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Onset of Darcy–Benard Penetrative Convection in Porous Media

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ABSTRACT

The onset of Darcy–Benard penetrative convection in a liquid saturated porous layer of high permeability of practical importance is investigated by employing the Brinkman–Forchheimer– Lapwood extended Darcy flow model with fluid viscosity different from effective viscosity. The lower boundary is taken to be rigid and isothermal and the upper surface is free and subject to the general thermal condition. The critical eigen values are obtained numerically, in general, using Galerkin method. The stability of the system is found to be dependent on the dimensionless internal heat source strength Ns , permeability parameter σ_e and the ratio of effective viscosity to fluid viscosity Λ. It is observed that the increase in the value of permeability parameter is to delay while increase in the value of internal heat source strength is to hasten the onset of convection in a fluid saturated porous layer.

Keywords: Penetrative convection; Volumetric heat source; Darcy–Benard convection.

NOMENCLATURE

A	ratio of heat capacity
D	differential operator
d_m	thickness of the porous layer
k_e	effective thermal conductivity of the porous medium
l, m	wave number in x and y
p	pressure
R	Rayleigh number
R_e	effective Rayleigh number
R_m	Rayleigh number in a porous medium
T	temperature
$\vec{V}$	velocity vector (u, v, w)
W	amplitude of perturbed vertical velocity
σ	permeability parameter
σ_e	effective permeability parameter
ρ	density
$(\rho_0 c)_e$	effective heat capacity of the porous medium
$(\rho_0 c_p)_l$	heat capacity of the liquid
Λ	ratio of viscosities

1. INTRODUCTION

The convective instability in a horizontal fluid saturated porous layer heated from below is referred to as Darcy–Benard (DB) convection in which the instability is due to buoyancy forces. The DB convection has been studied extensively since the pioneering works of Horton and Rogers (1945) and Lapwood (1948) owing to its natural occurrence and also its importance in many scientific, engineering and technological applications. The copious literature covering different developments in this field are well documented in (Char and Chiang 1994; Kaviany 1995; Vafai (2000, 2005); Nield and Bejan 2006; Barletta A and Rees, D.A.S. (2012) ; Barletta A (2013); Barletta A (2014)).

There are situations of great practical importance where the porous material offers its own source of heat. This gives a different way in which a convective flow can be set up through the local heat generation within the porous media. Such a situation can occur through radioactive decay or through, in the present perspective, a relatively weak exothermic reaction which can take place within the porous material. To be more specific, internal heat is the main source of energy for celestial bodies caused by nuclear fusion and decaying of radioactive materials, which keeps the celestial objects warm and active. It is due to the internal heating of the earth that there exists a thermal gradient between the interior and exterior of the earth's crust, saturated by multi components fluids, which helps convective flow, thereby transferring the thermal energy toward the surface of the earth. Therefore, the role of internal heat generation becomes very important in several applications that include geophysics, reactor safety analyses, metal waste form development for spent nuclear fuel, fire and combustion studies, and storage of radioactive materials. Penetrative convection in porous media has received great attention during the past few decades because of the importance of this process which occurs in many engineering and natural systems of practical interest such as geothermal energy utilization, thermal energy storage and recovery systems, petroleum reservoirs, industrial and agricultural water distribution, to name just a few applications. Very recent reviews by Nield and Bejan (2006); Carr (2004); Carr and Putter (2003); Hill (2004); Straughan (2008); Tse and Chasnov (1998); Straughan and Walker (1996); and Zhang and Schubert (2002).

For a high porosity porous medium, Givler and Altobelli (1994) have demonstrated experimentally that the effective viscosity is about 7.5 times the fluid viscosity. Therefore, the aim of the present study is essentially to investigate the linear stability analysis of DB convection in a sparsely packed porous medium with internal heat generation by employing the Brinkman–Forchheimer– Lapwood-extended–Darcy flow model with effective viscosity different from fluid viscosity. In the present work, the ratio of these two viscosities is taken as a separate parameter to know its influence on the critical stability parameters. Also, the values of the permeability parameter are suitably chosen in the range $\sqrt{0.1} \le \sigma \le \sqrt{10^3}$, where σ is the permeability parameter, as suggested by Walker and Homsy (1977). There exist several works on coupled Benard– Marangoni convection in a clear fluid layer (see C. Perez-Garcia and G. Carneiro(1991); Bragard and Velarde(1999); Orand Kelly(2002) ; Char and Chiang (1994); C. E. Nanjundappa *et al.* (2011) and references therein). Shivakumara *et al.* (2009) have investigated the criterion for the onset of coupled Darcy–Benard–Marangoni convection in a liquid saturated porous layer of high permeability. It is shown that increase in σ and Bi, and decrease in ratio of viscosities is to decrease the dimensions of the convective cell and thus the buoyancy force has a destabilizing effect on the system.

