



# Electrohydrodynamic Instability in a Heat Generating Porous Layer Saturated by a Dielectric Nanofluid

D. Yadav

*Athabasca River Basin Research Institute, Athabasca University, Canada*

*Email: dyadav@athabascau.ca*

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## ABSTRACT

In this paper, a theoretical investigation has been carried out to study the combined effect of AC electric field and temperature depended internal heat source on the onset of convection in a porous medium layer saturated by a dielectric nanofluid. The model used for nanofluid incorporates the combined effect of Brownian diffusion, thermophoresis and electrophoresis, while for porous medium Darcy model is employed. The flux of volume fraction of a nanoparticle with the effect of thermophoresis is taken to be zero on the boundaries and the eigenvalue problem is solved using the Galerkin method. Principle of exchange of stabilities is found to be valid and subcritical instability is ruled out. The results show that increase in the internal heat source parameter  $H_S$ , AC electric Rayleigh-Darcy number  $R_e$ , the Lewis number  $L_e$ , the modified diffusivity ratio  $N_A$  and the concentration Rayleigh-Darcy number  $R_n$  are to hasten the onset of convection. The size of convection cells decreases with increasing the internal heat source parameter  $H_S$  and the AC electric Rayleigh-Darcy number  $R_e$ .

**Keywords:** Electrohydrodynamic instability; Nanofluids; Internal heat source; Porous medium.

## 1. INTRODUCTION

Nanofluids have an important impact on heat transfer enhancement in modern years. They have been utilized in various technologies including cooling systems of electronic devices (Wong and Leon 2010; Saidur *et al.* 2011), porous media solar collectors (Bég *et al.* 2012), geothermics (Sheikholeslami and Ganji 2014), automotive applications (Srinivas *et al.* 2016) and chemical engineering coating processes (Malvandi *et al.* 2015). Further, remarkable implementations of nanofluids include crystal growth (Jayaraman *et al.* 2009), materials processing (Muthamilselvan and Rakkiyappan 2011), energy storage (Uddin *et al.* 2013) and automotive engine cooling (Leong *et al.* 2010). Nanofluids represent a significant class of heat transfer fluids obtained by dispersing very small amount of nanoparticles (<100 nanometers in diameter) in conventional poor thermal conductivity base fluids. Nanoparticles used in nanofluid are typically made of oxide ceramics ( $Al_2O_3$ , CuO), metal carbides (SiC) or metals (Al, Cu) and base fluids are water, oil, bio-fluids, polymer solutions and other common fluids. The presence of the nanoparticles in the fluid increased the effective thermal conductivity of the fluid and consequently enhanced the heat transfer characteristics. There are

some review papers that show detailed characteristic feature of nanofluids (Wang 2007; Murshed *et al.* 2008; Özerinç *et al.* 2010).

A nanofluid modelling was made by Buongiorno (2006) who clarified that the key mechanisms contributing to thermal enhancement are Brownian diffusion and thermophoresis. This model was applied to study the thermal instability problem by Tzou (2008 a, b) and observed that nanofluid is less stable than regular fluid. Later, this problem was revisited by Nield and Kuznetsov (2010) by taking different types of non-dimensional parameters. Thermal instability problem for rotating nanofluid layer was studied by Chand and Rana (2012), Yadav *et al.* (2011, 2013a, 2014a) and Rana *et al.* (2014). Thermal conductivity and viscosity variation on the onset of convection in nanofluid was studied by Nield and Kuznetsov (2012), Yadav *et al.* (2013b, 2014b, 2015a) and Umavathi *et al.* (2015). They obtained that the consequence of these factors increase the critical Rayleigh number.

In recent years, the study of the interaction of electromagnetic fields with fluids saturated porous medium started gaining attention with the promise of applications in many fields of science and engineering including nuclear fusion, chemical engineering and medicine (Allen and Karayiannis

1995; Yabe *et al.* 1996; Moreno *et al.* 1996; Lai and Lai 2002). Depending on the nature of fluids, the effects of magnetic and electric fields become important. The magnetic field effects are dominant if the fluid is highly electrically conducting. To the opposing, the electric forces play a key role in driving the motion if the fluid is dielectric with low electrical conductivity. The investigation of convective instability together with the electrical and magnetic forces in nanofluids is of practical importance. A systematic study through a proper theory is essential to understand the physics of the complex flow behaviour of nanofluids and also to obtain invaluable scaled up information for industrial applications.

Thermal convection in a dielectric fluid layer in the presence of an AC and/or DC electric field is known as electrothermal convection (ETC). Rudraiah and Gayathri (2009) investigated the effect of vertical electric field on the electroconvection in a horizontal dielectric fluid saturated densely packed porous layer and discussed the importance of ETC in porous media. El-Sayed (2008) analyzed the problem of electrohydrodynamic instability in a horizontal layer of the Oldroydian viscoelastic dielectric liquid through Brinkman porous medium under the simultaneous action of a vertical ac electric field and a vertical temperature gradient. Using linear stability theory, both stationary and oscillatory instabilities are discussed when the liquid layer is heated from below or above. It is shown that the electrical force is the sole agency causing instability. The effect of vertical AC electric field and boundaries on the onset of ETC in sparse porous layer heated from below or above was studied by Shivakumara *et al.* (2011a). Later, the onset of ETC in a rotating Brinkman porous layer was investigated by Shivakumara *et al.* (2011b). An extensive review on this topic has been made by Nield and Bejan (2013).

In some convective flow situations, one may encounter heat generated due to radioactive decay or weak exothermic reactions and nuclear reaction, which leads to change in the energy and thus the temperature profile of the system, like in the case of celestial bodies. Therefore, the role of internal heat generation becomes very important in several applications including food engineering, nuclear heat cores, nuclear energy, nuclear waste disposals, electrical equipment, oil extractions and crystal growth. Despite the importance of internal heat generation in nanofluids, there are few investigations of its effect. The previous works have only considered the much simpler case of uniform heat generation. For example, Hamad and Pop (2011), Yadav *et al.* (2012) and Nield and Kuznetsov (2013a,b) carried out numerical investigations and found that the basic temperature distribution and the basic volumetric fraction of nanoparticle distribution deviate from linear to non-linear in the presence of internal heating, and the critical Rayleigh number decreases with an increase in the internal heat source strength.

Under the circumstances, the study of electric field

with non-uniform internal heat source on the onset of dielectric nanofluid convection in a porous medium seems to be significance in electrical equipment such as distribution transformers, regulating transformers and shunt reactors (Sheikhbahai *et al.* 2012; Asadzadeh *et al.* 2012; El-Genk 2012), and has not been given any attention in the literature. Therefore, the purpose of the research reported here is to examine theoretically the combined effect of vertical AC electric field and temperature dependent internal heat source on the criterion for the onset of convection in a dielectric nanofluid saturated horizontal layer of porous medium. We used the Horton–Rogers–Lapwood model with effects due to the electric field, the internal heat generation and nanoparticles. The temperature is considered to be fixed at the boundaries, while the nanoparticles flux under the thermophoretic effects to be zero thereat (Nield and Kuznetsov 2014).

## 2. ANALYSIS

Consider an infinite horizontal layer of incompressible dielectric nanofluid-saturated porous layer of thickness  $d$  with internal heat generation and heated from below. A Cartesian coordinate system  $(x, y, z)$  is chosen in which  $z$  axis is taken at right angle to the boundaries. The nanofluid is confined between two parallel plates  $z = 0$  and  $z = d$ , where the temperatures at the lower and upper boundaries are taken to be  $T_0$  and  $T_1$ , respectively,  $T_0$  being greater than  $T_1$ . Nanofluid layer is subjected to a uniform vertical AC electric field applied across the layer; lower surface is grounded and upper surface is kept at an alternating potential whose root mean square is  $\psi_1$ . For simplicity, Darcy's law is assumed to hold and the Oberbeck--Boussinesq approximation is employed. Homogeneity and local thermal equilibrium in the porous medium is also assumed. According to the works of Buongiorno (2006), the governing equations under this model are:

The continuity equation for the nanofluid is

$$\nabla \cdot \bar{v} = 0, \tag{1}$$

where  $\bar{v} = (u, v, w)$  is the Darcy velocity.

