleration waves are important kind of solutions. These waves are characterized by a discontinuity in a normal derivative of the field but not in the field itself (Mentrelli *et al.* 2008). The general theory of wave interaction problem has been developed in the works of Jeffrey (Jeffrey 1973) and Boillat & Ruggeri (Boillat and Ruggeri 1979). Varley & Cumberbatch Varley and Cumberbatch (1965), Jeffrey (Jeffrey 1976) and Boillat & Ruggeri (Boillat and Ruggeri 1979) have studied the evolution of a weak discontinuity for a hyperbolic quasi-linear system of equations satisfying the Bernoulli's law. In (Landau and Lifshitz 1976), it has been shown that as a consequence of an interaction with a weak wave, the shock undergoes an acceleration jump; the same has also been seen in the works of Brun (Brun 1975) and Boillat & Ruggeri (Boillat and Ruggeri 1979). In (Radha *et al.* 1993), it has been shown that the general theory of wave interaction problem originated from the work (Jeffrey 1973) leads to the results obtained in (Boillat and Ruggeri 1979) and (Brun 1975). The theory has been successfully used in the works (Jena 2005) - (Pandey and Sharma 2013) to study the interaction of discontinuity wave with a characteristic shock or a strong shock in different mediums.

Since the study of Magneto-hydrodynamic (MHD) plasma is very complex, several simplifications have been introduced in many mod-els suggested in literatures. The confinement of plasma by magnetic pressure plays an impor-tant role in the MHD and plasma physics. A pinch is the compression of an electrically con-ducting filament by magnetic forces. There are two types of pinch i.e. z-pinch and θ-pinch. In a z-pinch, the current is axial and the mag-netic field azimuthal in a cylindrical coordi-nate system; however, in a θ -pinch the cur-rent is azimuthal (i.e. in θ direction in the cylindrical coordinate systems) and the mag-netic field is axial. In many kinds of pinch de-vice, an impulsive application of a large current seems to set up a shock wave propagating into plasma. Sinzi (Sinzi Kuwabara 1963)-(Sinzi Kuwabara 2013) considered a problem concern-ing the pinch of a cylindrical plasma due to magnetic pressure. The magnetic fields are in-duced by (i) axial currents corresponding to the self pinch in which the circumferential mag-netic field is generated by an axial sheet current on the plasma cylinder (assumed to be a perfect conductor), (ii) azimuthal currents corresponding to the induced pinch in which a coil surrounding a plasma cylinder generates an axial magnetic field, which pinches the plasma and an azimuthal sheet current being induced on its surface. For the system of equations describing transient pinched plasma, self-similar solutions are obtained in Jena (2012) and interaction of an acceleration wave with a characteristic shock is considered in Jena and Singh (2013b). Radiation effects on MHD flow past an exponentially accelerated isothermal vertical plate with uniform mass diffusion in the presence of heat source has been taken up in Reddy, Reddy, and Suneetha (2012). In Reddy (2013), Lie Group Analysis of heat and mass transfer effects on steady MHD free convection flow past an inclined surface with viscous dissipation has been considered.

In this paper the system of partial differential equations describing one dimensional unsteady axisymmetric motion of transient pinched plasma is considered. It is assumed that (i) the plasma is axisymmetric and infinitely long, (ii) the plasma is a perfect conductor so that the electric current appears only as a sheet current, (iii) initially the magnetic field is not present in the plasma and (iv) the infinitely conductivity prevents the diffusion of the outer magnetic field into the plasma. We have evaluated the amplitude of the acceleration wave propagating along the characteristic associated with the largest eigenvalue. Numerical calculations are carried out by considering the particular solution obtained in Jena (2012). The interaction of an acceleration wave with a strong shock has been considered. The jump in the shock acceleration and the amplitudes of reflected and transmitted waves after interaction are evaluated by using the results of general theory of wave interaction Radha and Jeffrey (1993).

