



## Influence of Magnetic Field on Fingering Instability in a Porous Medium with Cross-Diffusion Effect: a Thermal Non-equilibrium Approach

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### ABSTRACT

The purpose of this paper is to investigate the thermal instability problem under the influence of three diffusing components on fluid saturated horizontal porous media under local thermal non-equilibrium effect. These three components are vertical magnetic field, solute and heat. We considered an electrically conducting two components fluid and anisotropic porous medium. The physical system is heated and salted from above. Flow in the porous medium is characterized by Darcy model, whereas the fluid and solid phases are not in local thermodynamic equilibrium (LTNE). Linear stability analysis is used to calculate critical Darcy-Rayleigh number and corresponding wave number for onset of stationary convection. The study is based on mainly four controlling parameters, Darcy-Chandrasekhar number ( $Q$ ), dimensionless inter-phase heat transfer coefficient ( $H$ ), Soret ( $Sr$ ) and Dufour ( $Du$ ) cross-diffusion parameters, among several. We illustrated the effects of these parameters on thermal instability and found parameters may have stabilizing or destabilizing effects, thus may advance or delay the onset of convection. Various comparative studies are also presented between different cases and conditions, such as for anisotropic and isotropic cases, cross-diffusion and without cross-diffusion cases and for different values of  $Sr$  and  $Du$ .

**Keywords:** Thermal non-equilibrium; Magneto-convection; Darcy-Rayleigh Number; Fingering instability; Cross-diffusion.

### NOMENCLATURE

$a$	Horizontal wave number	$Q$	Darcy-Chandrasekhar number
$c$	specific heat capacity	$\vec{q}$	velocity
$C$	concentration	$R$	Darcy-Rayleigh number
$d$	length of the porous layer	$R_C$	Critical Rayleigh number
$D_M$	solute diffusivity	$R_s$	Solutal Rayleigh number
$D_F$	DuFour coefficient	$Sr$	soret parameter
$D_T$	soret coefficient	$T_f$	temperature of fluid,
$D_u$	duFour parameter	$T_s$	temperature of solid,
$\vec{g}$	acceleration due to gravity	$\Delta T$	temperature difference across the porous layer
$H$	non-dimensional inter-phase heat transfer coefficient	$T_l$	temperature of lower surface
$h_s$	inter-phase heat transfer coefficient	$T_u$	temperature of upper surface
$h_z$	perturbed magnetic field along z-axis	$t$	time
$K$	permeability of porous layer	$x, y, z$	space coordinates
$K_x, K_x, K_y$	characteristic permeabilities in the x, y and z directions	$\kappa$	thermal diffusivity
$Le$	Lewis number	$\kappa_f$	thermal diffusivity for fluid
$l, m$	x-component and y-component of wavenumber	$\kappa_s$	thermal diffusivity for solid
$p$	reduced pressure	$\beta_C$	Coefficient of concentration expansion
$Pm$	magnetic Prandtl number		
$\gamma$	magnetic field $(\gamma_1 \gamma_2 \gamma_3)$		

$\beta_T$	coefficient of thermal expansion	$\Psi$	perturbation amplitude of fluid concentration perturbation
$\Lambda$	magnetic viscosity		
$\eta_f$	fluid thermal conductivity ratio		
$\eta_s$	solid thermal conductivity ratio	<b>Other symbols</b>	
$\varepsilon$	porosity	$D$	$d/dz$
$\mu$	dynamic viscosity of the fluid	$\nabla_1^2$	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$
$\mu_m$	magnetic permeability		
$\nu$	kinematic viscosity	<b>Subscripts</b>	
$\rho$	density of fluid	$b$	basic state
$\rho_0$	reference density	$c$	Critical
$\gamma$	porosity modified conductivity ratio	$f$	Fluid
$\chi$	diffusivity ratio	$S$	Solid
$\xi$	anisotropy ratio	<b>Superscripts</b>	
$\Theta$	amplitude of fluid temperature perturbation	'	perturbed quantity
$\Phi$	amplitude of solid temperature perturbation	*	dimensionless quantity

### 1. INTRODUCTION

Double-diffusive convection in fluid-saturated porous media with thermo-diffusive effect is of practical interest in many engineering applications such as petrology, hydrology, solidification of binary alloys as well as many other applications, and well documented by Nield and Bejan (2013). When an imposed magnetic field is associated with double-diffusive convection in a fluid-saturated porous medium, then it has various applications in science, engineering and technology (Wallace *et al.* (1969)). One of the key importance of such type of convection is seen in petroleum reservoirs where the system comprises of both electrically inert solid and electrically conducting fluid phases. Due to its applications, as mentioned above, the problem has attracted many researchers to work in this field. Some of the recent literature in this area are: Sarvanan and Yamaguchi (2005), Bhadauria *et al.* (2008a,b,2010). Bhadauria *et al.* (2008 a, b, 2010) studied the effect of magnetic field on thermal modulated convection in porous medium. Specially in the paper of (2010), the present authors have studied the effect of thermal modulation in double-diffusive magneto-convection.