If one introduces a buoyancy force and a force of electrical origin, the momentum equation in a Darcy-porous medium can be written as

$$0 = -\nabla p - \frac{\mu}{K} \bar{v} + \rho \bar{g} + \bar{f}_e, \tag{2}$$

where  $p$  is the pressure,  $K$  is the permeability of the porous medium,  $\bar{g}$  is the gravitational acceleration,  $\bar{f}_e$  is the force of electrical origin,  $\mu$  and  $\rho$  are the viscosity and the mass density of the nanofluid, respectively. The mass density ( $\rho$ ) of nanofluid can be written as:

$$\rho = \phi\rho_p + (1-\phi)\rho_f, \quad (3)$$

where  $\phi$  is volumetric fraction of nanoparticle,  $\rho_p$  and  $\rho_f$  are the density of the nanoparticle and the base fluid, respectively.

Under the Boussinesq approximation, the density of base fluid can be approximated as

$$\rho_f = \rho_{f_0} \{1 - \beta(T - T_1)\}, \quad (4)$$

Where  $\rho_{f_0}$  is the density of the base fluid at reference temperature  $T_1$  and  $\beta$  is the thermal expansion coefficient. The density of the nanofluid can be approximated by that of base fluid, i.e.  $\rho_0 \cong \rho_{f_0}$ , since the volumetric fraction of nanoparticles is only a few percent. Hence, with the help of equations (3) and (4), the buoyancy term can be approximated as

$$\rho\bar{g} \cong [\phi\rho_p + \rho_0(1-\phi)\{1 - \beta(T - T_1)\}] \bar{g}. \quad (5)$$

On using equation (5), the momentum equation (2) can be written as

$$\nabla p + \frac{\mu}{K} \bar{v} = [\phi\rho_p + \rho_0(1-\phi)\{1 - \beta(T - T_1)\}] \bar{g} + \bar{f}_e. \quad (6)$$

The conservation equation for the nanoparticle is

$$\left[ \frac{\partial}{\partial t} + \frac{1}{\phi} (\bar{v} \cdot \nabla) \right] \phi = -\frac{1}{\rho_p} \nabla \cdot \bar{j}_p, \quad (7)$$

where  $\phi$  is the porosity of the porous medium and  $\bar{j}_p$  is the diffusion mass flux for the nanoparticle, given as the sum of two diffusion terms (Brownian motion and thermophoresis) by

$$\bar{j}_p = -\rho_p D_B \nabla \phi - \rho_p \left( \frac{D_T}{T_1} \right) \nabla T, \quad (8)$$

where  $D_B$  and  $D_T$  are the Brownian and the thermophoresis diffusion coefficients of the nanoparticle, respectively. According to Einstein formula (Einstein 1905),  $D_B$  and  $D_T$  are given as:

$$D_B = \frac{k_B T}{3\pi\mu d_p}, \quad D_T = \left( \frac{\mu}{\rho} \right) \left( \frac{0.26k}{2k + k_p} \right) \phi, \quad (9)$$

where  $T$  is the temperature,  $k_B$  is the Boltzmann's constant,  $d_p$  is the diameter of nanoparticle,  $k$  and  $k_p$  are the thermal conductivity of the fluid and the nanoparticle, respectively.

Substituting the expression of  $\bar{j}_p$  from the equation (8) into the equation (7), the conservation equation for the nanoparticle (7) becomes

$$\left[ \frac{\partial}{\partial t} + \frac{1}{\phi} (\bar{v} \cdot \nabla) \right] \phi = D_B \nabla^2 \phi + \frac{D_T}{T_1} \nabla^2 T, \quad (10)$$

The energy equation is

$$\left[ (\rho c)_m \frac{\partial}{\partial t} + (\rho c) (\bar{v} \cdot \nabla) \right] T = -\nabla \cdot \bar{q} + \phi h_p \nabla \cdot \bar{j}_p + Q(T - T_1), \quad (11)$$

where  $Q(T - T_1)$  is the strength of internal heat generation,  $(\rho c)$  is the heat capacity of nanofluid,  $\bar{q}$  is the energy flux relative to nanofluid velocity and  $h_p \nabla \cdot \bar{j}_p$  is the addition flow work due to the Brownian and the thermophoresis motions of nanoparticle relative to the flow velocities. The energy flux  $\bar{q}$  can be written as the sum of the conduction heat flux and the heat flux due to nanoparticle diffusion as

$$\bar{q} = -k_m \nabla T + \phi h_p \bar{j}_p, \quad (12)$$

where  $k_m$  is the effective thermal conductivity and  $h_p$  is the specific enthalpy. Equation (12) gives

$$\begin{aligned} \nabla \cdot \bar{q} &= -k_m \nabla^2 T + \nabla \cdot (\phi h_p \bar{j}_p) \\ &= -k_m \nabla^2 T + \phi (h_p \nabla \cdot \bar{j}_p + \bar{j}_p \cdot \nabla h_p) \\ &= -k_m \nabla^2 T + \phi \{ h_p \nabla \cdot \bar{j}_p + \bar{j}_p \cdot (c_p \nabla T) \}, \end{aligned} \quad (13)$$

where  $\nabla h_p$  is equal to  $c_p \nabla T$  and  $c_p$  is the specific heat of the material constituting the nanoparticles. Substituting the expression for  $\nabla \cdot \bar{q}$  from equation (13) and  $\bar{j}_p$  from equation (8), the energy equation (11) becomes as

$$\begin{aligned} \left[ (\rho c)_m \frac{\partial}{\partial t} + (\rho c) (\bar{v} \cdot \nabla) \right] T &= k_m \nabla^2 T \\ &+ \phi (\rho c)_p \left[ D_B \nabla \phi \cdot \nabla T + \left( \frac{D_T}{T_1} \right) \nabla T \cdot \nabla T \right] + Q(T - T_1) \end{aligned} \quad (14)$$

The force of electrical origin  $\bar{f}_e$  in equation (6) can be expressed by Landau and Lifshitz (1960) for incompressible nanofluid as

$$\bar{f}_e = \rho_e \bar{E} - \frac{1}{2} (\bar{E} \cdot \bar{E}) \nabla \epsilon, \quad (15)$$

where  $\bar{E}$  is the root mean square value of the electric field,  $\rho_e$  is the charge density and  $\epsilon$  is the dielectric constant. The first term on the right hand side is the Coulomb force due to a free charge and the second term depends on the gradient of  $\epsilon$ . If an AC electric field is applied at a frequency much higher than the reciprocal of the electrical relaxation time, the free charge does not have time to accumulate. Moreover, the electrical relaxation times of most dielectric liquids appear to be sufficiently long to prevent the buildup of free charge at standard power line frequencies. At the

same time, dielectric loss at these frequencies is so low that it makes no significant contribution to the temperature field. Under the circumstances, only the force induced by non-uniformity of the dielectric constant is considered. Furthermore, since the second term in the above equation depends on  $(\vec{E} \cdot \vec{E})$  rather than  $\vec{E}$  and the variation of  $\vec{E}$  is very rapid, the root mean square value of  $\vec{E}$  can be assumed as the effective value. In other words, we can treat the AC electric field as the DC electric field whose strength is equal to the root mean square value of the AC electric field. Assuming the free charge density is negligibly small, the relevant Maxwell equations are:

$$\nabla \times \vec{E} = 0, \tag{16}$$

$$\nabla \cdot (\epsilon \vec{E}) = 0. \tag{17}$$

In view of equation (16),  $\vec{E}$  can be expressed as

$$\vec{E} = -\nabla \psi, \tag{18}$$

where  $\psi$  is the root mean square value of the electric potential.

The dielectric constant is assumed to be a linear function of temperature in the form

$$\epsilon = \epsilon_0 [1 - \gamma(T - T_1)] = 0, \tag{19}$$

where  $\gamma (> 0)$  is the thermal expansion coefficient of dielectric constant and is assumed to be small.