2. Basic Equations

The matrix form of the governing equations describing axisymmetric motion of a polytropic gas with adiabatic index γ can be written in the matrix form as Jena and Singh (2013b)

$$U_t + AU_x = f, \tag{1}$$

where $U = (\rho,\ u,\ p,\ B)^{tr}$,
$f = (-\rho u/x,\ 0,\ -\gamma p u/x,\ -Bu/x)^{tr}$ and

$$A = \begin{pmatrix} u & \rho & 0 & 0 \\ 0 & u & 1/\rho & B/(\mu\rho) \\ 0 & \gamma p & u & 0 \\ 0 & B & 0 & u \end{pmatrix},$$

with u as the particle velocity along x-axis, t the time, ρ the density, p the pressure, B the magnetic induction and μ the magnetic permeability, respectively and 'tr' represents transpose. Considering T as the temperature of the gas and R the gas constant, the equation of state is given by

$$p = \rho RT.$$

We consider the motion of a shock front, $x = \chi(t)$, propagating into an inhomogeneous medium specified by $U_* = (\rho_0, u_0, p_0, B_0)^{tr}$ with the speed V where

$$\rho_0 = \rho_0(x), u_0 = 0,\ \ p_0 = \text{constant}, B_0 = B_0(x). \tag{2}$$

The conservation form of equation (1) in the regions behind and ahead of the shock (i.e. to the left and to the right of the discontinuity curve, $dx/dt = V$) can be written as

$$G_t(x,t,U) + F_x(x,t,U) = H(x,t,U),$$

$$G_t(x,t,U_*) + F_x(x,t,U_*) = H(x,t,U_*). \tag{3}$$

where U and U_* are the solution vectors behind and ahead of the shock and G, F and H have the following forms

$$G = \left(\rho, \rho u, \frac{p}{\gamma-1} + \frac{1}{2}\rho u^2 + \frac{B^2}{2\mu}, B\right)^{tr},$$

$$F = \left(\rho u, \rho u^2 + p + \frac{B^2}{2\mu}, u\left(\frac{\gamma p}{(\gamma-1)} + \frac{B^2}{\mu} + \frac{1}{2}\rho u^2\right), uB\right)^{tr},$$

$$H = \left(-\frac{\rho u}{x}, -\frac{\rho u^2}{x}, -\frac{u}{x}\left(\frac{\gamma p}{(\gamma-1)} + \frac{B^2}{\mu} + \frac{1}{2}\rho u^2\right), -\frac{uB}{x}\right)^{tr}. \tag{4}$$

and tr represents transpose.

In general if the conservation equation can be written in the form (3) where G and F are column vectors having n components, then the Rankine-Hugoniot equations are written as

$$|[G_i]|V = |[F_i]|,\ i = 1, 2, \ldots n. \tag{5}$$

Here, $|[A]| = A - A_0$ represents jump in the variable A, where A_0 is condition in the medium ahead of the shock and A is the condition in the medium behind the shock. Using (4) in (5), the Rankine-Hugoniot jump conditions across the shock front when $B_0 \neq 0$ and $\gamma \neq 2$ can be written as Jena (2012)

$$\frac{\rho(\chi(t),t)}{\rho_0(\chi(t))} = \frac{\mu}{(\gamma-2)B_0^2}\left(\{(\gamma-1)\rho_0(\chi(t))V^2 + \right.$$

$$\left. 2\gamma p_0 + \gamma B_0^2(\chi(t))/\mu\} \pm K\right),$$

$$u=\left(1-\frac{\rho_0(\chi(t))}{\rho(\chi(t),t)}\right)V,$$

$$p=\left(1-\frac{\rho_0(\chi(t))}{\rho(\chi(t),t)}\right)\rho_0(\chi(t))V^2-\frac{1}{2\mu}$$

$$\times\left(\frac{\rho^2(\chi(t),t)}{\rho_0^2(\chi(t))}-1\right)B_0^2(\chi(t))+p_0, \quad (6)$$

$$B=\frac{\rho(\chi(t),t)}{\rho_0(\chi(t))}B_0(\chi(t)),$$

where

$$K^2=\{(\gamma-1)\rho_0(\chi(t))V^2+2\gamma p_0+\gamma B_0^2/\mu\}^2$$

$$-4(\gamma+1)\rho_0V^2(\gamma-2)B_0^2(\chi(t))/\mu.$$

However, if $B_0=0$ then the conditions are

$$\rho(\chi(t),t)=\frac{(\gamma+1){\rho_0}^2(\chi(t))V^2}{(\gamma-1)\rho_0(\chi(t))V^2+2\gamma p_0},$$

