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Experimental Investigations on Oscillatory Couette Taylor Flow Wall Shear Stress Behavior using Electrochemical Technique: Low Modulation Effect

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ABSTRACT

In the simplest and original case of study of the Taylor–Couette TC problems, the fluid is contained between a fixed outer cylinder and a concentric inner cylinder which rotates at constant angular velocity. Much of the works done has been concerned on steady rotating cylinder(s) i.e. rotating cylinders with constant velocity and the various transitions that take place as the cylinder(s) velocity (ies) is (are) steadily increased. On this work, we concentrated our attention in the case in which the inner cylinder velocity is not constant, but oscillates harmonically (in time) clockwise and counter-clockwise while the outer cylinder is maintained fixed. Our aim is to attempt to answer the question if the modulation makes the flow more or less stable with respect to the vortices apparition than in the steady case and if there are any reversing or non reversing flows apparition. If the modulation amplitude is large enough to destabilize the circular Couette flow, two classes of axisymmetric Taylor vortex flow are possible: reversing Taylor Vortex Flow (RTVF) and Non-Reversing Taylor Vortex Flow (NRTVF). Our work presents an experimental investigation of the effect of oscillatory Couette-Taylor flow on the instantaneous and local mass transfer and wall shear rates evolutions, i.e. the impact of vortices at wall; and the detection of any RTVF and/or NRTVF apparition. The vortices may manifest themselves by the presence of time-oscillations of mass transfer and wall shear rates; this generally corresponds to an instability apparition even for steady rotating cylinder. On laminar CT flow, the time-evolution of wall shear rate is linear. It can be presented as a linear function of the angular velocity. For a mean Taylor number corresponding to a laminar Couette flow, a modulation frequency F = 0.1 Hz and an amplitude respectively $\beta = 0.53$ and$\beta = 1.08$ are sufficient to destabilize the laminar CT flow, Taylor vortices appear. Comparing to a steady rotational velocity case, oscillatory flow accelerates the instability apparition, i.e. the mean critical Taylor number corresponds to the transition is smaller than that of the steady rotational case. The vortices direction can be deduced from the sign of the instantaneous wall shear rate time evolution.

Key Words*:* Couette-Taylor(CT); Electrochemical technique; Experiments; Transition; Instability; Taylor number.

NOMENCLATURE

a	inner cylinder radius	Ω	angular velocity of the rotating plate
A	surface of the probe	υ	kinetic viscosity
C	concentration	δ	$\delta = R_2\text{-}R_1$
D	coefficient of diffusion	η	radial ratio
F	frequency of oscillation	Γ	aspect ratio

H	height of the CTS
I	current
l	probe length
Pe	Péclet number
R	radius
Re	Reynolds number
S	wall shear stress
Sh	sherwood number
t	time
Ta	Taylor number
u	velocity
x ,z	coordinates
Greek symbols	
β	amplitude of the oscillation

Superscripts

*	dimensionless
_	temporal average

Subscripts

a	inertia effect
ax	axial
c	critical
cent	centrifugal force
i	inner
q.l.	quasi-linear
Lev	lévêque approach
Sob	Sobolik *et al*. approach
Osc	oscillatory
Max	Maximum

1. INTRODUCTION

Couette-Taylor (CT) flows are frequently re-encountered on engineering processes (such on rotor-stator systems) and medical applications. It is widely used for the depleted uranium enrichment, as a membrane or filter...Most of the studies were interested to the Couette-Taylor simple case where the fluid is contained between a fixed outer cylinder and a concentric inner cylinder which rotates at constant angular velocity. We called this case "the steady rotating cylinder(s)"case such as the recent work of Abcha *et al*. (2013)who studied spiral vortex flow in counter-rotating Couette-Taylor system (SCT) with a large aspect ratio and an intermediate gap using the laser technique "Particle Image Velocimetry" (PIV) which is a global technique for velocity field determination. However, few studies focus on flow dynamic perturbation effects such as axial and/or radial flow imposed to rotating flow (Tilton *et al.*, 2010; Martinand *et al*., 2009) or heat transfer combined to flow dynamics (Kedia *et al.*, 1999; Gilchrist *et al*., 2005). Less works were interested to mass transfer combined to rotating flow dynamics and wall interaction on Couette Taylor system such as Berrich *et al.*, 2012 (a); 2012 (b), 2013 (a); Kristiawan *et al.*, (2011)... However, Kristiawan *et al.*, (2011) work has concerned the determination of wall shear rate from mass transfer for steady rotating cylinder. While our works focused on steady rotating flow combined to axial flow (Berrich *et al.*, 2012 (a) and (b)) and oscillating flows (Berrich *et al.*, 2013 (a)) effects on mass transfer and wall shear rate.