According to fact that the fluid density depends on solute concentration, it leads to a competition between thermal and compositional gradients. So it is obvious to ask what will happen when cross-diffusion takes place in double-diffusive magneto-convection? In a system where three diffusing properties (magnetic field, solute, heat) are present, instabilities can occur only if at least one of the component is destabilizing. In the present paper, we are attempting to answer the above question. Lots of work is available on the onset of double-diffusive convection in a porous medium with cross-diffusion effects (Nield and Bejan (2013)). Thermal convection in a binary fluid driven by the Soret and DuFour effects, has been investigated by Knobloch (1980). He has shown that the equations are identical to the thermo-solutal problem except for a relation between the thermal and solute Rayleigh numbers. The double diffusive convection in a porous medium in the presence of Soret and DuFour coefficients has

been analyzed by Rudraiah and Malashetty (1986). It is important to mention that most of the work related to double-diffusive convection with and without cross-diffusion is considered when system is heated and salted from below. What will happen if we consider the case of heating and salting from above? From the available literature, it can be stated that due to its occurrence in many situations such as in oceanography, it is matter of attraction for researchers. In this case double-diffusive convection differ from classical Rayleigh Be´nard convection. There is two fundamental new flow structures can appear: layers and fingers. Fingers are vertically oriented long and narrow regions of up or down welling fluid which can occur if the fluid property with large diffusion coefficient (for example temperature) imposes a stability gradient on the fluid, whereas the property with the small diffusion coefficient (the salt concentration in oceanography) is unstably stratified (Huge and Tilgner (2010)). A study of convective instability in a fluid mixture heated from above with negative separation ratio (Soret coefficient) was performed experimentally by La Porta and Surko (1998). This topic is well documented in the Huge and Tilgner (2010) and reference of it.

Thermal equilibrium is assumed between solid porous matrix and saturated fluid in most of the available literatures related to thermal convection in porous media. But for sufficiently large Rayleigh number or rapid heat transfer for high speed flow, it can be expected that the equilibrium will break down, so that the respective local mean values of the temperature of solid and fluid phases are not same. Local thermal non-equilibrium (LTNE) state will be more suitable than local thermal equilibrium (LTE) state in this situation. A two-phase model for energy equations was well documented in the book of Nield and Bejan (2013). Rees and Pop (2005), Banu and Rees (2002), Rees *et al.* (2008) have investigated thermal non-equilibrium effect on natural convection in horizontal porous layer. Recently Kuznetsov *et al.* (2015) is investigated analytically, the effect of local thermal non-equilibrium on the onset of double-diffusive convection in a porous medium consisting of two horizontal layers, each internally heated.

It is also noticeable that most of works related to convective flow with LTNE effect have dealt with mechanical and thermal isotropic case. Mechanical term is used in the sense when porous matrix allow fluid flow with same permeability in all possible directions. Whereas thermal term indicates that heat is equally distributed in all possible directions in fluid as well as solid. However, in many practical situations the porous materials are anisotropic in their mechanical and thermal properties. Onset of thermal convection in horizontal porous layer with anisotropic permeability was first investigated by Castinel *et al.* (1974). Later on Epherre (1975) extended the stability analysis to media with anisotropic thermal diffusivity. Malashetty *et al.* (2005) investigated onset of convection in an anisotropic porous medium under LTNE effect, when porous medium is heated from below. The effect of mechanical and thermal anisotropic on the stability of gravity driven convection in rotating porous media in the presence of thermal non-equilibrium is investigated by Govender and Vadasz (2007). They considered the physical system heated from below.

The idea of this paper is influence by the recent work of Saravanan and Jegajothi (2010) and Srivastava *et al.* (2011). In the first paper, they have investigated the stationary fingering instability in a non-equilibrium porous medium with coupled molecular diffusion. They focused on the effects of cross-diffusion parameters and inter-phase heat transfer coefficient. Where as in the second paper, authors presented impact of magnetic field and anisotropic effect on onset of convection in horizontal porous media. A question arise, what will happen if we consider the magnetic field effect (i.e. three diffusing components) with mechanical and thermal anisotropic properties Motivated by the importance and impact of three diffusing components with more realistic anisotropic condition on the onset of convection, we would like to investigate the problem theoretically in this article. An outline of the paper is as follows. In section 2, the mathematical formulation and solution of the physical problem are given. Linear stability analysis presented in section 3. Results and discussions are reported in section 4. Finally, some important features of the analysis are concluded in section 5.

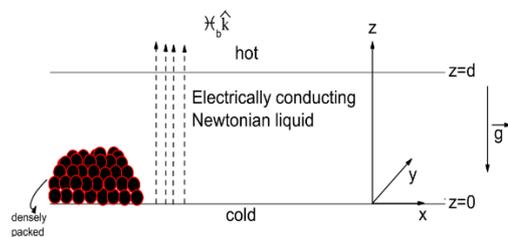


Fig. 1. Schematic of physical configuration.

2. GOVERNING EQUATIONS

We consider an electrically conducting two component fluid-saturating anisotropic porous

layer of depth  $d$ , which is heated and salted from above confined between two parallel horizontal planes at  $z = 0$  and  $z = d$ . Cartesian coordinates have been taken with the origin at the bottom of the porous medium, and the  $z$ -axis vertically upwards. The surfaces are extended infinitely in  $x$  and  $y$  directions and maintained at a constant gradient  $\Delta T$  and salinity  $\Delta S$  across the porous layer. A constant magnetic field  $\vec{\kappa}_b = \kappa_b \hat{k}$  is maintained externally in the vertical upward direction. We use a two-phase model for heat equation with separate thermal conductivities for the solid and fluid phases respectively, because the solid and liquid phases of the medium are considered not to be in local thermal equilibrium. A schematic digram of physical configuration of the problem has been shown in Fig. 1. Under the Boussinesq approximation, the dimensional governing equations for the study of magneto-convection in an electrically conducting-fluid-saturated anisotropic porous layer are:

$$\nabla \cdot \vec{q} = 0, \tag{1}$$

$$\nabla \cdot (\vec{\kappa} \cdot \nabla T) = 0, \tag{2}$$

$$\frac{1}{\rho_0} \nabla P + \frac{\rho_f}{\rho_0} \vec{g} - \nu K \cdot \nabla \vec{q} + \frac{\mu_m}{\rho_0} (\nabla \cdot \nabla) \vec{q} = 0, \tag{3}$$

$$\begin{aligned} \epsilon (\rho_f c_f) \frac{\partial T_f}{\partial t} + \rho_f c_f (\vec{q} \cdot \nabla) T_f = \nabla \cdot (\epsilon \kappa_f \nabla T_f) \\ + D_f \nabla^2 C + h_s (T_s - T_f), \end{aligned} \tag{4}$$

$$(1 - \epsilon) (\rho_s c_s) \frac{\partial T_s}{\partial t} = \nabla \cdot ((1 - \epsilon) \kappa_s \nabla T_s) - h_s (T_s - T_f), \tag{5}$$

$$\epsilon \frac{\partial C}{\partial t} + (\vec{q} \cdot \nabla) C = \epsilon D_m \nabla^2 C + \epsilon D_T \nabla^2 T_f, \tag{6}$$

$$\frac{\partial}{\partial t} (\nabla \cdot \vec{q}) + \vec{q} \cdot \nabla (\nabla \cdot \vec{q}) = \eta \nabla^2 (\nabla \cdot \vec{q}). \tag{7}$$

The constants and variables in the above equations have their usual meanings and are given in the nomenclature. The relation between the reference density and temperature is given by

$$\rho_f = \rho = \rho_0 [1 - \beta_T (T_u - T_f) + \beta_C (C_u - C_f)]. \tag{8}$$

where  $\beta_T > 0$  and  $\beta_C > 0$  are the coefficient of thermal and concentration expansions, respectively. The boundary conditions are

$$\begin{aligned} T_f = T_S = T_l, C = C_l \text{ at } z = 0 \text{ and } T_f = T_S = T_u, \\ C = C_u \text{ at } z = d. \end{aligned} \tag{9}$$

Eqs. (1) – (7) may be non-dimensionalised using the transformation

$$\begin{aligned} (x, y, z) = d(x^*, y^*, z^*), (u, v, w) = \left( \frac{\epsilon \kappa_f z}{(\rho_f c_f) d} \right) (u^*, v^*, w^*) \\ p = \left( \frac{\kappa_f \mu}{(\rho_f c_f) K z} \right) p^*, T_f = \Delta T T_f^*, T_s = \Delta T T_s^* \end{aligned}$$

$$C = \Delta C C^*, ) - (=) - (b) - (*, t = \left( \frac{d^2(\rho_f c_f)}{\kappa_{fz}} \right) t^* \quad (10)$$

where '\*' refers to the non-dimensional quantity. Taking the curl twice of momentum equation and considering its z-component, after dropping the asterisk, we get

$$\left( \nabla_1^2 + \frac{1}{\xi} D^2 \right) w = -R \nabla_1^2 T_f + R_S \nabla_1^2 C + Q Pm \nabla^2 D h_z, \quad (11)$$

$$\frac{\partial T_f}{\partial t} + w \frac{\partial T_f}{\partial z} = \eta_f \nabla_1^2 T_f + \frac{\partial^2 T_f}{\partial z^2} + Du \frac{R_S}{R} (\nabla_1^2 + D^2) C + H(T_s - T_f), \quad (12)$$

$$\chi \frac{\partial T_s}{\partial t} = \eta_s \nabla_1^2 T_s + \frac{\partial^2 T_s}{\partial z^2} - \gamma H(T_s - T_f), \quad (13)$$

$$\frac{\partial C}{\partial t} + w \frac{\partial C}{\partial z} = \frac{1}{Le} (\nabla_1^2 + D^2) C + \frac{SrR}{R_S Le} (\nabla_1^2 + D^2) T_f, \quad (14)$$

$$\frac{\partial h_z}{\partial t} - \frac{\partial w}{\partial z} = Pm \nabla^2 h_z. \quad (15)$$

together with the boundary conditions

$$T_f = T_s = C = 0 \text{ at } z = 0 \text{ and } T_f = T_s = C = 1, \text{ at } z = 1. \quad (16)$$

where

$$R = \frac{\rho_f g \beta_T \Delta T K_z d}{\varepsilon \mu \kappa_{fz}}, R_S = \frac{\rho_f g \beta_C \Delta C K_z d}{\varepsilon \mu \kappa_{fz}}, Q = \frac{\mu_m ?? K_z}{\rho_0 \nu \Lambda}, Pm = \frac{\Lambda}{\kappa_{fz}}, \xi = \frac{K_x}{K_z}, H = \frac{h_s d^2}{\varepsilon \kappa_{fz}}, Du = \frac{D_f \beta_T}{\varepsilon \kappa_{fz} \beta_C}, Le = \frac{\kappa_{fz}}{D_m}, Sr = \frac{D_s \beta_C}{D_m \beta_T}, \gamma = \frac{\varepsilon \kappa_{fz}}{(1 - \varepsilon) \kappa_{fz}}, \chi = \frac{(\rho_s c_s) \kappa_{fz}}{(\rho_f c_f) \kappa_{sz}}, \eta_f = \frac{\kappa_{fx}}{\kappa_{fz}} = \frac{\kappa_{fy}}{\kappa_{fz}}, \eta_s = \frac{\kappa_{sx}}{\kappa_{sz}} = \frac{\kappa_{sy}}{\kappa_{sz}}. \quad (17)$$