$$u(\chi(t),t)=\frac{2}{(\gamma+1)}\frac{\rho_0(\chi(t))V^2-\gamma p_0}{\rho_0(\chi(t))V},$$

$$p(\chi(t),t)=\frac{2\rho_0(\chi(t))V^2-(\gamma-1)p_0}{(\gamma+1)}, \quad (7)$$

$$B(\chi(t),t)=0,$$

and if $\gamma=2$ then the conditions are

$$\rho(\chi(t),t)=\frac{(\gamma+1){\rho_0}^2(\chi(t))V^2}{(\gamma-1)\rho_0(\chi(t))V^2+2\gamma p_0+\gamma B_0^2(\chi(t))/\mu},$$

$$u(\chi(t),t)=\frac{2\{\rho_0(\chi(t))V^2-\gamma p_0\}-\gamma B_0^2(\chi(t))/\mu}{(\gamma+1)\rho_0(\chi(t))V},$$

$$p(\chi(t),t)=\frac{2\{\rho_0(\chi(t))V^2-\gamma p_0\}-\gamma B_0^2(\chi(t))/\mu}{(\gamma+1)}$$

$$-\frac{B_0^2(\chi(t))}{2\mu}\left(\frac{\{(\gamma+1)\rho_0(\chi(t))V^2\}^2}{\{(\gamma-1)\rho_0(\chi(t))V^2+2\gamma p_0+\gamma B_0^2(\chi(t))/\mu\}^2}-1\right), \quad (8)$$

$$B(\chi(t),t)=\frac{(\gamma+1)\rho_0(\chi(t))V^2}{(\gamma-1)\rho_0(\chi(t))V^2+2\gamma p_0+\gamma B_0^2(\chi(t))/\mu}B_0(\chi(t)).$$

In case of a strong shock $B_0<<V$ and $p_0<<V$ and hence the conditions are as per following

$$\rho(\chi(t),t)=\frac{\gamma+1}{\gamma-1}\rho_0(\chi(t)),$$

$$u=\frac{2}{\gamma+1}V,\ p=\frac{2}{\gamma+1}\rho_0(\chi(t),t)V^2, \quad (9)$$

$$B=\frac{\gamma+1}{\gamma-1}B_0(\chi(t)).$$

3. Evolution of the Acceleration Wave

The eigenvalues of matrix A in (1) are given by

$$\lambda^{(1)}=(u+a),\ \lambda^{(2)}=u(\text{a double root}), \quad (10)$$

$$\lambda^{(3)}=(u-a),$$

where $a=\left(\frac{\gamma p}{\rho}+\frac{B^2}{\rho\mu}\right)^{1/2}$ is the frozen speed of sound in the medium. The left and right eigenvectors corresponding to these eigenvalues are

$$L^{(1)}=\left(0,\rho a,1,\frac{B}{\mu}\right),\ R^{(1)}=\left(\frac{1}{2a^2},\frac{1}{2\rho a},\frac{\gamma p}{2\rho a^2},\frac{B}{2\rho a^2}\right)^{tr},$$

$$L^{(2,1)}=\left(-a^2,0,1,\frac{B}{\mu}\right),\ R^{(2,1)}=\left(-a^{-2},0,\frac{-B}{\mu},1\right)^{tr},$$

$$L^{(2,2)}=(1,0,-\frac{\rho}{2\gamma p},-\frac{\rho}{2B}),\ R^{(2,2)}=(u^{-2},0,0,0)^{tr},$$

$$L^{(3)}=(0,-\rho a,1,\frac{B}{\mu}),\ R^{(3)}=(\frac{1}{2a^2},-\frac{1}{2\rho a},\frac{\gamma p}{2\rho a^2},\frac{B}{2\rho a^2})^{tr}. \quad (11)$$