The aim of this paper is, therefore, to study penetrative convection via internal heating in a fluid saturated porous medium with fluid viscosity different from effective viscosity. This is achieved by performing the linear stability analysis. The lower boundary is taken to be rigid and isothermal and the upper surface is free and subject to the general thermal condition. The eigen value problem is solved numerically, in general, using Galerkin method. A wide-ranging parametric study is under taken to explore their impact on the stability characteristics of the system.

2. MATHEMATICAL FORMULATION

We consider a Boussinesquian liquid saturated horizontal sparsely packed porous layer of thickness d with no lateral boundaries (see Fig. 1). The lower boundary is assumed to be rigid, while the upper free surface which is in contact with air and subjected to temperature-dependent surface tension forces is assumed to be flat and undeformable. A temperature difference of T is maintained between the boundaries of the porous layer with the lower boundary at a higher temperature than the upper boundary. A Cartesian coordinate system (x, y, z) is chosen such that the origin is at the lower boundary and the z axis is taken vertically upward. The gravity acts in the negative z direction.

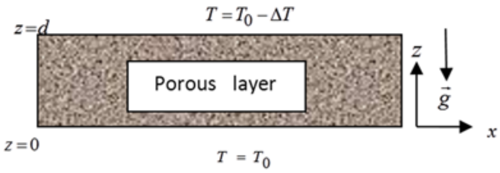

Fig. 1. Physical configuration.

The fluid density ρ is assumed to vary linearly with temperature in the form

$$\rho = \rho_0\left[1-\alpha\left(T - T_0\right)\right] \tag{1}$$

where α is the positive coefficient of the thermal liquid expansion and ρ_0 is the value at the reference temperature T_0.

Thus, the governing equations for the porous layer in the Boussinesq approximation are:

$$\nabla\cdot\vec{V} = 0 \tag{2}$$

$$\frac{\rho_0}{\phi}\left(\frac{\partial\vec{V}}{\partial t}+\frac{1}{\phi}\left(\vec{V}.\nabla\right)\vec{V}+\frac{C_b}{\sqrt{\kappa}}\vec{V}\left|\vec{V}\right|\right)= -\nabla p-\frac{\mu}{K}\vec{V}+\rho\vec{g}+\mu_e\nabla^2\vec{V} \tag{3}$$

$$\left(\rho_0 c\right)_e\frac{\partial T}{\partial t}+\left(\rho_0 c_p\right)_l\left(\vec{V}\cdot\nabla\right)T = k_e\nabla^2 T+q. \tag{4}$$

The temperature and pressure distributions in the basic state is given by

$$T_b\left(z\right)=T_0-\left[\left(\frac{\Delta T}{d}-\frac{q\,d}{2\kappa}\right)z+\frac{q}{2\kappa}z^2\right] \tag{5}$$

$$P_b\left(z\right)=P_0-\rho_0 gz-\alpha\rho_0 g\,\frac{\Delta T}{d}z^2 \tag{6}$$

where the subscript b denotes the basic state. The pressure distribution is of no consequence here as we are going to eliminate the same. To study the stability of DB convection, we superimpose infinitesimal disturbances on the basic state solution and substitute into the governing Eqs. (2)–(4). Employing the well known standard linear stability analysis procedure and eliminating the pressure from the momentum equation by operating curl twice and retaining the z-component, we arrive at the following dimensionless equations

$$\left(\frac{1}{\Pr}\frac{\partial}{\partial t}+\sigma^2-\Lambda\nabla^2\right)\nabla^2 w = R_e\ \nabla_h^2 T \tag{7}$$

$$\left[A\frac{\partial}{\partial t}-\nabla^2\right]T = w\left[1-Ns(1-2z)\right] \tag{8}$$

where $R=\alpha g_0 \Delta T d^3/\nu\kappa$ is the Rayleigh number, $Ns = qd^2/2\kappa\Delta T$ is the dimensionless heat source strength, $\sigma_e=\sigma/\sqrt{\Lambda}$ is the effective porous parameter and $R_e = R/\Lambda$ is the effective Rayleigh number.