are the Darcy-Rayleigh number, solutal Rayleigh number, Darcy-Chandrasekhar number, magnetic Prandtl number, non-dimensional inter-phase heat transfer coefficient, Dufour parameter, Lewis number, Soret parameter, porosity modified conductivity ratio and diffusivity ratio. Eqs. (11) – (16) allow a basic conduction state given by

$$\bar{q}_b = (0, 0, 0), T_{fb} = T_{sd} = C_b = z \quad (18)$$

Combining Eqs. (11) and (15), we get

$$\left[ \left( \nabla_1^2 + \frac{1}{\xi} D^2 \right) \left( \frac{\partial}{\partial t} - Pm \nabla^2 \right) - Q Pm \nabla^2 D^2 \right] w + R \left( \frac{\partial}{\partial t} - Pm \nabla^2 \right) \nabla_1^2 T_f - R \left( \frac{\partial}{\partial t} - Pm \nabla^2 \right) \nabla_1^2 C = 0. \quad (19)$$

### 3. LINEAR STABILITY ANALYSIS

We are concerned only with finding thresholds for the onset of convection, hence, it is sufficient to intigate the stability of the basic state by linear analysis. We perturbed the basic state by considering the infinitesimal perturbations. The perturbation variables are defined as:

$$[w, T_f, T_s, C] = [0, T_{fB}, T_{sB}, C_B] + [W, \Theta, \Phi, \Psi]. \quad (20)$$

Substituting expression (20) into the Eqs. (12) – (14) and (19), the linearised equations are

$$\left[ \left( \nabla_1^2 + \frac{1}{\xi} D^2 \right) \left( \frac{\partial}{\partial t} - Pm \nabla^2 \right) - Q Pm \nabla^2 D^2 \right] W + R \left( \frac{\partial}{\partial t} - Pm \nabla^2 \right) \nabla_1^2 \Theta - R_S \left( \frac{\partial}{\partial t} - Pm \nabla^2 \right) \nabla_1^2 \Psi = 0, \quad (21)$$

$$W + \left( \frac{\partial}{\partial t} + \eta_f \nabla_1^2 - D^2 + H \right) \Theta - H \Phi + Du \frac{R_S}{R} (\nabla_1^2 + D^2) \Psi = 0, \quad (22)$$

$$-\gamma H \Theta + \left( \chi \frac{\partial}{\partial t} - \eta_s \nabla_1^2 - D^2 + \gamma H \right) \Phi = 0, \quad (23)$$

and

$$W - \frac{SrR}{R_S Le} (\nabla_1^2 + D^2) \Theta + \left( \frac{\partial}{\partial t} - \frac{1}{Le} (\nabla_1^2 + D^2) \right) \Psi = 0 \quad (24)$$

The corresponding boundary conditions reduce to

$$W = 0 \text{ at } z = 0, 1 \text{ and } \Theta = \Psi = 0 \text{ at } z = 0, 1. \quad (25)$$

In this study we restrict our self to investigate only stationary stability, as discussed by Saravanan and Jegajothi (2010). To solve our system of Eqs. (21) – (25), we apply normal mode technique, thus express the perturbed variables as given below 5)

$$[W, \Theta, \Phi, \Psi] = [A_1, A_2, A_3, A_4] e^{i(lx + my)} \sin \pi z \quad (26)$$

where  $A_1, A_2, A_3, A_4$  are constants and l, m are wave numbers in x, y directions respectively. We Substitute Eq. (26) into Eqs. (21) – (24) and get the matrix equation  $XA = 0$  where

$$X = \begin{pmatrix} \left( a^2 + \frac{\pi^2}{\xi} \right) & a^2 R & 0 & -a^2 R_S \\ 1 & (\eta_f a^2 + \pi^2 + H) & -H & Du \frac{R_S(a^2 + \pi^2)}{R} \\ 0 & -\gamma H & (\eta_s a^2 + \pi^2 + \gamma H) & 0 \\ 1 & \frac{SrR(a^2 + \pi^2)}{R_S Le} & 0 & \frac{(a^2 + \pi^2)}{Le} \end{pmatrix} \quad (27)$$

For a non-trivial solution we have  $Det(X) = 0$ . Thus we get

$$R = \frac{1}{(1 + Sr)} [Du \{R' - \alpha_5\} \frac{\alpha Sr}{a^2} + R' \frac{\alpha_2}{a} + \frac{\alpha_2 \alpha_5}{a^2} + \frac{R' \alpha_3 H}{\alpha \alpha_4} + \frac{\alpha_3 \alpha_5 H}{a^2 \alpha_4}]. \quad (28)$$

where

$$R' = R_S Le, \alpha = (a^2 + \pi^2), \alpha_1 = a^2 + \frac{\pi^2}{\xi}, \alpha_2 = \eta_f a^2 + \pi^2, \alpha_3 = \eta_s a^2 + \pi^2, \alpha_4 = \eta_s a^2 + \pi^2 + \gamma H$$

The minimum value of  $R$  in Eq. (28) can be obtained by  $\frac{\partial R}{\partial a} = 0$ . However it is difficult to obtain it in closed form, therefore, we have used the multidimensional Newton-Raphson iteration scheme as used by Banu and Rees (2002).