We assume that the temperature is constant and nanoparticles flux including the effect of thermophoresis is zero on the boundaries. Thus, boundary conditions are:

$$w = 0, T = T_0, D_B \frac{d\phi}{dz} + \frac{D_T}{T_1} \frac{dT}{dz} = 0 \text{ at } z = 0, \tag{20a}$$

$$w = 0, T = T_1, D_B \frac{d\phi}{dz} + \frac{D_T}{T_1} \frac{dT}{dz} = 0 \text{ at } z = d. \tag{20b}$$

### 2.1. Basic State

Assuming the basic state to be quiescent, the quantities at the basic state are given by:

$$\begin{aligned} \vec{v} &= 0, T = T_b(z), p = p_b(z), \phi = \phi_b(z), \\ \epsilon &= \epsilon_b(z), \psi = \psi_b(z), E = E_b(z). \end{aligned} \tag{21}$$

The solution of the basic flows is:

$$T_b = T_1 + \Delta T \frac{\sin \left[ \sqrt{\frac{Q}{k_m}} (d - z) \right]}{\sin \left[ d \sqrt{\frac{Q}{k_m}} \right]},$$

$$\phi_b = \left( -\frac{D_T}{T_1 D_B} \right) T_b + \phi_0, E_b = \frac{E_0}{1 + \gamma \Delta T z / d} \hat{k},$$

$$\psi_b(z) = -\frac{E_0 d}{\gamma \Delta T} \log \left( 1 + \frac{\gamma \Delta T}{d} z \right) \hat{k},$$

$$\epsilon_b = \epsilon_0 \left( 1 + \frac{\gamma \Delta T}{d} z \right) \hat{k},$$

where subscript  $b$  denote the steady state,  $\phi_0$  is a reference scale for the nanoparticle fraction,  $\Delta T = (T_0 - T_1)$  and  $E_0 = -\frac{\psi_1 \gamma \Delta T / d}{\log(1 + \gamma \Delta T)}$  is the

root mean square value of the electric field at  $z = 0$ .

### 2.2. PERTURBATION SOLUTION

Let the initial basic state as described by equation be slightly perturbed so that the perturbed state is given by:

$$\begin{aligned} \mathbf{v} &= \mathbf{v}', p = p_b(z) + p', T = T_b(z) + T', \\ \phi &= \phi_b(z) + \phi', \epsilon = \epsilon_b + \epsilon', \vec{E} = \vec{E}_b + \vec{E}', \\ \psi &= \psi_b + \psi', \end{aligned} \tag{22}$$

where the prime denote the perturbed quantities. On substituting the equation (22) into the equations (1), (6), (10), (14) -(20) and linearizing by neglecting the products of primed quantities, eliminating the pressure term from the momentum equation by operating curl twice, and retaining the vertical component and converting the resulting equations to non-dimensional form by introducing the following dimensionless variables:

$$(x'', y'', z'') = \left( \frac{x', y', z'}{d} \right), t'' = \frac{\alpha_m}{\sigma d^2} t',$$

$$(u'', v'', w'') = \left( \frac{u', v', w'}{\alpha_m} \right) d, T'' = \frac{T' - T_1}{\Delta T},$$

$$\phi'' = \frac{\phi' - \phi_0}{\phi_0}, \psi'' = \frac{\psi'}{\gamma E_0 \Delta T d}, \text{ where } \phi_0 \text{ is a}$$

reference scale for the nanoparticle fraction and  $\alpha_m = \frac{k_m}{(\rho c)}$ ,  $\sigma = \frac{(\rho c)_m}{(\rho c)}$ , we obtain the linear stability equations (dropping the dashes (')) for simplicity) in non dimensional form as:

$$\nabla^2 w = R_D \nabla_H^2 T - R_n \nabla_H^2 \phi + R_e \nabla_H^2 \left( T - \frac{\partial \psi}{\partial z} \right), \tag{23}$$

$$\begin{aligned} \frac{\partial T}{\partial t} + wf(z) &= (\nabla^2 + H_S) T + \frac{N_A N_B}{L_e} f(z) \frac{\partial T}{\partial z} \\ &+ \frac{N_B}{L_e} f(z) \frac{\partial \phi}{\partial z}, \end{aligned} \tag{24}$$

$$\frac{1}{\sigma} \frac{\partial \phi}{\partial t} - \frac{N_A}{\phi} wf(z) = \frac{1}{L_e} \nabla^2 \phi + \frac{N_A}{L_e} \nabla^2 T, \tag{25}$$

$$\nabla^2 \psi = \frac{\partial T}{\partial z}, \tag{26}$$

$$\text{where } f(z) = \frac{-\sqrt{H_S} \cos \left[ \sqrt{H_S} (z - 1) \right]}{\sin \left[ \sqrt{H_S} \right]}.$$

In the above equations the following non-

dimensional parameters are given as:

$L_e = \frac{\alpha_m}{D_B}$  is the Lewis number,

$R_D = \frac{\rho_0 g \beta \Delta T K d}{\mu \alpha_m}$  is the thermal Rayleigh-Darcy

number,  $R_n = \frac{(\rho_p - \rho_0) \phi_0 g K d}{\mu \alpha_m}$  is the

nanoparticle Rayleigh-Darcy number,

$R_e = \frac{\gamma^2 \epsilon E_0^2 (\Delta T)^2 K}{\mu \alpha_m}$  is the AC electric

Rayleigh-Darcy number,  $H_S = \frac{Q d^2}{k_m}$  is the

internal heat source parameter,

$N_A = \frac{D_T \Delta T}{D_B T_1 \phi_0}$  is the modified diffusivity

ratio,  $N_B = \phi \frac{(\rho c)_p \phi_0}{(\rho c)}$  is the modified particle-density increment.

In non-dimensional form, the boundary conditions become:

$$w = \frac{\partial \psi}{\partial z} = T = 0, \frac{\partial \phi}{\partial z} + N_A \frac{\partial T}{\partial z} \text{ at } z = 0, 1 \quad (27)$$

Analyzing the disturbances into the normal modes and assuming that the perturbed quantities are of the form:

$$[w, T, \phi, \psi] = [W(z), \Theta(z), \Phi(z), \Psi(z)] \times \exp(ik_x x + ik_y y + nt), \quad (28)$$

where,  $n$  is the growth rate of disturbances,  $k_x$  and  $k_y$  are wave numbers in  $x$  and  $y$  directions, respectively.

On using equation (28), into equations (23) - (26), we have:

$$(D^2 - a^2)W + R_D a^2 \Theta - R_n a^2 \Phi + R_e a^2 \left( \Theta - \frac{\partial \Psi}{\partial z} \right) = 0, \quad (29)$$

$$-Wf(z) + \left[ D^2 - a^2 + H_S - n + \frac{N_A N_B f(z)}{L_e} D \right] \Theta + \frac{N_B f(z)}{L_e} D \Phi = 0, \quad (30)$$

$$\frac{N_A}{\phi} Wf(z) + \left[ \frac{1}{L_e} (D^2 - a^2) - \frac{n}{\sigma} \right] \Phi + \frac{N_A}{L_e} (D^2 - a^2) \Theta = 0, \quad (31)$$

$$(D^2 - a^2) \Psi - D \Theta = 0, \quad (32)$$

where,  $D = \frac{d}{dz}$ ,  $f(z) = \frac{-\sqrt{H_S} \cos[\sqrt{H_S}(z-1)]}{\sin[\sqrt{H_S}]}$

and  $a = \sqrt{k_x^2 + k_y^2}$  is the resultant dimensionless wave number. The boundary conditions in view of normal mode analysis are:

$$W = 0, \Theta = 0, D \Psi = 0, D \Phi + N_A D \Theta = 0 \text{ at } z = 0, 1. \quad (33)$$

The growth rate  $n$  is in general a complex quantity such that  $n = \omega_r + i \omega_i$ , the system with  $\omega_r < 0$  is always stable, while for  $\omega_r > 0$  it will become unstable. For neutral stability, the real part of  $\omega$  is zero. Hence, we write  $n = i \omega_i$ , where  $\omega_i$  is real dimensionless frequency.