The occurrence of modulated flows in nature and in technological applications such as the stability of periodic blood flow in the aorta (Skalak *et al.*, 1989), the alternation of heating and cooling of planetary atmospheres, encourages researchers to study parametric excitation effect on hydrodynamic (Feugaing *et al.*, 2011). Donnelly *et al.*, (1962) enumerated some theoretical (Hall, 1975; Hall, 1983; Riley and Laurence, 1976), experimental (Barenghi and Jones, 1989; Barenghi, 1991; Ganske *et al.*, 1994) and numerical (Walsh and Donnelly, 1988) studies. These works illustrate that the oscillation motion destabilizes the flow but reveal a disagreement on the fact that a low frequency modulation produces a large or small destabilizing effect. However, few experimental studies have investigated the effect of modulation on mass transfer and wall shear rate. On this work, we concentrated our attention to the case in which the inner cylinder velocity is not constant, but oscillates harmonically (in time) clockwise and counter-clockwise while the outer cylinder is maintained fixed. Our aim is to attempt to answer the questions if the modulation makes the flow less stable with respect to the vortices apparition than in the steady rotating case and if there are any Reversing Taylor Vortex Flow (RTVF) or/and Non-Reversing Taylor Vortex Flow (NRTVF).

If the modulation amplitude is large enough to destabilise the circular Couette flow, two classes of axisymmetric Taylor vortex flow are possible: Reversing Taylor Vortex Flow (RTVF) and Non-Reversing Taylor Vortex Flow (NRTVF). For Reversing Flow, the inner cylinder is rotating anti-clockwise for the first part of the cycle; the vortices respond by rotating in the same cylinder motion direction. For the second part of the cycle, the inner cylinder rotates in the opposite direction, the vortices respond by changing their rotation direction. For Non-Reversing Flow, the inner cylinder and the vortices rotate in the same direction. However, when the inner cylinder reverses its rotation direction, the vortices don't follow the new cylinder direction i.e. they continue to rotate in the same direction as in the first half-cycle. Youd *et al*. (2003) studied numerically the two flow types. They demonstrated that for relatively high oscillation frequencies, the Taylor vortices direction does not change every half-cycle, i.e. presence of NRTVF. Youd *et al*. (2005) proved that, for Couette-Taylor system characterized by a radial ratio $\eta = R_{int}/R_{out} = 0.75$, there was a transition to an axisymmetric time-modulated flow, i.e. the Non-Reversing Taylor Vortex Flow (NRTVF) in which Taylor vortices rotation direction is independent on the inner cylinder rotation direction. They illustrated that if the oscillation amplitude is large enough, the resulting time-dependent Taylor vortex can rotate in a direction independent on the direction of the azimuthal motion which drives it. Taylor vortex pairs always rotate in the same direction, despite the inner cylinder drives the flow in the opposite direction. According to Youd *et al*., (2005) work, NRTVF takes place at sufficiently high modulation frequencies for which there is not enough time for toroïdal motion to vanish to small values. It can be caused by linear instability or finite-amplitude effects (Youd *et al.* 2003). It may be affected by the weak Ekman circulation (absent in the calculation of Youd *et al*. (2003) which is induced by the fixed top and bottom ends of the Taylor–Couette System. Carmi and Tustaniwskyj (1981) numerically studied oscillatory Couette flow. They did not detect the existence of NRTVF. Their approach was based on studying the effects of infinitesimal perturbations over a cycle, i.e. the Floquet theory. More details about the theory are available on (Youd *et al*., 2003; Carmi and Tustaniwskyj, 1981; Barenghi and Jones, 1989). It is possible that NRTVF is due to finite-amplitude effects, whereas in RTVF, the radial velocity becomes negative for a part of the cycle. However, NRTVF could be due to a linear instability too according to Lopez and Marques (2002) Floquet analysis.

Our work presents an experimental investigation of oscillatory Couette-Taylor flow behaviour, i.e. modulation effect on the apparition of RTVF and NRTVF by analyzing the instantaneous and local mass transfer and wall shear rate evolutions, i.e. the impact of vortices at wall.