We consider that the acceleration wave (or $C^{(1)}$ discontinuity) is propagating along the characteristic curve associated with the largest eigenvalue i.e. determined by $\frac{dx}{dt}=\lambda^{(1)}$, originating from the point (x_0,t_0) in the region $U=(\rho,u,p,B)^{tr}$ swept by the strong shock. Using the transport equation obtained by Radha and Jeffrey (1993)

$$L^{(1)}\left\{\frac{d\Lambda_1}{dt}+(U_x+\Lambda_1)(\nabla\lambda^{(1)})\Lambda_1\right\}+\{(\nabla L^{(1)})\Lambda_1\}^{tr}\frac{dU}{dt}$$

$$+(L^{(1)}\Lambda_1\left\{(\nabla\lambda^{(1)})U_x+\lambda_x^{(1)}\right\}-(\nabla(L^{(1)}f))\Lambda_1=0, \quad (12)$$

the Bernoulli type of transport equation for amplitude of the acceleration wave $\alpha^{(1)}$ has been obtained in Jena and Singh (2013b) as

$$\frac{d\alpha^{(1)}}{dt}+J_1{\alpha^{(1)}}^2+J_2\alpha^{(1)}=0, \quad (13)$$

where $\Lambda_1=\alpha^{(1)}(t)R^{(1)}$ denotes the jump in U_x across the $C^{(1)}$ discontinuity and is collinear to the right eigenvector $R^{(1)}$ and

$$J_1=\frac{1}{2\rho a}\left(\frac{1}{2}+\frac{\gamma^2 p}{2\rho a^2}+\frac{B^2}{\rho a^2\mu}\right),$$

$$J_2=\frac{1}{2\rho a}\left(\frac{1}{2}+\frac{\gamma^2 p}{2\rho a^2}+\frac{B^2}{\rho a^2\mu}\right)\left(\rho a u_x+p_x+\frac{BB_x}{\mu}\right) \quad (14)$$

$$+2(u_x+a_x)\left\{-\frac{\rho}{2}-\frac{\gamma^2 p}{2a^2}-\frac{B^2}{a^2\mu}-\frac{\gamma^2 p}{\rho a^2}+\right.$$

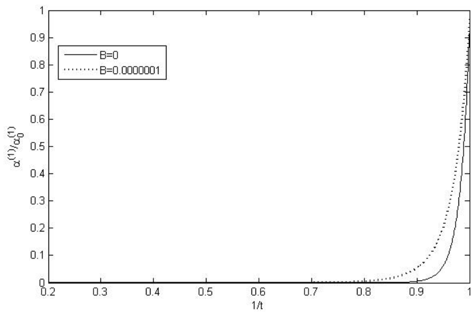

Fig. 1. Variation with B

$\gamma = 2,\ \ \mu = 1.2566371 * 10^{(-6)},\ \ \delta = 1.5$
$\alpha = -1/3,\ \ \sigma = 1.5$

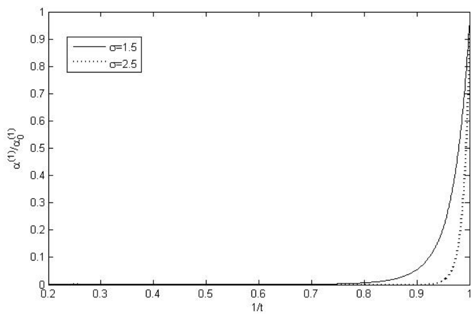

Fig. 2. Variation with σ

$\gamma = 2,\ \ \mu = 1.2566371 * 10^{(-6)},\ \ \delta = 1.5,$
$\alpha = -1/3,\ \ B = 0.0000001$

$$\frac{\gamma^3 p^2}{(\rho a^2)^2} + \frac{2\gamma B^2 p}{(\rho a^2)^2\mu} - \frac{B^2}{\mu\rho a^2} + \frac{B^2}{\mu(\rho a^2)^2}\left(\gamma^2 p + \frac{2B^2}{\mu}\right) -$$
$$\left(\frac{1}{4} + \frac{\gamma^2 p}{4\rho a^2} + \frac{B^2}{2\rho a^2\mu}\right)\Big\}\left(u_x + \frac{u}{x}\right) + \frac{a}{2x} +$$
$$+ \frac{u}{x\rho a^2}\left(\frac{\gamma^2 p}{2} + \frac{B^2}{\mu}\right).$$