When we take  $Q = 0$  and isotropic case, we get

$$R = \frac{1}{(1 + Sr)} [Du \{R' - \frac{(\pi^2 + a^2)^2}{a^2} Sr\} + \{R' + \frac{(\pi^2 + a^2)^2}{a^2}\} \{1 + \frac{H}{(\pi^2 + a^2 + \gamma H)}\}] \quad (29)$$

same as result of Saravanan and Jegajothi (2010).

#### 4. RESULTS AND DISCUSSIONS

In this section, we have attempted to built up a picture of how LTNE affects the onset of double-diffusive thermal convection in electrically conducting fluid saturated anisotropic porous medium under cross-diffusion effects. The porous medium is considered as mechanically and thermally anisotropic. We will emphasis only the effects of Darcy-Chandrasekhar number ( $Q$ ) (magnetic effect), non-dimensional inter-phase heat transfer coefficient ( $H$ ) (LTNE effect), DuFour ( $Du$ ) and Soret ( $Sr$ ) effects (cross-diffusion effects) out of several. A comparative study has also been performed on the basis of isotopic and anisotropic effects, cross-diffusion and without crossdiffusion (pure double diffusive) and for different values of  $Sr$  and  $Du$ .

Since  $H$  is a quantity that is not easily measurable, therefore, we take nearly the same set of values of the parameters as taken by Govender and Vadasz (2007). The values of the  $Q$  are taken to be very small

due to porous medium. Also due to the non-availability of accurate experimental values of  $Sr$  and  $Du$ , it is impossible to proceed to a practical quantitative study of cross-diffusion effects. The results obtained from Eq. (28) have been presented graphically in Figs. 2 – 15.

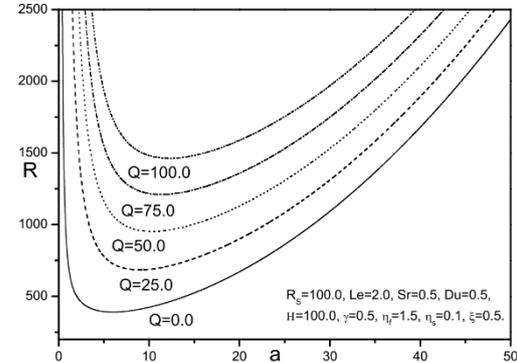


Fig. 2. Variation of  $R$  with  $a$ .

Initially we consider the neutral stability curves in Figs. 2 – 5 and depict the variation of the Rayleigh number  $R$  with respect to the wave number  $a$  for fixed values of the parameters  $Q = 25.0$ ,  $\eta_f = 1.5$ ,  $s = 0.1$ ,  $\xi = 0.5$ ,  $H = 100.0$ ,  $Du = 0.5$ ,  $Sr = 0.5$ ,  $R_S = 100.0$ ,  $Le = 2.0$  and  $\gamma = 0.5$ , while varying one of the parameters. From the figures we see that initially the value of  $R$  is large and decreases on increasing the wave number. It touches the minimum value at some value of  $a$  and then increases as the values of  $a$  increases. The minimum value of  $R$  is known as the critical value of Rayleigh number, denoted by  $R_c$ . The corresponding value of the wave number  $a$  is known as the critical value of the wave number and is denoted by  $a_c$ .

In Fig. 2, we find the effect of  $Q$  on thermal instability. From the figure it is observed that as the value of  $Q$  increases, the minimum value of  $R$  also increases, thus showing the stabilizing effect on the system. This can be explained for observing the behaviour of magnetic field lines. Increasing the value of  $Q$  implies that magnetic field strength permeating the medium is considerably strong. It induces viscosity into fluid, and the magnetic lines are distorted by convection. Then these magnetic lines hinder the growth of disturbances leading to the delay in onset of instability.

In Fig. 3, we investigate the effect of variation of dimensionless inter phase heat transfer coefficient ( $H$ ) on the onset of convection. We observed that as  $H$  increases the minimum value of  $R$  increases thus the system becomes more stable. To explain this: if

$$\text{we increases the value of } H = \frac{h_s d^2}{\epsilon \kappa_f}, \text{ only the value}$$

of inter phase heat transfer coefficient  $h_s$  increases as  $d$ ,  $\epsilon$  and  $\kappa_f$  are used in fixed parameters. That indicates heat transfer between solid and fluid is increases (maximum heat involve in transferring not

in convection), therefore delay the onset of convection.

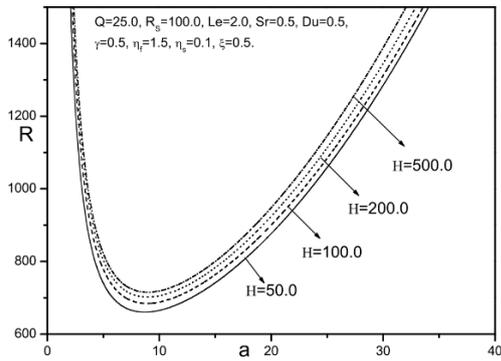


Fig. 3. Variation of R with a.

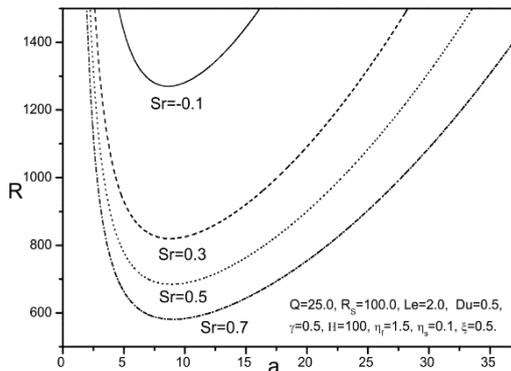


Fig. 4. Variation of R with a.