2. EXPERIMENTAL INSTALLATION

The experimental installation (Fig. 1) is composed of an inner cylinder with a radius R_1=97±0.2 mm and an outer cylinder with a radius R_2=100±0.2 mm. The radius ratio is η=0.97. The aspect ratio is Γ=150. The whole apparatus is made in Plexiglas. The inner cylinder was driven by a α-Step motor (marketed by the Omeroncompany). The electrical motor imposed to the inner cylinder a maximum speed of 120 rpm. It was supplied by a signal generator. The revolutions were controlled by a speed controller. The engine assures to impose sinusoidal motion to the inner cylinder.

3. POLAROGRAPHY TECHNIQUE

Polarography (electrochemical) technique, known as the electro-diffusion method, has been used. This requires the use of Electro-Diffusion (ED) probe which delivers the Limiting Diffusion current while it is polarized by a well polarization voltage. The local mass transfer rate is determined from the current. Then, wall shear rate is determined from mass transfer signals. In addition, a series of platinum probes have been mounted flush to the inner surface of the outer cylinder of the Couette-Taylor System (CTS). ED results are presented for the probe G situated at the mid-height of the CTS (Fig. 1).

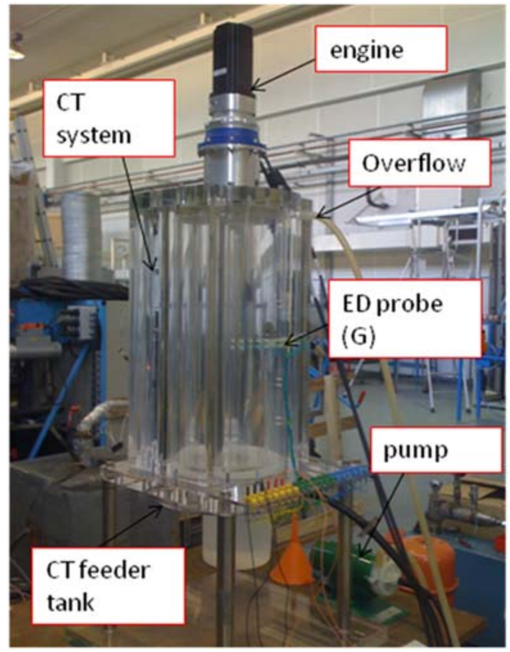

Fig. 1. Experimental installation.

In our tests, a ferri-ferrocyanure of potassium is used as electrochemical solution with a concentration of 25 mol/m^3. An access of the sulfate of potassium K_2SO_4 (130 mol/m^3) is added as supporting electrolyte and 40% of Glycerin to relay the instability apparition (viscosity effect).

4. STEADY TAYLOR VORTEX FLOW CHARACTERIZATION

To attempt to answer the question if the modulation makes the flow more or less stable with respect to the vortices apparition than in the steady rotational case, we begin by characterizing the flow for the case where the inner cylinder rotates at constant angular velocity i.e. steady rotational case.

The Taylor number is generally used to characterize modes transition. It describes the competition between the viscous dissipation and the centrifugal force. It is defined as:

$$Ta = \frac{\tau_\upsilon}{\tau_{cent}} = \frac{\Omega R_{int}\delta}{\upsilon}\sqrt{\frac{\delta}{R_{int}}} = Re_i\sqrt{\frac{\delta}{R_{int}}} \qquad (1)$$

where the Reynolds number describes the competition between the viscous dissipation and the inertia effect:

$$Re = \frac{\tau_\upsilon}{\tau_a} = \frac{\Omega R\delta}{\upsilon} \qquad (2)$$

The time-evolution of the Taylor number is presented on Fig. 2. It shows that the Taylor number is constant and that the average Taylor number is equal to 8±0.4. For this Taylor number, we studied the time-evolution of mass transfer and wall shear rates. This allows flow instability detection because the vortices manifest themselves by the apparition of variations on mass transfer and wall shear rates evolutions.