Integrating equation (13) with respect to t we obtain

$$\alpha^{(1)} = \frac{\alpha_0^{(1)} M(t)}{1 + \alpha_0^{(1)} N(t)}, \qquad (15)$$

where $N(t) = \int_{t_0}^{t} J_1(s)M(s)ds$, $M(t) = \exp\left(-\int_{t_0}^{t} J_2(s)ds\right)$ and $\alpha_0^{(1)} = \alpha^{(1)}(t_0)$. The solution at (15) breaks down at some critical time $t = t_c$ where $1 + \alpha_0^{(1)} N(t) = 0$, which indicates the appearance of a shock wave at an instant t_c (i.e. a compression wave culminates into a shock in a finite time only when the initial discontinuity associated with the wave exceeds a critical value).

For numerical computations we considered the particular solution for $\gamma = 2$ to the system of equations obtained in Jena (2012), which is given by

$$\rho = \rho_{0c}\xi^{-\eta} t^{\delta\sigma},\ u = \frac{2}{3}\xi t^{\delta-1},$$

$$p = \frac{2}{3}\rho_{0c}\delta^2\xi^{-\phi} t^{\delta\sigma + 2(\delta-1)}, \qquad (16)$$

$$B = 3B_{0c}\xi^{-\psi} t^{(\delta-1)+\delta\sigma/2},$$

where

$$\xi = xt^{-\delta},\ \eta = -(4+3\sigma),$$
$$\delta = 3/2,\ \phi = -3(2+\sigma),$$
$$\psi = \phi/2,\ 2 + 3\frac{B_{0c}^2}{\rho_{0c}\mu} = 0,$$

and σ is the ambient density exponent (i.e. $\rho_0 = \rho_{0c} x^{\sigma}$).

Using (16), equation (13) is solved numerically for $\alpha^{(1)}$ and the results are depicted in Figures 1 and 2. It has been observed that $\alpha^{(1)}$ decreases as t increases and tends to zero as $t \to \infty$. The amplitude of the incident wave $\alpha^{(1)}$ increases with an increase in the magnetic induction B, but decreases with an increase in the ambient density exponent.

4. Collision of Acceleration Wave with the Strong Shock

For evaluating the amplitudes of the reflected and transmitted waves after interaction, we consider the conservation equation (3) in the regions behind and ahead of the shock propagating with speed V. Let $P(x_p, t_p)$ be the point at which the fastest $C^{(1)}$ discontinuity of $(3)_1$, moving along the characteristic $\phi(x,t) = 0$ and originating from the point (x_0, t_0) intersects the shock path. As we have considered the case of strong shock, the value of a in (10) at $t = t_p$ i.e. at the shock, can be given by $\frac{\Gamma V}{\gamma+1}$ where $\Gamma^2 = 2\gamma(\gamma-1)$. Hence, the eigenvalues at $t = t_p$ on both the sides of the shock are of the following form

$$\lambda^{(1)} = \frac{2+\Gamma}{(\gamma+1)}V,\ \lambda^{(2)} = \frac{2}{(\gamma+1)}V,$$

$$\lambda^{(3)} = \frac{2-\Gamma}{(\gamma+1)}V, \qquad (17)$$

and

$$\lambda_*^{(1)} = \left(\frac{\gamma p_0}{\rho_0} + \frac{B_0^2}{\rho_0\mu}\right)^{1/2},$$

$$\lambda_*^{(2)} = 0,$$

$$\lambda_*^{(3)} = -\left(\frac{\gamma p_0}{\rho_0} + \frac{B_0^2}{\rho_0\mu}\right)^{1/2}. \qquad (18)$$

Since, the pressure ahead of the shock wave (where the variables are designated by asterisk) is very small when compared with the pressure behind, the Lax evolutionary conditions for a physical shock can be given by Radha and Jeffrey (1993), Lax (1957)

$$\lambda^{(3)} < \lambda^{(2)} < V < \lambda^{(1)} \text{ and } \lambda_*^{(3)} < \lambda_*^{(2)} < \lambda_*^{(1)} < V. \quad (19)$$