The Galerkin weighted residuals method is used to obtain an analytical solution to the system of equations (29) - (32). Accordingly, the base functions  $W, \Theta, \Phi$  and  $\Psi$  are taken in the following way:

$$W = \sum_{p=1}^N A_p W_p, \Theta = \sum_{p=1}^N B_p \Theta_p, \Phi = \sum_{p=1}^N C_p \Phi_p, \Psi = \sum_{p=1}^N D_p \Psi_p, \quad (34)$$

where  $W_p = \Theta_p = \sin p\pi z$ ,  $\Phi_p = -N_A \sin p\pi z$ ,  $\Psi_p = \cos p\pi z$ , (satisfying the corresponding boundary conditions),  $A_p, B_p, C_p$  and  $D_p$  are unknown coefficients, and  $p = 1, 2, 3, \dots, N$ . On using above expression for  $W, \Theta, \Phi$  and  $\Psi$  into equations (29)-(32) and multiplying the resulting first equation by  $W_p$  second equation by  $\Theta_p$ , third equations by  $\Phi_p$  and fourth equation by  $\Psi_p$  and integrating in the limits from zero to unity, we obtained a system of  $4N$  linear algebraic equations in the  $4N$  unknowns  $A_p, B_p, C_p$  and  $D_p$ ,  $p = 1, 2, 3, \dots, N$ . For the existence of non trivial solution, the determinant of coefficients matrix must vanish, which gives the characteristic equation for the system, with the thermal Rayleigh-Darcy number  $R_D$  as the eigenvalue of the characteristic equation. For a first approximation, we take  $N = 1$ ; this produces the result

$$\begin{vmatrix} \frac{-J}{2}, & \frac{a^2(R_D + R_e)}{2}, & \frac{a^2 N_A R_n}{2}, & \frac{a^2 \pi R_e}{2} \\ -F(z), & \frac{[-J - iaq + H_S - 2G(z)]}{2}, & G(z), & 0 \\ \frac{-N_A^2 F(z)}{\phi}, & \frac{N_A^2 J}{2L_e}, & \frac{N_A^2(-iaqL_e - J\sigma)}{2L_e\sigma}, & 0 \\ 0, & \frac{-\pi}{2}, & 0, & \frac{-J}{2} \end{vmatrix} = 0, \quad (35)$$

Where  $J = (a^2 + \pi^2)$ ,

$$F(z) = \int_0^1 f(z) \sin^2 \pi z \, dz = \frac{2\pi^2}{H_S - 4\pi^2},$$

$$G(z) = \frac{\sqrt{H_S} N_A N_B \pi^2 \tan[\sqrt{H_S}/2]}{L_e (H_S - 4\pi^2)}$$

Generally when we employ a single-term Galerkin approximation in this situation we get a value overestimate by about 3%. But in the present case, the single-term Galerkin approximation gives the exact result. Because the terms containing  $N_B$  involves as a function of  $N_B/L_e$  and the value of  $N_B/L_e$  is too small of order  $10^{-2} \sim 10^{-5}$ , pointing to the zero contribution of the nanoparticle flux in the thermal energy conversation.

### 3. RESULTS AND DISCUSSION

#### 3.1 Stationary Convection

First, consider the case of stationary convection, i.e.  $\omega_i = 0$ . Then, from equation (35), the expression of thermal Rayleigh-Darcy number  $R_D$  for the stationary convection can be obtained as

$$R_D = \frac{\phi (H_S - J) J^2}{2a^2 F(z) [\phi J + 2L_e G(z)]} - \frac{a^2 R_e}{J} - \frac{N_A R_n [\phi J + \{2G(z) - H_S + J\} L_e]}{\phi J + 2L_e G(z)}. \quad (36)$$

It is obvious from equation (36) that the thermal Rayleigh-Darcy number  $R_D$  decreases with increasing the internal heat source parameter  $H_S$ , AC electric Rayleigh-Darcy number  $R_e$  and nanofluid parameters while increases with porosity parameter  $\phi$ .

When  $H_S = 0$  and  $R_e = 0$ , i.e. in the absence of internal heat source and electric field, therefore equation (36) becomes

$$R_D = \frac{(a^2 + \pi^2)^2}{a^2} - N_A R_n \left(1 + \frac{L_e}{\phi}\right). \quad (37)$$

This result is identical with the result of Yadav and Lee (2015) for a thermal equilibrium case with  $D_a = T_D = 0$ .

In the absence of nanoparticles and electric field, i.e.  $R_n = N_B = R_e = 0$ , equation (36) gives

$$R_D = \frac{(H_S - a^2 - \pi^2)(a^2 + \pi^2)}{2a^2 F(z)}. \quad (38)$$

The above is the same result obtained by Bhadauria (2012) for homogeneous single component convection and absence of electric field in a porous layer.

Now rescaling the parameter as  $\chi = a_c^2/\pi^2$  and putting into the equation (36) and then taking  $\partial R_D / \partial \chi = 0$ , the expression for critical thermal Rayleigh-Darcy number  $R_{D,c}$  is obtained, corresponding to the critical value of the wave number  $\chi = \chi_c$  which satisfies the following equation

$$4F(z)G(z)L_e^2\chi^2 \left\{ 2G(z)R_e + N_A\pi^2 R_n(1+\chi)^2 \right\} + \phi^2\pi^4(1+\chi)^2 \left\{ 2F(z)R_e\chi^2 + H_S(1+\chi)^2 + \pi^2(-1+\chi)(1+\chi)^3 \right\} + 2\phi L_e\pi^2(1+\chi) \times [F(z)H_S N_A R_n\chi^2(1+\chi) + G(z)\{4F(z)R_e\chi^2 - H_S(-1+\chi)(1+\chi)^2 + \pi^2(1+\chi)^3(-1+2\chi)\}] = 0. \quad (39)$$

Here  $\chi$  refer to critical value.

In the absence of nanoparticles and electric field, i.e.  $R_n = N_B = R_e = 0$ , equation (39) gives:

$$\pi^2\chi^2 - \pi^2 + H_S = 0. \quad (40)$$

Equation (40) coincides with that of Bhadauria (2012) for homogeneous single component convection and absence of electric field in a porous layer.

#### 3.2 OSCILLATORY CONVECTION

Oscillatory convection is possible only if some supplementary constraints, such as rotation, magnetic field, salinity gradient etc., are present in the system. For oscillatory convection  $\omega_i \neq 0$ , the real and imaginary parts of equation (35) yield:

$$2a^2 F(z) L_e [J(-H_S + J) N_A R_n + 2G(z) \times \{-\pi^2 R_e + J(R_{D,Osc} + R_e + N_A R_n)\}] \sigma + \phi J [-H_S J^2 \sigma + J^3 \sigma - 2a^2 F(z) \pi^2 \times R_e \sigma + J\{-L_e \omega_i^2 + 2a^2 F(z) \times (R_{D,Osc} + R_e + N_A R_n) \sigma\}] = 0, \quad (41)$$

$$\phi L_e [J\{J\{2G(z) - H_S + J\} + 2a^2 F(z) R_{D,Osc}\} + 2a^2 F(z) a^2 R_e] + \phi J^3 \sigma + 2a^2 F(z) J L_e N_A R_n \sigma = 0. \quad (42)$$

Equations (41) and (42) give the following expressions for the thermal Rayleigh-Darcy number  $R_{D,Osc}$  and the frequency of oscillation  $\omega_i$ :

**Table 1 Comparison of critical thermal Rayleigh-Darcy number  $R_{D,c}$  and critical wave number  $a_c$  with different values of internal heat source parameter  $H_S$  and AC electric Rayleigh-Darcy number  $R_e$  for regular fluid**

$H_S$	$R_e$	$R_{D,c}$	$a_c$	$H_S$	$R_e$	$R_{D,c}$	$a_c$
0	0	39.48	3.14	1.0	0	36.50	3.06
	10	34.32	3.35		10	31.47	3.27
	20	28.85	3.55		20	26.09	3.50
	30	23.10	3.76		30	20.41	3.72
	40	17.08	3.96		40	14.44	3.93
	50	10.83	4.15		50	8.22	4.13
0.5	0	37.98	3.10	2.0	0	33.57	2.97
	10	32.89	3.31		10	28.67	3.20
	20	27.47	3.53		20	23.40	3.44
	30	21.74	3.74		30	17.79	3.67
	40	15.75	3.94		40	11.87	3.90
	50	9.52	4.14		50	5.68	4.11

**Table 2 Comparison of critical thermal Rayleigh-Darcy number  $R_{D,c}$  and critical wave number  $a_c$  with different values of internal heat source parameter  $H_S$  and AC electric Rayleigh-Darcy number  $R_e$  for nanofluid at  $N_A = 2, L_e = 5, N_B = 0.01, \varphi = 0.7, R_n = 0.5$**