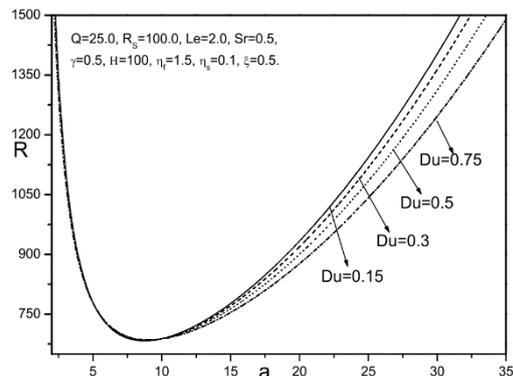


Fig. 5. Variation of R with a.

In Fig. 4, we show the effect of Soret parameter ( $Sr$ ). It is observed that on increasing the value of  $Sr$ , decreasing the minimum (critical) value of  $R$ , thus destabilizes the system. It is also remarkable that maximum stability obtained for negative value of  $Sr$ . It can be explain as: positive  $Sr$  means solute diffuse towards cooler plates, whereas negative  $Sr$  represents that solute diffuse towards hotter plate. Therefore negative  $Sr$  opposes the convection and delay the onset of convection.

In Fig. 5, we considered the effect of DuFour parameter  $Du$ . We find that critical value of Rayleigh number ( $R_c$ ) increases on increasing the values of  $Du$ , thus making the system stable.

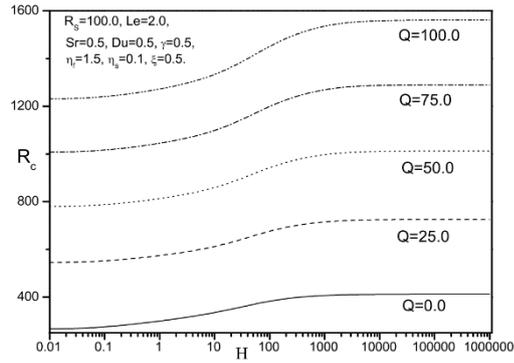


Fig. 6. Variation of  $R_c$  with  $H$ .

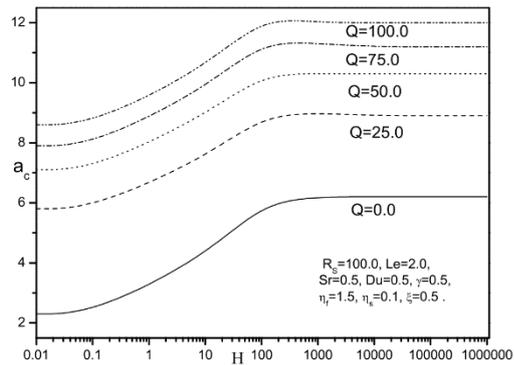


Fig. 7. Variation of  $a_c$  with  $H$ .

Now we consider the Figs. 6 – 11, and depict the variation of  $R_c$  and the corresponding  $a_c$  with respect to the inter-phase heat transfer coefficient  $H$  for the fixed values of the parameters  $Q = 25.0$ ,  $\eta_f = 1.5$ ,  $\eta_s = 0.1$ ,  $\xi = 0.5$ ,  $Du = 0.5$ ,  $Sr = 0.5$ ,  $R_s = 100.0$ ,  $Le = 2.0$  and  $\gamma = 0.5$ , while varying one of the parameters. From these figures we observe that initially when  $H$  is small the value of  $R_c$  is also small, increases for intermediate values of  $H$  and then becomes constant as the values of  $H$  increased further. Figs. 6 and 10, depict the effects of  $Q$  and  $Du$  respectively, on thermal instability. It is observed from the figures that on increasing  $Q$  and  $Du$ , the value of  $R_c$  increases, thus stabilizing the system. Further from Fig. 8 we find that on increasing  $Sr$  the value of  $R_c$  decreases. This shows that increase in the value of  $Sr$  makes the system unstable. From the Figs. 7, 9, and 11, we find that initially when  $H$  is small  $a_c$  is also small. At some particular value of  $H$  we see that  $a_c$  takes on a maximum value and then decreases and becomes constant at very large values of  $H$ . From Fig. 7, which depicts the effect of  $Q$  on magneto-convection, we find that value of  $a_c$  increases on increasing  $Q$ . However from Figs. 9 and 11 having same patterns, which show the effect of  $Sr$  and  $Du$  on thermal instability of the system, we observed that initially  $a_c$  increases on decreasing  $Sr$  and  $Du$ , attains its maximum values at some  $H$  and then decreases on increasing  $Sr$  and  $Du$  respectively. To understand the above phenomena, we can recall the physics of inter phase heat transfer coefficient. It states that for small values (i.e.  $H \rightarrow 0$ ) and large

values (i.e.  $H \rightarrow \infty$ ) of  $H$  system behaves like closed to local thermal equilibrium (CLTE) not to LTNE. A simple physics involve behind this, when  $H$  is small that indicates that heat transfer between solid and fluid is so small that it cannot influence the convection. Similarly when  $H$  is very large, heat transfer between solid and fluid is going to negligible and cannot influence the convection. So that we can see the effect of LTNE only for intermediate value of  $H$ .