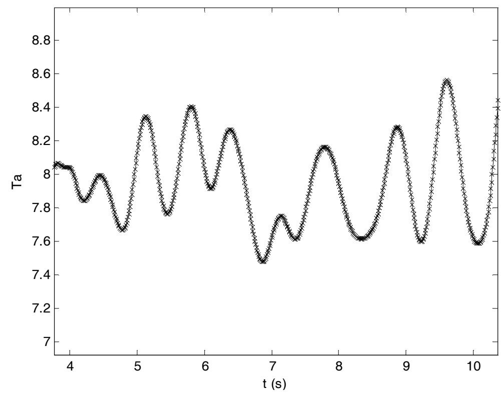

Fig. 2. Evolution of the Taylor number for steady rotational flow case.

The dimensionless mass transfer rate is almost constant (Fig. 3). The average value is equal to 1±0.02. The flow regime is thus laminar. It corresponds to Couette flow. The time-evolution of

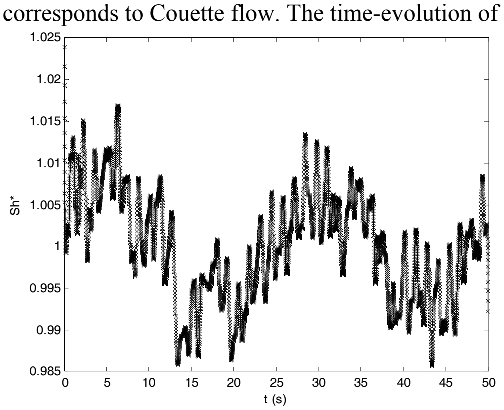

Fig. 3. Evolution of dimensionless mass-transfer rate for Ta_m=8.

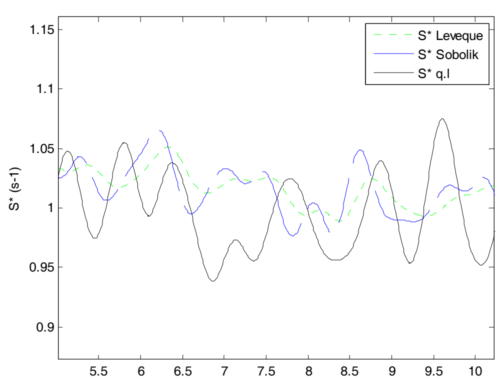

Fig. 4. Evolution of dimensionless wall shear stress for Ta_m=8.

dimensionless wall shear rate for Ta=8 is presented on Fig. 4. The dimensionless wall shear rate remains constant. The average value is equal to 1±0.05. The flow is thus stable and no vortices appear. On laminar CT flow, the time-evolution of wall shear rate is linear. It may be presented as a linear function of the angular velocity.

For the same average inner cylinder velocity, we studied the effect of oscillations on flow.

5. MODULATION EFFECT ON RTVF AND/OR NRTVF

5.1. Protocols and Taylor number evolutions

As the RTVF and the NRTVF strongly depend on the inner cylinder rotation direction, we studied two principle protocols:

- CASE A: The first one corresponds to non-stop advancing inner cylinder i.e. the inner cylinder always advances while oscillating.
- CASE B: The second protocol corresponds to advancing – stopped inner cylinder.

The time-evolutions of the Taylor number are shown respectively on Fig. 5 (case A) and Fig. 6 (case B).As we can see, for the different oscillatory cases, the Taylor number presents a sinusoidal evolution characterizing the oscillatory flow presence. The

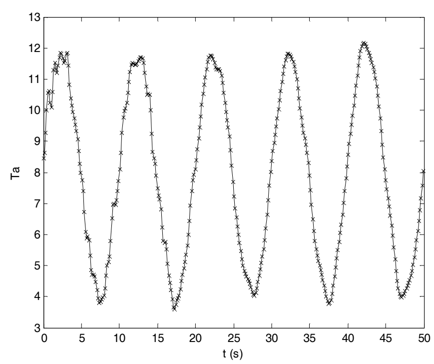

Fig. 5. Evolution of the Taylor number for F=100 mHz and β=0.53 (case A).

average Taylor number is equal to 8 which correspond to Taylor number for the steady case studied on the first section. On case A, the modulation amplitude β is strictly lesser than the unit and the Taylor number remains strictly positive. The Taylor number varies between 12 and 4. On case B, β is equals to the unit. The Taylor number time-evolution presents a zero Taylor number when the inner cylinder is stopped and strictly positive values when it advances. The cylinder oscillates between Ta=0 and Ta=16.

The average Taylor number, for all cases; is equals to 8 which correspond to Taylor number for the steady laminar Couette flow case studied on the first section.