The boundary conditions at the bottom are for a rigid boundary insulated to temperature perturbations:

$$w = \frac{\partial w}{\partial z} = T = 0 \quad at \quad z=0 \tag{9}$$

$$w = \frac{\partial T}{\partial z} = \frac{\partial^2 w}{\partial z^2} = 0 \quad at \quad z=1. \tag{10}$$

The principle of exchange instabilities holds good even for the present configuration as well, hence the time derivatives will be dropped conveniently from Eqs. (7) and (8).Then performing a normal mode expansion and seek solutions for the dependent variables as

$$(w,T) = \left[W(z),\Theta(z)\right]\exp\left[i\left(lx+my\right)\right] \tag{11}$$

and substituting them in Eqs. (7) and (8) (with $\partial/\partial t$ = 0), we obtain the following ordinary differential equations

$$\left[\left(D^2-a^2\right)^2-\sigma_e^{\,2}\left(D^2-a^2\right)\right]W = -R\,a^2\Theta \tag{12}$$

$$\left(D^2-a^2\right)\Theta = f(z)W \tag{13}$$

Where W is the amplitude of perturbed vertical velocity and Θ is the amplitude of perturbed temperature. In the above equations, $D=d/dz$, $a=\sqrt{l^2+m^2}$ is the overall horizontal wave numbers and

$$f(z) = \left[Ns(1-2z)-1\right]. \tag{14}$$

The boundary conditions take the form

$$W = DW = \Theta = 0 \quad at \quad z=0 \tag{15}$$

$$W = D\Theta = D^2 W = 0 \quad at \quad z=1. \tag{16}$$

3. NUMERICAL SOLUTIONS

Eqs. (12) and (13) together with the boundary conditions given by Eqs. (15) and (16) constitute an eigen value problem with R as the eigen value. The Galerkin method is employed to solve the eigen value problem as explained in the book by Finlayson (1972). Accordingly, the unknown variables are written in a series of basis functions as

$$W = \sum_{i=1}^{n} A_i W_i\,, \qquad \Theta = \sum_{i=1}^{n} B_i\,\Theta_i \tag{17}$$

where A_i and B_i are constants and the basis functions W_i and Θ_i will be represented by the power series satisfying the boundary conditions. Substituting Eq. (17) into Eqs. (12) and (13) and the Galerkin procedure of demanding the residues be orthogonal to the basis functions are applied, we get the following system of homogeneous algebraic equations.

$$C_{ji}A_i + D_{ji}\,B_i = 0 \tag{18}$$

$$E_{ji}A_i + F_{ji}\,B_i = 0 \tag{19}$$

The coefficients C_{ji} to F_{ji} involve inner products of the basis functions and are given by

$$C_{ji} = \langle D^2 W_j D^2 W_i + (2a^2+\sigma_e^2)DW_j DW_i + a^2(a^2+\sigma_e^2)W_j W_i\rangle \tag{20a}$$

$$D_{ji} = -R\,a^2\langle W_j\Theta_i\rangle \tag{20b}$$

$$E_{ji} = f(z)\langle W_i\Theta_j\rangle \tag{20c}$$

$$F_{ji} = \langle D\Theta_j D\Theta_i + a^2\Theta_j\Theta_i\rangle \tag{20d}$$

where the inner product is defined as

$$\langle f\ g\rangle = \int_0^1 f\ g\,dz \tag{21}$$

The system of homogeneous equations given by Eq. (18) will have a nontrivial solution if and only if

$$\begin{vmatrix} C_{ji} & D_{ji} \\ E_{ji} & F_{ji} \end{vmatrix} = 0 \tag{22}$$

The base functions W_i and Θ_i are generally chosen such that they satisfy the corresponding boundary conditions but not the differential equations. We select the trial functions as

$$W_i = \frac{3}{2}z^{i+1}-\frac{5}{2}z^{i+2}+z^{i+3} \tag{23}$$

$$\Theta_i = z^i - \frac{1}{2}z^{i+1} \tag{24}$$

The inner products are evaluated analytically to avoid errors in the numerical integration. The minimum point of R_e as a function of wave number a gives the critical effective Rayleigh number R_{ec} and the corresponding critical wave number ac. This procedure is repeated for different values of Λ, Ns and σ^2 .and the results are discussed in Section 4.