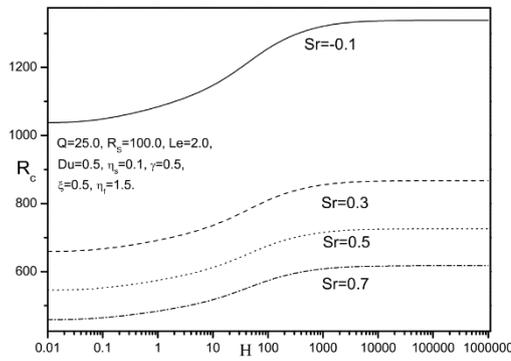


Fig. 8. Variation of  $R_c$  with  $H$ .

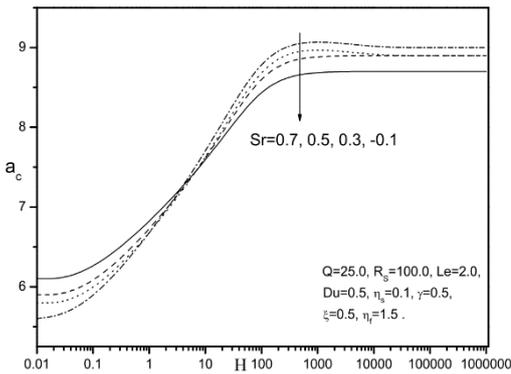


Fig. 9. Variation of  $a_c$  with  $H$ .

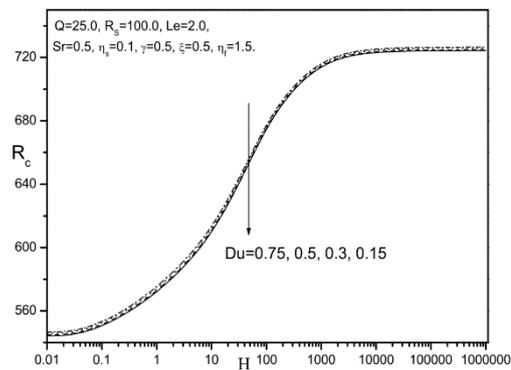


Fig. 10. Variation of  $R_c$  with  $a_c$ .

Now in upcoming figures, we are going to discuss a comparative study between different conditions as isotropic (mechanical as well as thermal) and anisotropic porous medium, cross diffusion and without cross diffusion and in last for different values of  $Du$  and  $Sr$ .

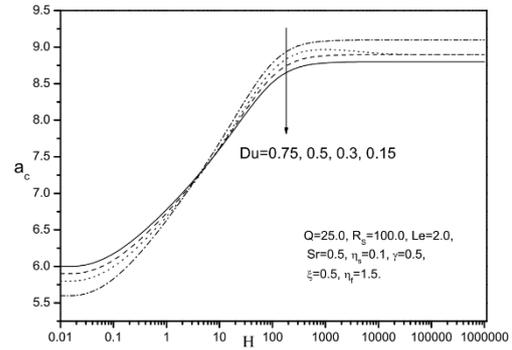


Fig. 11. Variation of  $a_c$  with  $H$ .

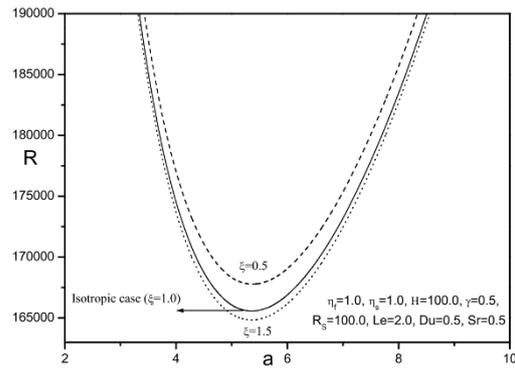


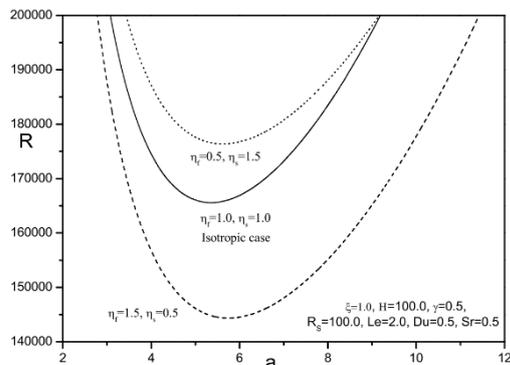
Fig. 12. Comparison between isotropic and anisotropic case in terms of  $\xi$ .

In Figs. 12 and 13, we compare the results of isotropic and anisotropic cases with the help of graphs. In Fig. 12, mechanical anisotropic and thermal isotropic cases are considered. From the figure, it is clear that as mechanical anisotropic parameter ( $\xi$ ) increases from 0.5 to 1.5, the value of critical Rayleigh number decreases and thus destabilize the system. We represent our study as isotropic case ( $\xi=1$ ) and anisotropic case ( $\xi < 1$ ) and ( $\xi > 1$ ). To understand the phenomena, we recall definition of  $\xi = \frac{k_x}{k_z}$ . As  $k_z$  is used in other fixed parameters, so  $k_z$