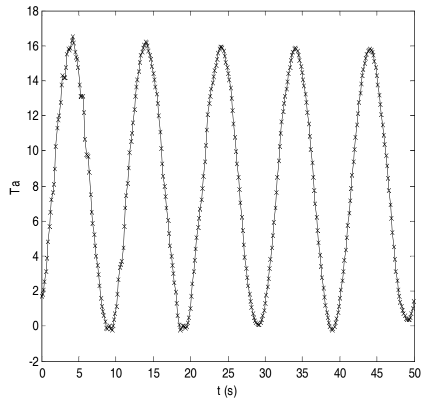

Fig. 6. Evolution of the Taylor number for Ta_m=8, F=100 mHz and β=1.08 (case B).

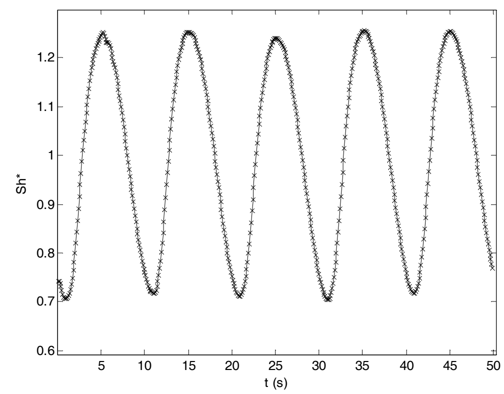

Fig. 8. Evolution of dimensionless experimental mass transferrate for Tam=8, F=100 mHz and β=1.08.

As we have proved that for Taylor number equal to 8, the flow corresponds to laminar Couette Flow. The question now is to know if the oscillation disturbs the flow enough by accelerating the transition apparition and if there is any RTVF or NRTVF. We propose an answer by analyzing the mass transfer and the wall shear rates.

5.2. Mass transfer rates analysis

Mass transfer rates time-evolution i.e. the Sherwood number for the different oscillatory flow are presented respectively on Fig. 7 (case A), Fig. 8 (case B).

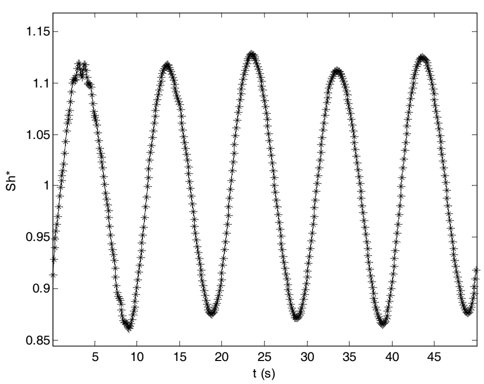

Fig. 7. Evolution of dimensionless experimental mass transfer rate for Ta_m= 8, F=100 mHz and β=0.53.

The Sherwood number is defined by:

$$Sh = \frac{i\,l}{n\,F\,C_0 A D} \tag{3}$$

Where i is the Limiting Diffusion current delivered by the Electro-Diffusion (ED) probe, l the probe length, n the ions number, F the Faraday number, C_0 the initial concentration, A the probe area and D the coefficient of Diffusion.

The instantaneous and local mass transfer rates evolutions demonstrate the apparition and development of Taylor vortices which manifest themselves by the presence of oscillations having constant amplitudes for β = 0.53 (case A) and β = 1.08; despite that the mean Taylor number corresponds to a steady laminar case i.e. Couette flow. So, we can deduce that oscillations accelerate the transition apparition and thus the critical mean Taylor number corresponding to the transition from laminar Couette flow to Wavy Vortex flow is lesser than that of the steady case. For β = 0.53 and β = 1.08, the mass transfer rate is no longer linear, it has a sinusoidal evolution. It follows the cylinder motion. It remains always positive.

5.3. Wall shear rates analysis

Basing on mass transfer evolutions, we have found that, if the oscillation amplitude is 0.53 ≤ β ≤ 1.08, laminar Couette flow can be destabilized, Taylor vortices appear.

The wall shear rate was determined using three approaches: the "Lévêque solution" (1928), Sobolik *et al.* (1987) method which is a correction of the Lévêque (1928) solution and the inverse method (2006). The different approaches are well developed on our previous work (Berrich *et al.*, 2013 b). The inverse method was experimentally validated in our previous work (Berrich *et al.*, 2013 c). It gives satisfactory results even for reversing flows.