$H_S$	$R_e$	$R_{D,c}$	$a_c$	$H_S$	$R_e$	$R_{D,c}$	$a_c$
0	0	31.34	3.14	1.0	0	28.74	3.07
	10	26.18	3.35		10	23.68	3.29
	20	20.71	3.55		20	18.28	3.51
	30	14.96	3.76		30	12.58	3.73
	40	8.94	3.96		40	6.59	3.94
	50	2.69	4.15		50	0.35	4.14
0.5	0	30.03	3.11	2.0	0	26.22	3.00
	10	24.92	3.32		10	21.26	3.23
	20	19.49	3.53		20	15.94	3.47
	30	13.76	3.75		30	10.28	3.70
	40	7.75	3.95		40	4.32	3.93
	50	1.51	4.15		50	-1.91	4.14

$$R_{D,osc} = \frac{-J \{2G(z) - H_S + J\}}{2a^2 F(z)} - \frac{a^2 R_e}{J} - \frac{N_A R_n \sigma}{\varphi} - \frac{J^2 \sigma}{2a^2 L_e F(z)} \quad (43)$$

$$\omega_i^2 = \frac{\sigma}{L_e^2 \varphi^2 J} [-2\varphi L_e \{\varphi J + \{2G(z) - H_S + J\} L_e\}] \times \{G(z) J - a^2 F(z) N_A R_n\} - \{\varphi J + 2G(z) L_e\} \times \{\varphi J^2 + 2a^2 F(z) L_e N_A R_n\} \sigma \quad (44)$$

From equation (44), it is interesting to note that the vertical AC electric field does not influence the existence of oscillatory convection. Following Buongiorno (2006), Nield and Kuznetsov (2013a,b) and Yadav (2014) for most of nanofluids, the Lewis number  $L_e$  is on the order of  $10^1 \sim 10^3$ ,  $N_A$  is on the order of  $1 \sim 10$ , the nanoparticle Rayleigh-Darcy number  $R_n$  and  $\sigma$  are on the order of

$1 \sim 10$ , and Hence from equation (44), the value of  $\omega_i^2$  will be always negative. Since  $\omega_i$  is real for oscillatory convection, therefore oscillatory convection cannot occur and the principle of the exchange of stability is valid for the case of nanofluid.

The results which are given in the equations (36) and (39) are presented graphically in the Figs.1-12 and also tabled in the Tables 1-6 for fixed values of  $H_S = 1, N_A = 2, L_e = 5, N_B = 0.01, \varphi = 0.7, R_n = 0.5$  except the varying parameters. The range of parameters fall in these figures is taken from the available literature (Buongiorno 2006; Nield and Kuznetsov 2013a,b; Yadav 2014; Kefayati 2013; Kuznetsov 2012, Yadav *et al.* 2015c, 2016a,b,c,d,e,f,g).

The linear stability theory expresses the criteria of stability in terms of the critical thermal Rayleigh-Darcy number  $R_{D,c}$ , below which the system is

**Table 3 Comparison of critical thermal Rayleigh-Darcy number  $R_{D,c}$  and critical wave number  $a_c$  with different values of nanoparticles Rayleigh-Darcy number  $R_n$  and AC electric Rayleigh-Darcy number  $R_e$  for nanofluid at  $H_S = 1, N_A = 2, L_e = 5, N_B = 0.01, \varphi = 0.7$**

$R_n$	$R_e$	$R_{D,c}$	$a_c$	$R_n$	$R_e$	$R_{D,c}$	$a_c$
0.2	0	33.41	3.07	0.6	0	27.19	3.08
	10	28.36	3.28		10	22.12	3.29
	20	22.98	3.50		20	16.72	3.52
	30	17.28	3.72		30	11.01	3.74
	40	11.30	3.94		40	5.01	3.95
	50	5.08	4.13		50	-1.23	4.15
0.4	0	30.30	3.07	1.0	0	20.97	3.09
	10	25.24	3.29		10	15.88	3.31
	20	19.85	3.51		20	10.46	3.53
	30	14.14	3.73		30	4.73	3.75
	40	8.16	3.94		40	-1.28	3.96
	50	1.93	4.14		50	-7.53	4.16

**Table 4 Comparison of critical thermal Rayleigh-Darcy number  $R_{D,c}$  and critical wave number  $a_c$  with different values of Lewis number  $L_e$  and AC electric Rayleigh-Darcy number  $R_e$  for nanofluid at  $H_S = 1, N_A = 2, N_B = 0.01, \varphi = 0.7, R_n = 0.5$**

$L_e$	$R_e$	$R_{D,c}$	$a_c$	$L_e$	$R_e$	$R_{D,c}$	$a_c$
2	0	32.81	3.07	8	0	24.68	3.08
	10	27.76	3.28		10	19.60	3.30
	20	22.38	3.50		20	14.19	3.52
	30	16.68	3.72		30	8.47	3.74
	40	10.70	3.94		40	2.47	3.95
	50	4.48	4.13		50	-3.78	4.15
5	0	28.74	3.07	11	0	20.61	3.09
	10	23.68	3.29		10	15.52	3.31
	20	18.28	3.51		20	10.09	3.53
	30	12.58	3.73		30	4.36	3.75
	40	6.59	3.94		40	-1.65	3.96
	50	0.35	4.14		50	-7.90	4.16

stable, while above which it is unstable. To determine the accuracy of the numerical procedure used to obtain the critical stability parameters, first the test computations are carried out for different values of internal heat source parameter  $H_S$  under the limiting case of nanoparticle and electric field i.e.  $R_n = R_e = 0$  and results are tabled in Table 1. From the Table 1, we identify that in the absence of nanoparticles, internal heat source and electric field, we recover the exactly well-known result that the critical Rayleigh-Darcy number  $R_{D,c}$  is equal to  $4\pi^2$  and the corresponding wave number  $a_c$  is  $\pi$ . This verifies the accuracy of the numerical method used.

The critical thermal Rayleigh-Darcy number  $R_{D,c}$

and the corresponding critical wave number  $a_c$  as a function of AC electric Rayleigh-Darcy number  $R_e$  for different values of internal heat source parameter  $H_S$  are shown in Figs. 1 and 2, respectively. From Fig. 1, it is observed that on increasing the value of internal heat source parameter  $H_S$ , the critical value of thermal Rayleigh-Darcy number decreases; thus the effect of  $H_S$  is to advance the onset of convection. This is because increase in  $H_S$ , amounts to increase in energy supply to the system which in turn improve the disturbances in the fluid layer and thus system is more unstable. From Fig.1, it is also found that the critical thermal Rayleigh-Darcy number  $R_{D,c}$  decreases with the AC electric Rayleigh-Darcy number  $R_e$ .

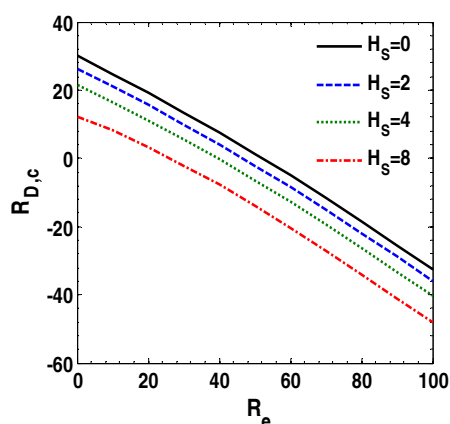


**Table 5 Comparison of critical thermal Rayleigh-Darcy number  $R_{D,c}$  and critical wave number  $a_c$  with different values of modified diffusivity ratio  $N_A$  and AC electric Rayleigh-Darcy number  $R_e$  for nanofluid at  $H_S = 1, L_e = 5, N_B = 0.01, \varphi = 0.7, R_n = 0.5$**

$N_A$	$R_e$	$R_{D,c}$	$a_c$	$N_A$	$R_e$	$R_{D,c}$	$a_c$
1	0	32.62	3.07	3	0	24.86	3.08
	10	27.57	3.28		10	19.79	3.30
	20	22.19	3.51		20	14.38	3.52
	30	16.49	3.73		30	8.66	3.74
	40	10.51	3.94		40	2.66	3.95
	50	4.29	4.14		50	-3.59	4.15
2	0	28.74	3.07	4	0	20.98	3.09
	10	23.68	3.29		10	15.89	3.31
	20	18.28	3.51		20	10.47	3.53
	30	12.58	3.73		30	4.74	3.75
	40	6.59	3.94		40	-1.27	3.96
	50	0.35	4.14		50	-7.52	4.16

**Table 6 Comparison of critical thermal Rayleigh-Darcy number  $R_{D,c}$  and critical wave number  $a_c$  with different values of porosity parameter  $\varphi$  and AC electric Rayleigh-Darcy number  $R_e$  for nanofluid at  $H_S = 1, N_A = 2, L_e = 5, N_B = 0.01, \varphi = 0.7, R_n = 0.5$**