Table 1 Comparison of critical Rayleigh number R_{ec} and critical wave number a_c for different values of Ns and $\sigma_e = 0$ (i.e., in the absence of a porous medium)

Ns	Char and Chiang (1994)		Present analysis	
	R_{ec}	a_c	R_{ec}	a_c
0	669.013	2.086	668.998	2.086
0.5	608.758	2.070	608.746	2.070
1. 5	557.618	2.060	557.607	2.060
5	328.590	2.035	328.582	2.035
10	215.415	2.033	215.409	2.035
15	159.957	2.034	159.952	2.034
20	127.144	2.036	127.14	2.036
30	90.116	2.038	90.114	2.038
40	69.778	2.039	69.776	2.039
70	41.598	2.042	41.597	2.042
100	29.629	2.043	29.628	2.043

4. RESULTS AND DISCUSSION

The linear stability analysis has been carried out to investigate the effect of internal heat generation on the onset of the convection in a horizontal saturated porous layer. The lower boundary is rigid-isothermal, while the upper boundary is free with general thermal convection boundary condition. The critical Rayleigh number R_{ec} and the corresponding critical wave number a_c are obtained numerically using the Galerkin technique for various values of physical parameters Ns, σ_e^2 and Λ . The results presented here are for $i = j = 6$; the order at which the convergence is achieved, in general.

To validate the numerical procedure used in the present study, the critical Rayleigh number R_{ec} and the corresponding critical wave number a_c obtained for different values of Ns and $\sigma_e = 0$ (i.e., in the absence of a porous medium) are compared with those of Char and Chiang (1994) in Table 1. We note that the agreement is good and thus verify the accuracy of the numerical procedure employed.

The presence of internal heating makes the basic temperature distribution to deviate from linear to nonlinear which in turn have significant influence on the stability of the system. To assess the impact of internal heat source strength Ns on the criterion for the onset of convection, the distribution of dimensionless basic temperature $\hat{T}_b(z)$, is exhibited graphically in Fig. 2 for different values of Ns . From the figure it is observed that increase in the internal heat source strength amounts to large deviations in the distribution which in turn enhance the disturbances in the porous layer and thus reinforce instability on the system.

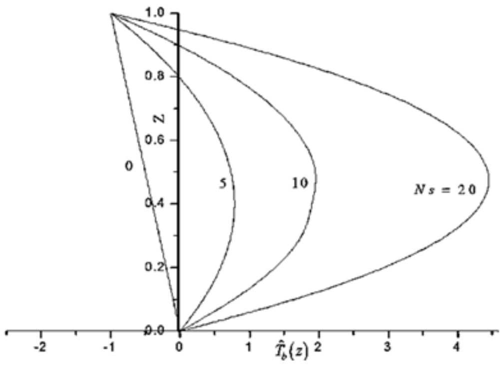

Fig. 2. Basic state temperature distributions for different values of Ns .

Figures 3 and 4 respectively show the variation of critical Rayleigh number R_c and the corresponding wave number a_c as a function of dimensionless internal heat source strength Ns for different values effective permeability parameter $\sigma_e{}^2$. Fig. 3 clearly indicates that R_{ec} decreases monotonically with Ns indicating the influence of increasing internal heating is to decrease the value of R_{ec} and thus destabilize the system. This is because; increasing Ns amounts to increase in energy supply to the system. Eventually, this leads to large deviations in the basic state temperature distribution (see Fig. 2) of the parabolic type which in turn enhances the thermal

Table 2 Comparison of critical Rayleigh number R_{ec} and critical wave number a_c for different values of Λ and σ^2