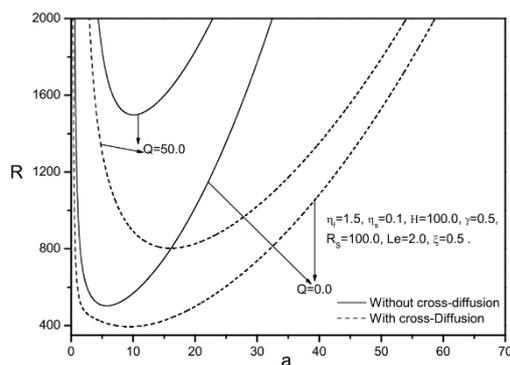
increasing  $\xi$  from 1 means increasing the value of  $k_x$  ( $k_x > k_z$ ) that is flow permeability is more in horizontal direction than vertical and similarly if the value of  $\xi$  decreases from 1, decreasing the value of  $k_x$  ( $k_x < k_z$ ). From figure, system is most stable at  $\xi = 0.5$ , then for  $\xi = 1.0$  and least stable at  $\xi = 1.5$ . When  $\xi < 1$ , horizontal permeability is smaller and slow down horizontal motion thus conduction solution is stabilized. Whereas when  $\xi > 1$ , horizontal permeability is larger and enhance horizontal motion thus conduction solution is destabilized.

In Fig. 13, we compare the stability criteria for mechanical isotropic and thermal anisotropic porous medium. The study based on the results for three conditions (i) when thermal conductivity for solid is greater than fluid i.e. ( $\eta_s > \eta_f$ ), (ii) isotropic case ( $\eta_s = \eta_f$ ) and (iii) when  $\eta_s < \eta_f$ . It is clear from figure that system is most stable for condition (i)

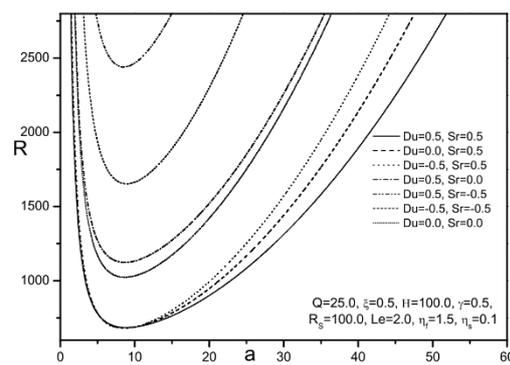
(maximum value of the  $R_c$ ) and least stable for condition (iii) (minimum value of the  $R_c$ ). This behaviour of system is explained as: if thermal conductivity of solid is more than fluid then most of the heat transfer in solid and delay onset of convection. Whereas when conductivity of fluid is more than solid, it enhance the onset of convection.



**Fig. 13. Comparison between isotropic and anisotropic case in terms of  $\eta_f$  and  $\eta_s$ .**



**Fig. 14. Comparison between with and without cross-diffusion.**



**Fig. 15. Comparison of  $R_c$  for different cases related with value of Du and Sr.**

In Fig. 14, we compare the results for two conditions (i) with cross-diffusion ( $Sr = 0.5, Du = 0.5$ ) and (ii) without cross-diffusion ( $Sr = 0, Du = 0$ ) that is purely double-diffusive convection. It is observed from figure that system is more stable for purely double-diffusion effect than with diffusion effect. The reason of this behaviour is explained as: we

considered the positive  $Sr$  and we mentioned above that has destabilizing effect. It is also well known fact that the effect of  $Du$  on onset of convection in porous media is negligible in comparison to  $Sr$ .

In Fig. 15, we considered different conditions related to the values of  $Sr$  and  $Du$ . We conclude from

figure that  $R_{c(Soret+, Dufor+)} < R_{c(Soret+)} < R_{c(Soret-, Dufor+)} < R_{c(nocross-diffusion)} < R_{c(Dufour+)} < R_{c(Soret-, Dufour+)} < R_{c(Soret-, Dufour-)}$ , thus for fixed value of other parameters, positive  $Sr$  and positive  $Du$  highly destabilize the system where as negative  $Sr$  and negative  $Du$  stabilize the system. These patterns of our results are similar to those obtained by Saravanan and Jegajothi (2010).

### 5. CONCLUSIONS

In the present article, linear stability analysis is performed to investigate the LTNE effect on finger type double-diffusive, magneto-convection in an anisotropic porous medium, heated and salted from above. The Darcy model is used for the momentum equation. We divided our study into two parts. In first part, we studied the effect of main controlling parameters in  $R - a$  and  $R_c / a_c - \log H$  planes. Whereas in second part, we illustrated a comparative study between different cases and conditions.

From the first part, following conclusions are drawn:

- The main controlling parameter  $Q$ , which represents the effect of magnetic field on the onset of convection, strengthening the stability of system.
- Inter-phase heat transfer coefficients ( $H$ ) and DuFour parameter ( $Du$ ) stabilize the system that is delay the onset of convection.
- Soret parameter ( $Sr$ ) destabilizes the system, i. e. enhance the onset of convection, and system is most stable for negative value of  $Sr$ .
- From  $R_c - \log H$  plane, we conclude that for  $H \rightarrow 0$  and  $H \rightarrow \infty$ , system behaves like CLTE than LTNE, whereas for intermediate values of  $H$ , the effect of LTNE is reasonable.

From the second part, the conclusions are:

- We compared the results of mechanical anisotropic and thermally isotropic case and find the stability criteria as, most stable for  $\xi < 1$  then for  $\xi = 1$  (isotropic case) and least stable for  $\xi > 1$ .
- We also compared the results of thermally anisotropic and mechanically isotropic case and conclude stability criteria as: most stable when  $\eta_s > \eta_f$  then  $\eta_s = \eta_f$  and least stable  $\eta_s < \eta_f$ .
- It is observed that system is more stable for pure double diffusive convection than with

cross-diffusion case.

- We presented the stability criteria for different conditions based on values of  $Sr$  and  $Du$ .

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