$\varphi$	$R_e$	$R_{D,c}$	$a_c$	$\varphi$	$R_e$	$R_{D,c}$	$a_c$
0.1	0	-11.97	3.17	0.7	0	28.74	3.07
	10	-17.18	3.39		10	23.68	3.29
	20	-22.71	3.61		20	18.28	3.51
	30	-28.54	3.82		30	12.58	3.73
	40	-34.64	4.03		40	6.59	3.94
	50	-40.98	4.22		50	0.35	4.14
0.4	0	23.67	3.09	1.0	0	30.77	3.07
	10	18.59	3.30		10	25.72	3.29
	20	13.17	3.53		20	20.33	3.51
	30	7.45	3.74		30	14.63	3.73
	40	1.44	3.96		40	8.64	3.94
	50	-4.80	4.15		50	2.41	4.14



**Fig. 1. Effect of AC electric Rayleigh-Darcy number  $R_e$  on the critical thermal Rayleigh-Darcy number  $R_{D,c}$  for different values of internal heat source parameter  $H_S$  at  $N_A = 2, L_e = 5, N_B = 0.01, \varphi = 0.7, R_n = 0.5$ .**

Therefore, the effect of imposed AC electric field results in the reduction of stability of the system. Fig 2 shows that increase in the values of internal heat source parameter  $H_S$  and AC electric Rayleigh-Darcy number  $R_e$  tend to increase  $a_c$  and thus its effect is to decrease the size of convection cells. Also, it is interesting to note that the effect of internal heat source parameter  $H_S$  on the critical wave number  $a_c$  is reversed for higher values of AC electric Rayleigh-Darcy number  $R_e (> 80)$ .

Figures 3 and 4 indicate the influence of nanoparticles Rayleigh-Darcy number  $R_n$  on the stability characteristics.

From Fig. 3, it is observed that the critical thermal Rayleigh-Darcy number  $R_{D,c}$  decreases as nanoparticles Rayleigh-Darcy number  $R_n$  increases. Therefore, nanoparticles Rayleigh-Darcy

number  $R_n$  has a destabilizing effect on the stability of the system. This is because as an increase in nanoparticles Rayleigh-Darcy number  $R_n$ , increases the Brownian motion of the nanoparticles and thus system is more unstable. The corresponding critical wave number  $a_c$  is plotted in Fig. 4 and found that the nanoparticles Rayleigh-Darcy number  $R_n$  has no significant effect on the critical wave number.

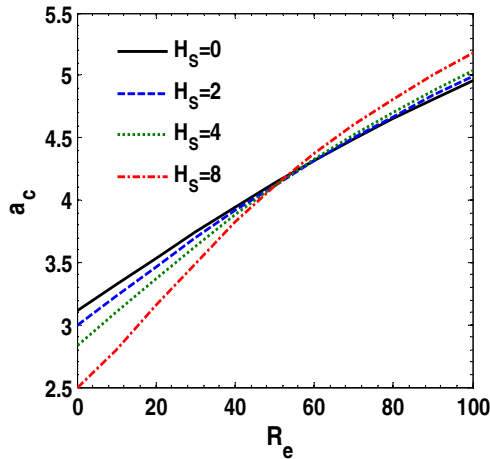


Fig. 2. Effect of AC electric Rayleigh-Darcy number  $R_e$  on the critical wave number  $a_c$  for different values of internal heat source parameter  $H_S$  at  $N_A = 2$ ,  $L_e = 5$ ,  $N_B = 0.01$ ,  $\phi = 0.7$ ,  $R_n = 0.5$ .

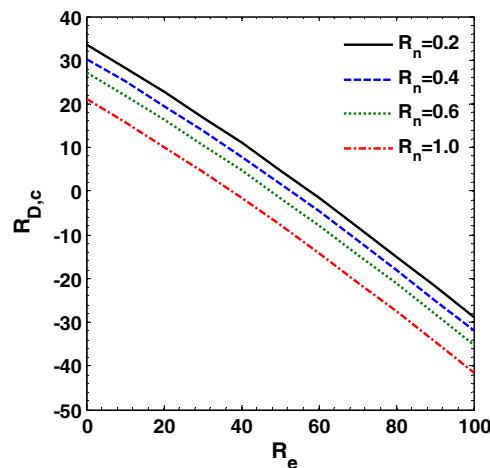


Fig. 3. Effect of AC electric Rayleigh-Darcy number  $R_e$  on the critical thermal Rayleigh-Darcy number  $R_{D,c}$  for different values of concentration Rayleigh-Darcy number  $R_n$  at  $N_A = 2$ ,  $L_e = 5$ ,  $N_B = 0.01$ ,  $\phi = 0.7$ ,  $H_S = 1$ .

Figs. 5-10 show the effect of Lewis number  $L_e$ , the modified diffusivity ratio  $N_A$  and porosity parameter  $\phi$  on the stability of the system. From Figs. 5, 7 and 9, we found that the Lewis number

$L_e$  and the modified diffusivity ratio  $N_A$  accelerate the onset of convection, while porosity parameter  $\phi$  delays the convection in a nanofluid layer. It may be happened because the thermophoresis at a higher value of thermophoretic diffusivity is more supportable to the disturbance in nanofluids, while both thermophoresis and Brownian motion are driving forces in favour of the motion of nanoparticles. From Figs. 6, 8 and 10, we found that the Lewis number  $L_e$ , the modified diffusivity ratio  $N_A$  and porosity parameter  $\phi$  have no significant effect on the critical wave number  $a_c$ .

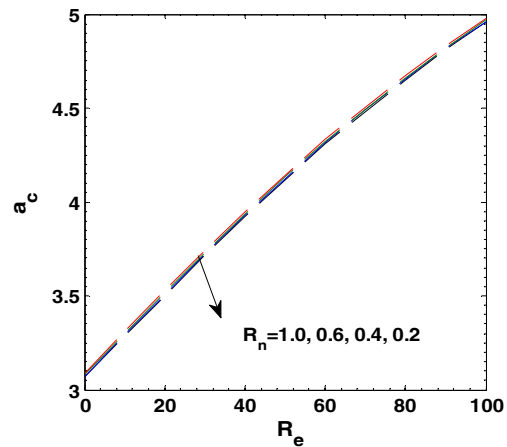


Fig. 4. Effect of AC electric Rayleigh-Darcy number  $R_e$  on the critical wave number  $a_c$  for different values of concentration Rayleigh-Darcy number  $R_n$  at  $N_A = 2$ ,  $L_e = 5$ ,  $N_B = 0.01$ ,  $\phi = 0.7$ ,  $H_S = 1$ .

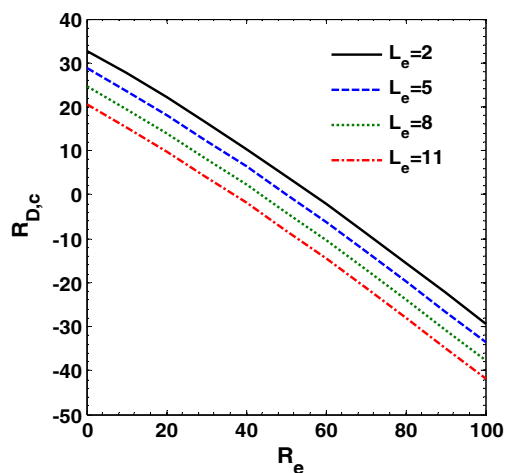
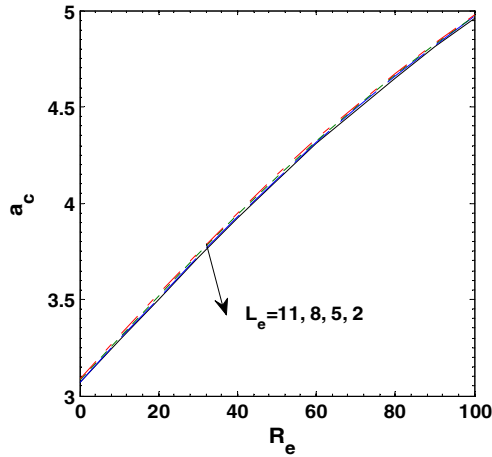


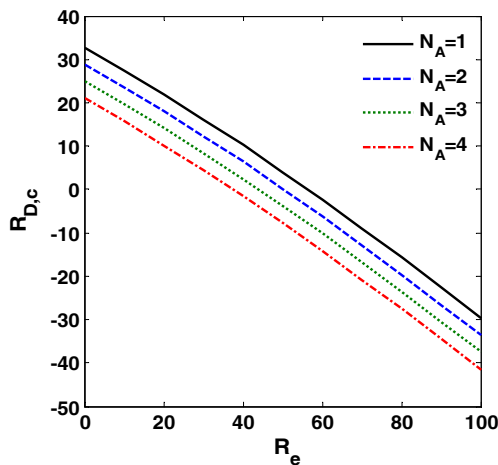
Fig. 5. Effect of AC electric Rayleigh-Darcy number  $R_e$  on the critical thermal Rayleigh-Darcy number  $R_{D,c}$  for different values of Lewis number  $L_e$  at

$N_A = 2$ ,  $N_B = 0.01$ ,  $\phi = 0.7$ ,  $H_S = 1$ ,  $R_n = 0.5$ .