Ns	σ^2	$\Lambda=1$		$\Lambda=3$		$\Lambda=5$	
		R_{ec}	a_c	R_{ec}	a_c	R_{ec}	a_c
	10	964.362	2.154	768.145	2.113	729.235	2.102
0	50	2119.232	2.264	1158.123	2.188	964.568	2.154
	100	3541.012	2.303	1640.984	2.236	1255.345	2.200
	10	806.694	2.129	641.123	2.085	608.908	2.074
1	50	1782.723	2.232	970.234	2.159	806.290	2.129
	100	2988.241	2.275	1377.123	2.207	1052.765	2.171
	10	600.900	2.111	508.457	2.112	453.237	2.055
3	50	1328..323	2.229	723.237	2.143	601.439	2.111
	100	2226.071	2.280	1026.432	2.196	784.568	2.156
	10	476.118	2.107	378.798	2.064	359.123	2.049
5	50	1049.324	2.240	572.891	2.143	476.453	2.107
	100	1754.982	2.302	812.786	2.203	620.657	2.157

disturbances in the saturated porous layer to hasten the onset of DB convection. Besides, increase in the value of effective permeability parameter σ_e^2 is to increase the critical Rayleigh number and thus its effect is to delay the onset of DB convection. The numerically calculated critical effective Rayleigh number R_{ec} and the corresponding wave number a_c are shown in Fig. 4 as a function of σ_e for different values of Ns. Increase in σ_e^2 is to increase ac and thus its effect is to decrease the dimensions of the convective cell and increasing internal heating is to destabilize the system.

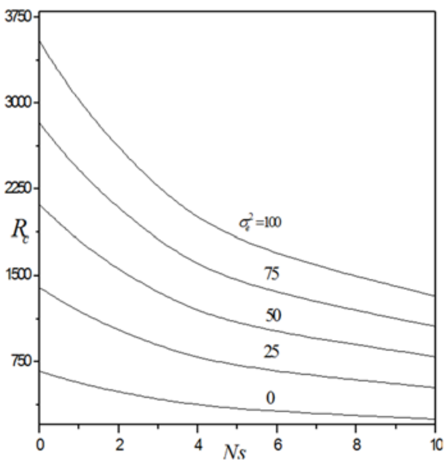

Fig. 3. Plots of effective critical Rayleigh number R_{ec} versus internal heat source strength Ns for different values of effective permeability parameter σ_e^2.

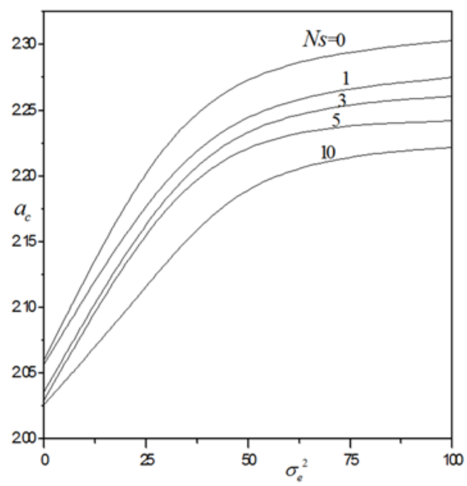

Fig. 4. Plots of effective critical wave number a_c versus effective permeability parameter σ_e^2 for different values of internal heat source strength Ns.

The critical Rayleigh number and wave numbers obtained numerically for different values of σ^2 and Ns are presented in Table 3. From the table we note that an increase in the value of Λ is to decrease the critical Rayleigh number and thus making the system unstable. Nevertheless, increase in Λ is to decrease the critical wave number and hence its effect is to increase the dimension of convection cells. Further, increase in σ^2 and Ns is to make the system more stable.

5. CONCLUSIONS

The effect of internal heat generation and thereby the influence of non-uniform basic temperature gradient on the onset of convection in a fluid saturated porous layer is investigated. The lower boundary is considered to be rigid – isothermal ,while the upper boundary is free and subject to a general thermal condition on the perturbed temperature. The resulting eigen value problem is solved numerically by employing the Galerkin technique. The following conclusions can be drawn from the present study:

1. The effect of increase in the internal heat source strength Ns is to decrease critical Rayleigh number R_{ec} and hence to hasten the onset of DB convection in a fluid saturated porous layer.
2. The effect of increase effective permeability parameter σ_e^2 is to delay the onset of DB convection.
3. The effect increase ratio of viscosities Λ has destabilizing effect on the system.

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