The experimental mass transfer signals were smoothed (filtered) before using it to determine the wall shear rate because the different approaches are very sensitive to noise.

The temporal evolutions of wall shear rates, for relatively low modulation frequency F = 100 mHz, are presented respectively for modulation amplitude β = 0.53 and β = 1.08 on Fig. 9 and Fig. 10.The vortices manifest themselves by the presence of time-oscillations on wall shear rate evolution. For low modulation frequency (F = 0.1 Hz), and for

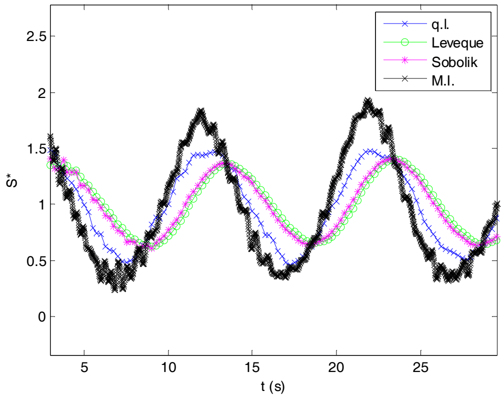

Fig. 9. Evolution of dimensionless wall shear rate for Ta$_m$=8, F=100 mHz and β=0.53.

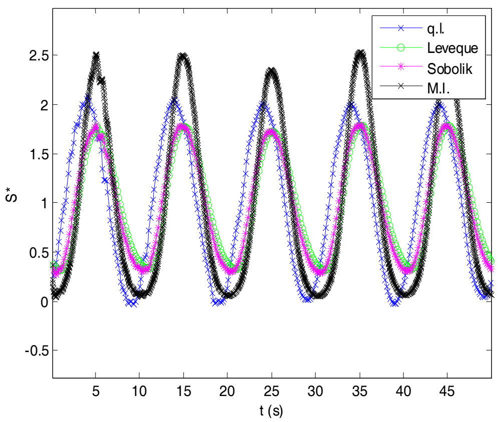

Fig. 10. Evolution of dimensionless wall shear rate for Ta$_m$=8, F=100 mHz and β=1.08.

modulation amplitudes β = 0.53 (i.e. nonstop advancing CT inner cylinder) and β = 1.08 (i.e. advancing - stop advancing CT inner cylinder), wall shear rates still positive. We proved that if the oscillation amplitude is "small" enough, the resulting Taylor vortex rotates in a direction which do not depend on the direction of the azimuthal motion which drives it i.e. NRTVF.

6. DETECTION AND CHARACTERIZATION OF OSCILLATORY FLOWS USING FREQUENCY SPECTRUM

The periodic signals usually used, whether low or high frequency, are rarely purely sinusoidal. They are in fact a mixture of sinusoidal signals whose frequencies are multiples of the fundamental frequency which is the lowest frequency. The multiple frequencies of the fundamental frequency are called harmonic. In general, any periodic function of frequency f_0 i.e. the fundamental frequency, can be decomposed into a sum of sinusoidal waves (harmonics) whose frequencies are integer multiples of f_0 (2 f_0 3 f_0 ... Nf_0). Its amplitudes and the respective

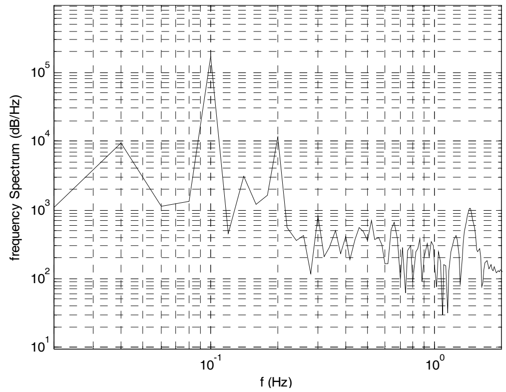

Fig. 11. Frequency spectrum Ta$_m$ = 8, F = 0.1 Hz, β = 0.53 (case A).