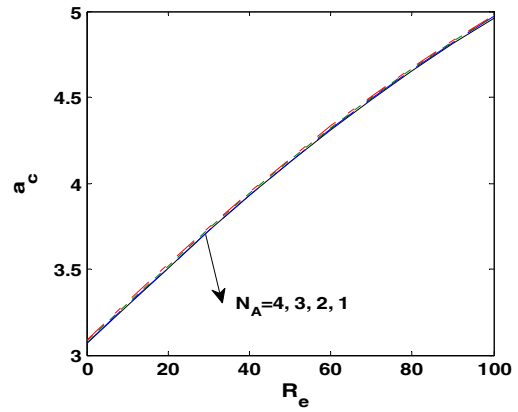
The effect of modified specific heat increment  $N_B$  on the onset of convection is shown in Figs. 11 and 12. It is found that the modified specific heat increment  $N_B$  has no significant effect on the stability of system. This is occurred due to the terms containing  $N_B$  involves as a function of  $N_B/L_e$  and the value of  $N_B/L_e$  is too small of order  $10^{-2} \sim 10^{-5}$ , pointing to the zero contribution of the nanoparticle flux in the thermal energy conversation.



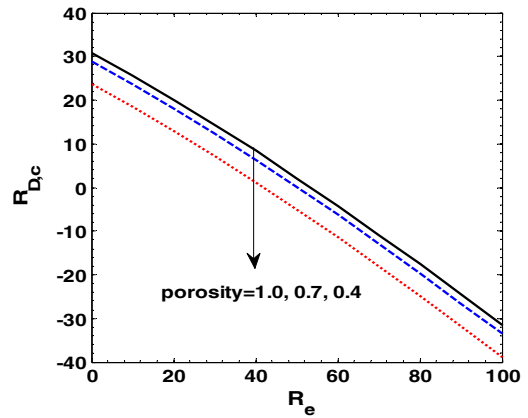
**Fig. 6.** Effect of AC electric Rayleigh-Darcy number  $R_e$  on the critical wave number  $a_c$  for different values of Lewis number  $L_e$  at  $N_A = 2$ ,  $N_B = 0.01$ ,  $\varphi = 0.7$ ,  $H_S = 1$ ,  $R_n = 0.5$ .



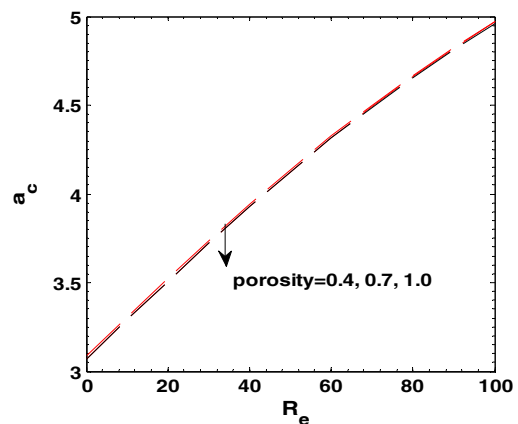
**Fig. 7.** Effect of AC electric Rayleigh-Darcy number  $R_e$  on the critical thermal Rayleigh-Darcy number  $R_{D,c}$  for different values of modified diffusivity ratio  $N_A$  at  $L_e = 5$ ,  $N_B = 0.01$ ,  $\varphi = 0.7$ ,  $H_S = 1$ ,  $R_n = 0.5$ .



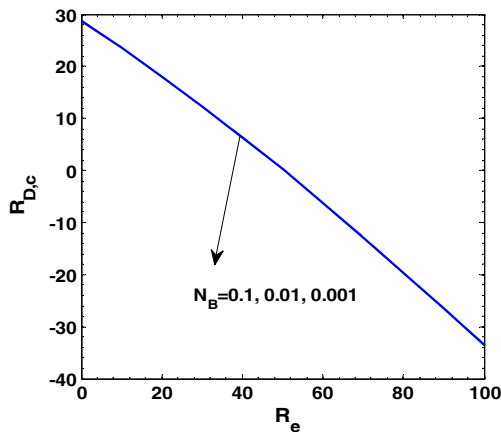
**Fig.8.** Effect of AC electric Rayleigh-Darcy number  $R_e$  on the critical wave number  $a_c$  for different values of modified diffusivity ratio  $N_A$  at  $L_e = 5$ ,  $N_B = 0.01$ ,  $\varphi = 0.7$ ,  $H_S = 1$ ,  $R_n = 0.5$ .



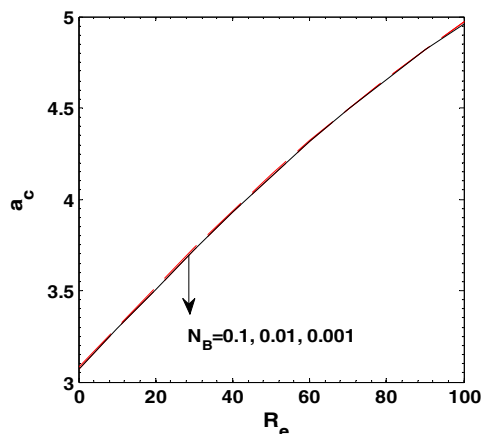
**Fig. 9.** Effect of AC electric Rayleigh-Darcy number  $R_e$  on the critical thermal Rayleigh-Darcy number  $R_{D,c}$  for different values of porosity  $\varphi$  at  $N_A = 2$ ,  $L_e = 5$ ,  $N_B = 0.01$ ,  $R_n = 0.5$ ,  $H_S = 1$ .



**Fig. 10.** Effect of AC electric Rayleigh-Darcy number  $R_e$  on the critical wave number  $a_c$  for different values of porosity  $\varphi$  at  $N_A = 2$ ,  $L_e = 5$ ,  $N_B = 0.01$ ,  $R_n = 0.5$ ,  $H_S = 1$ .



**Fig. 11. Effect of AC electric Rayleigh-Darcy number  $R_e$  on the critical thermal Rayleigh-Darcy number  $R_{D,c}$  for different values of modified specific heat increment  $N_B$  at  $N_A = 2, L_e = 5, \varphi = 0.7, R_n = 0.5, H_S = 1$ .**



**Fig. 12. Effect of AC electric Rayleigh-Darcy number  $R_e$  on the critical wave number  $a_c$  for different values of modified specific heat increment  $N_B$  at  $N_A = 2, L_e = 5, \varphi = 0.7, R_n = 0.5, H_S = 1$ .**

#### 4. CONCLUSIONS

The combined effect of vertical AC electric field and temperature depended internal heat source on the onset of convection in a nanofluid-saturated porous layer has been studied analytically with the assumption that the nanoparticle flux is zero under the thermophoresis at the boundaries. The results have been obtained in terms of the critical thermal Rayleigh-Darcy number  $R_{D,c}$ . We have the following observations:

- 1) The instability of the nanofluid is reinforced with an increase in the value of internal heat source  $H_S$ , AC electric Rayleigh-Darcy number  $R_e$ , the Lewis number  $L_e$ , the modified diffusivity ratio  $N_A$  and the concentration

Rayleigh-Darcy number  $R_n$ .

- 2) The size of convection cells decreases with increasing the AC electric Rayleigh-Darcy number  $R_e$ .
- 3) For lower values of AC electric Rayleigh-Darcy number  $R_e (< 80)$ , the size of convection cells decreases with increasing the internal heat source  $H_S$ , while reverse for higher values of AC electric Rayleigh-Darcy number  $R_e$ .
- 4) The vertical AC electric field does not influence the existence of oscillatory convection and it is ruled out for nanofluid convection due to absence of the two opposing agencies who affect the instability.