From the above relations imply that as the incident wave encounters the shock with velocity $\lambda^{(1)}$ at $t = t_p$, two reflected waves in the characteristics with velocities $\lambda^{(2)}$ and $\lambda^{(3)}$ are generated from the collision point and no transmitted waves formed. As in Radha and Jeffrey (1993), the jump in U_x of the incident and reflected waves (as there are no transmitted waves) on the discontinuity line are given by the relations

$$\Lambda_1(P) = \alpha^{(1)}(t_p)R_s^{(1)},$$

$$\Lambda_2^{(R)}(P) = \alpha_1^{(2)}(t_p)R_s^{(2,1)} + \alpha_2^{(2)}(t_p)R_s^{(2,2)}, \quad (20)$$

$$\Lambda_3^{(R)}(P) = \alpha^{(3)}(t_p)R_s^{(3)},$$

where a subscript s refers to the values evaluated at the shock. The evolutionary equations to determine the jump in the shock acceleration $[\dot{V}] = \dot{V}_{t_p^+} - \dot{V}_{t_p^-}$ at the collision time $t = t_p$ and the coefficients of the amplitudes of reflected waves $\alpha_1^{(2)}$, $\alpha_2^{(2)}$ and $\alpha^{(3)}$ can be determined from the algebraic system of equations

$$(G - G_*)_s[\dot{V}] + (\nabla G)_s R_s^{(2,1)}(V - \lambda_s^{(2)})^2\alpha_1^{(2)} + (\nabla G)_s R_s^{(2,2)}(V - \lambda_s^{(2)})^2\alpha_2^{(2)} + (\nabla G)_s R_s^{(3)}$$

$$\times (V - \lambda_s^{(3)})^2\alpha^{(3)} = -(\nabla G)_s R_s^{(1)}(V - \lambda_s^{(1)})^2\alpha^{(1)}. \quad (21)$$

In view of relations (10), (11) and (21) the balance equation at the time t_p can be written as the following system of algebraic equations in the unknowns $[\dot{V}]$, $\alpha_1^{(2)}$, $\alpha_2^{(3)}$ and $\alpha^{(3)}$

$$\frac{2\rho_0(\chi(t))}{(\gamma-1)}[\dot{V}] - \frac{(\gamma-1)^2}{\Gamma^2}\alpha_1^{(2)} + \frac{(\gamma-1)^2}{4}\alpha_2^{(2)} + \frac{(\gamma+\Gamma-1)^2}{2\Gamma^2}\alpha^{(3)} = -\frac{(\gamma-\Gamma-1)^2}{2\Gamma^2}\alpha^{(1)},$$

$$\frac{2\rho_0(\chi(t))}{(\gamma-1)}[\dot{V}] - \frac{2(\gamma-1)^2}{\Gamma^2(\gamma+1)}\alpha_1^{(2)} + \frac{(\gamma-1)^2}{2(\gamma+1)}\alpha_2^{(2)} \quad (22)$$

$$+\frac{(\gamma+\Gamma-1)^2(\Gamma-2)}{2\Gamma^2(\gamma+1)}\alpha^{(3)} = -\frac{(\gamma-\Gamma-1)^2(\Gamma+2)}{2\Gamma^2(\gamma+1)}\alpha^{(1)},$$

$$\frac{4\rho_0(\chi(t))}{(\gamma-1)}[\dot{V}] - \left(\frac{2(\gamma-1)^2}{\Gamma^2(\gamma+1)} + \frac{B_0}{\mu}\right)\alpha_1^{(2)} + \frac{(\gamma-1)^2}{2(\gamma+1)}\alpha_2^{(2)}$$

$$+\frac{(\gamma+\Gamma-1)^3}{\Gamma^2(\gamma+1)}\alpha^{(3)} = -\frac{(\gamma-\Gamma-1)^2(\gamma+\Gamma+1)}{\Gamma^2(\gamma+1)}\alpha^{(1)},$$

$$\frac{2}{(\gamma-1)}B_0[\dot{V}] + \alpha_1^{(2)} = 0.$$

The solution of the algebraic system of equations (22) can be obtained as per the following

$$[\dot{V}] = \frac{-2(\gamma-\Gamma-1)^2(\Gamma+2\gamma)}{\Gamma^2\{2(\gamma+\Gamma-1)(\gamma+1)A_2 - (\Gamma-4)(\gamma+1)A_3 - 2(\Gamma+2\gamma)A_1\}}\alpha^{(1)},$$