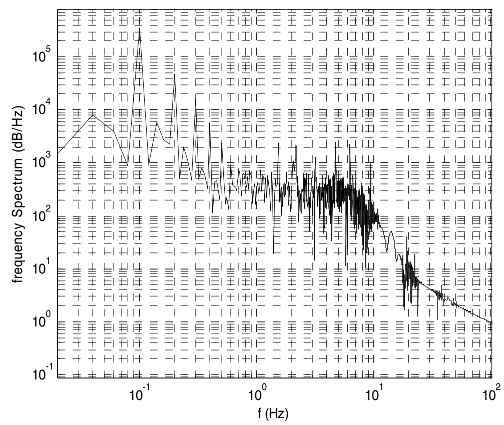

Fig. 12. Frequency spectrum Ta$_m$ = 8, F = 0.1 Hz, β = 1.08 (case B).

phases are the resulting of Fourier series decomposition.

Swinney and Zhang (2013) studied Taylor Vortex Flow dynamics in the case of rotating cylinders (steady rotational case). They defined the Reynolds number as $R=(2\pi.f_{cyl}.a).d/\nu$; where f_{cyl} is the cylinder frequency, a is the inner cylinder radius, b is the outer cylinder radius, $d=b-a$, and ν is the kinematic viscosity. They found that the transition from laminar flow to Taylor vortex flow, is an example of a "pitchfork" bifurcation, that is, a transition from one time-independent state to another with different symmetry. Modulated wavy vortex flow is characterized by two disproportionate characteristic frequencies. Wavy vortex state is characterized by spectra with a single fundamental frequency f_1 and its harmonics. In the wavy vortex state, an increase in R will lead to a transition where characterized by a frequency f_2. Their visual observations revealed that the Taylor vortex is modulated. The frequency f_2 corresponds to an azimuthal traveling wave similar to the wavy vortices; the angular speed of the modulated waves is about 0.44f_{cyl}. Proportionate and in proportionate frequencies in MWVF can never be distinguished experimentally. A bifurcation from a time independent system to a periodic state is called a Hopf bifurcation.

Coughlin and Marcus (1992) studied the Modulated waves in TC flow. They resumed that in the experiments of Zhang and Swinney (1985) who used a Taylor-Couette system with radius ratio 0.883, the flow state was determined by flow visualization, and spectra were taken from the time series of scattered light intensity. With this technique, there is no precise quantitative relationship between the amplitude of a spectral peak and the power of the corresponding mode (1985). Zhang and Swinney (1985) observed a quasi-periodic flow in which the spectral peak associated with the modulation, was frame independent and thus not associated with an azimuthally travelling mode.

We imposed a sinusoidal motion characterized by a fixed frequency and amplitude while conserving a mean Taylor number equals to Ta = 8 in order to compare the characteristics of the flow obtained to the steady rotational one, characterized also by Ta = 8, in term of vortices appearance and the flow regime obtained. What are the characteristic frequencies of these vortices?

The Figs 11 and 12 present a logarithmic presentation of the absolute values of Fourier Transform i.e. the frequency spectrum of mass transfer signals function of frequencies, for an imposed frequency oscillatory motion $f_{cyl} = F = 0.1$ *Hz*.

For steady rotational flow characterized by Ta=8, the flow is laminar i.e. Couette flow is characterized by vortices absence. Thus, the spectral peaks detected while imposing an oscillatory motion are the vortices imprints i.e. the frequencies correspond to those of vortices which appear due to the modulation. There are commensurate (proportionate) and incommensurate frequencies (Table .1).

Table 1 Frequency spectrum

β	0.53	1.08
F_{cyl}	0.1	0.1
Harmonics (Hz)	0.2 0.3	0.2 0.3 0.4
	1.5	1.7 2.2

7. CONCLUSIONS

This paper presents an experimental investigation on oscillatory Couette-Taylor flows. The vortices may manifest themselves by the presence of time-oscillations on mass transfer. The instantaneous and local mass transfer rates evolutions demonstrate the apparition and development of Taylor vortices which manifest themselves by the presence of oscillations having constant amplitudes for β = 0.53 and β = 1.08. The vortices direction can be deduced from the sign of the instantaneous wall shear rate time evolution. The wall shear rate was determined using three approaches: the “Lévêque solution” (1923), Sobolik *et al*. (1987) and the inverse method (2006). The results illustrate that the modulation can destabilize laminar Couette flow even for low oscillation frequencies and relatively low oscillation amplitudes. Oscillatory flows accelerate the flow transition. The Taylor number of the steady case to obtain the same mean shear rate is greater than the mean Taylor number relative to the oscillatory case. The frequencies spectra allows to characterize the increasingly complex dynamics that emerges with the increasing of β.

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