$$\alpha^{(3)} = \frac{(\gamma-\Gamma-1)^2}{(\gamma+\Gamma-1)^2}$$

$$\times \frac{2(\Gamma-\gamma-2)(\Gamma+2\gamma)A_1 - 2(\Gamma+\gamma-1)(\gamma+1)A_2 + \gamma+1)\Gamma(\gamma+\Gamma-2)A_3}{(\gamma+\Gamma-2)\{-2(\Gamma+2\gamma)A_1 + 2(\gamma+\Gamma-1)(\gamma+1)A_2 - (\Gamma-4)(\gamma+1)A_3\}}\alpha^{(1)},$$

$$\alpha_2^{(2)} = \frac{-2(\gamma-\Gamma-1)^2}{(\gamma-1)^2\Gamma^2} \quad (23)$$

$$\times \frac{4(\Gamma+2\gamma)(\Gamma-2)A_1 + 2(\gamma+1)(\Gamma+\gamma-1)(\gamma+\Gamma-3)A_2 + 4(\gamma+1)(\Gamma+\gamma-2)A_3}{(\gamma+\Gamma-2)\{-2(\Gamma+2\gamma)A_1 + 2(\gamma+\Gamma-1)(\gamma+1)A_2 - (\Gamma-4)(\gamma+1)A_3\}}\alpha^{(1)},$$

$$\alpha_2^{(1)} = \frac{2}{(\gamma-1)}B_0\frac{-2(\gamma-\Gamma-1)^2(\Gamma+2\gamma)}{\Gamma^2\{2(\gamma+\Gamma-1)(\gamma+1)A_2 - (\Gamma-4)(\gamma+1)A_3 - 2(\Gamma+2\gamma)A_1\}}\alpha^{(1)},$$

where $\alpha^{(1)}$ is given by (15) and

$$A_1 = \frac{2\rho_0}{\gamma-1} + \frac{2(\gamma-1)B_0}{\Gamma^2},$$

$$A_2 = \frac{2\rho_0}{\gamma-1} + \frac{4(\gamma-1)B_0}{\Gamma^2(\gamma+1)},$$

$$A_3 = \frac{4\rho_0}{\gamma-1} + \frac{2B_0}{\gamma-1}\left(\frac{2(\gamma-1)^2}{\Gamma^2(\gamma+1)} + \frac{B_0}{\mu}\right).$$

Equations (23) show that the amplitudes of the reflected waves are proportional to the amplitude of the incident wave. In the absence of the incident wave (i.e. $\alpha^{(1)} = 0$), there are neither reflected waves nor jump in the shock acceleration. An increase in the magnitude of the incident wave causes the reflection coefficients and the jump in shock acceleration to increase in magnitude.

5. Results and Conclusion

In this paper, a system of equations describing one dimensional, unsteady, axisymmetric motion of a transient pinched plasma has been considered to study evolution of an acceleration wave and its interaction with a strong shock. The amplitude of the acceleration wave $\alpha^{(1)}$ traveling on the characteristics satisfy the Bernoulli type equation. Using solutions obtained in Jena and Singh (2013b) the equation is solved numerically for $\alpha^{(1)}$ and it has been observed that $\alpha^{(1)}$ decreases as t increases and tends to zero as $t \to \infty$. Also, the amplitude of the incident wave $\alpha^{(1)}$ increases with an increase in the magnetic induction B, but decreases with an increase in the ambient density exponent.

It has also been observed that after interaction of the acceleration wave through a strong shock, there are two reflected waves along the characteristic lines with velocities $\lambda^{(2)}$ and $\lambda^{(3)}$ and no transmitted waves. The amplitudes of reflected waves and the jump in shock acceleration after interaction are evaluated and are found to be proportional to the amplitude of the incident wave. In the absence of the incident wave (i.e. $\alpha^{(1)} = 0$), there are neither reflected waves nor the jump in the shock acceleration. An increase in the magnitude of the incident wave causes the reflection coefficients and the jump in shock acceleration to increase in magnitude.

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