#### REFERENCES

- Allen, P. H. G. and T. G. Karayiannis (1995). Electrohydrodynamic enhancement of heat transfer and fluid flow. *Heat Recov. Syst. CHP* 15, 389–423.
- Asadzadeh, F., M. N. Esfahany and N. Etesami (2012). Natural convective heat transfer of Fe3O4/ethylene glycol nanofluid in electric field. *International Journal of Thermal Sciences* 62, 114–119.
- Bég, O. A., T. A. Bég, M. M. Rashidi and M. Asadi (2012). Homotopy semi-numerical modelling of nanofluid convection boundary layers from an isothermal spherical body in a permeable regime. *International Journal of Microscale and Nanoscale Thermal Fluid Transport Phenomena* 3, 237–266.
- Bhadauria, B. S. (2012). Double-diffusive convection in a saturated anisotropic porous layer with internal heat source. *Transp. Porous Med.* 92, 299-320.
- Buongiorno, J. (2006). Convective transport in nanofluids. *ASME J. Heat Transf.* 128, 240-250.
- Chand, R, G. C. Rana (2012). On the onset of thermal convection in rotating nanofluid layer saturating a Darcy-Brinkman porous medium. *Int. J. Heat and Mass Transf.* 55, 5417–5424.
- Einstein, A. (1905). On the movement of small particles suspended in a stationary liquid demanded by the molecular-kinetic theory of heat. *Annalen der Physik* 322(8), 549–560.
- El-Genk, M. S. (2012). Immersion cooling nucleate boiling of high power computer chips. *Energy conversion and management* 53, 205–218.
- El-Sayed, M. F. (2008). Onset of electroconvective instability of Oldroydian viscoelastic liquid layer in Brinkman porous medium. *Arch. Appl. Mech.* 78, 211-224.
- Hamad, M. A. A. and I. Pop (2011). Unsteady MHD free convection flow past a vertical

- permeable flat plate in a rotating frame of reference with constant heat source in a nanofluid. *Heat Mass Transfer* 47, 1517–1524.
- Jayaraman K, K. V. Anand, S. R. Chakravarthy and R. Sarathi (2009). Effect of nano-aluminium in plateau-burning and catalyzed composite solid propellant combustion. *Combustion and Flame* 156, 1662–1673.
- Kefayati, G. H. R. (2013). Lattice Boltzmann simulation of natural convection in nanofluid-filled 2D long enclosures at presence of magnetic field. *Theor. Comput. Fluid Dyn.* 27, 865–883.
- Kuznetsov, A. V. (2012). Nanofluid bioconvection: interaction of microorganisms oxytactic upswimming, nanoparticle distribution, and heating/cooling from below. *Theor. Comput. Fluid Dyn.* 26, 291–310.
- Lai, F. C. and K. W. Lai (2002). EHD-enhanced drying with wire electrode. *Drying Technol.* 20, 1393-1405.
- Landau, L. D. and E. M. Lifshitz (1960). *Electrodynamics of Continuous Media*. Pergamon Press, London.
- Leong, K. Y., R. Saidur, S. N. Kazi and A. H. Mamun (2010). Performance investigation of an automotive car radiator operated with nanofluid-based coolants (nanofluid as a coolant in a radiator). *Applied Thermal Engineering* 30, 2685–2692.
- Malvandi, A., D. D. Ganji and M. H. Kaffash (2015). Magnetic field effects on nanoparticle migration and heat transfer of alumina/water nanofluid in a parallel-plate channel with asymmetric heating. *The European Physical Journal Plus* 130(4), 1-21.
- Moreno, R. Z., E. J. Bonet and O. V. Trevisan (1996). Electric alternating current effects on flow of oil and water in porous media. *Proceedings of the International Conference on Porous Media and Their Applications in Science, Engineering and Industry*, Hawaii, 147–172.
- Murshed, S. M. S., K. C. Leong and C. Yang (2008). Thermophysical and electrokinetic properties of nanofluids-a critical review. *Applied Thermal Engineering* 28, 2109–2125.
- Muthamilselvan, M. and R. Rakkiyappan (2011). Mixed convection in a lid-driven square cavity filled with nanofluids. *International Journal of Nanomechanics Science and Technology* 2, 275–294.
- Nield, D.A. and A. Bejan (2013). *Convection in Porous Media*. Springer.
- Nield, D. A. and A.V. Kuznetsov (2010). The onset of convection in a horizontal nanofluid layer of finite depth. *Eur. J. Mech. B Fluids* 29, 217–223.
- Nield, D. A. and A. V. Kuznetsov (2012). The onset of convection in a layer of a porous medium saturated by a nanofluid: Effects of conductivity and viscosity variation and cross-diffusion. *Transp. Porous Med* 92, 837-846.
- Nield, D. A. and A.V. Kuznetsov (2013a). Onset of convection with internal heating in a porous medium saturated by a nanofluid. *Transp. Porous Med.* 99, 73-83.
- Nield, D. A. and A. V. Kuznetsov (2013b). The onset of convection in an internally heated nanofluid layer. *ASME Journal of Heat Transfer* 136, 014501.
- Nield, D. A. and A. V. Kuznetsov (2014). Thermal instability in a porous medium layer saturated by a nanofluid: A revised model. *Int. J. Heat and Mass Transf.* 68, 211–214.
- Özeringç, S., S. Kakaç and A. Yazicioğlu (2010). Enhanced thermal conductivity of nanofluids: a state-of-the-art review. *Microfluidics and Nanofluidics* 8, 145–170.
- Rana, G. C., R. C. Thakur and S. K. Kango (2014). On the onset of double-diffusive convection in a layer of nanofluid under rotation saturating a porous medium. *J Porous Med.* 17, 657-667.
- Rudraiah, N. and M. S. Gayathri (2009). Effect of thermal modulation on the onset of electrothermoconvection in a dielectric fluid saturated porous medium. *ASME Journal of Heat Transfer* 131, 101009–101015.
- Saidur, R., K. Y. Leong and H. A. Mohammad (2011). A review on applications and challenges of nanofluids. *Renewable and Sustainable Energy Reviews* 15, 1646-1668.
- Sheikhabhai, M., M. N. Esfahany and N. Etesami (2012). Experimental investigation of pool boiling of Fe<sub>3</sub>O<sub>4</sub>/ethylene glycol–water nanofluid in electric field. *International Journal of Thermal Sciences* 52, 149–153.
- Sheikholeslami, M. and D. D. Ganji (2014). Heated permeable stretching surface in a porous medium using Nanofluids. *Journal of Applied Fluid Mechanics* 7(3), 535-542.
- Shivakumara, I. S., C. O. Ng and M. S. Nagashree (2011a). The onset of electrothermoconvection in a rotating Brinkman porous layer. *International Journal of Engineering Science* 49, 646–663.
- Shivakumara, I. S., N. Rudraiah, J. Lee and K. Hemalatha (2011b). The onset of Darcy–Brinkman electroconvection in a dielectric fluid saturated porous layer. *Transp. Porous Med.* 90, 509-528.
- Srinivas, V., C. V. K. N. S. N. Moorthy, V. Dedeepya, P. V. Manikanta and V. Satish (2016). Nanofluids with CNTs for automotive applications. *Heat Mass Transfer* 52, 701–712.
- Tzou, D. Y. (2008a). Instability of nanofluids in natural convection. *ASME J Heat Transfer*

- 130, 372–401.
- Tzou, D. Y. (2008b). Thermal instability of nanofluids in natural convection. *Int. J. Heat Mass Transf.* 51, 2967–2979.
- Uddin, M. J., N. H. M. Yusoff, O. A. Bég and A. I. Ismail (2013). Lie group analysis and numerical solutions for non-Newtonian nanofluid flow in a porous medium with internal heat generation. *Physica Scripta* 87, 025401.
- Umavathi, J. C., D. Yadav and M. B. Mohite (2015). Linear and nonlinear stability analyses of double-diffusive convection in a porous medium layer saturated in a Maxwell nanofluid with variable viscosity and conductivity. *Elixir Mech. Engg.* 79, 30407-30426.
- Wang, X. Q. (2007). Heat transfer characteristics of nanofluids: a review. *Int. J. Therm. Sci.* 46, 1–19.
- Wong, K. V., O. D. Leon (2010). Applications of nanofluids: Current and Future. *Advances in Mechanical Engineering* 10, 1-11.
- Yabe, A., Y. Mori and K. Hijikata (1996). Active heat transfer enhancement by utilizing electric fields. *Ann. Rev. Heat Transf.* 7, 193-244.
- Yadav, D, G. S. Agrawal and R. Bhargava (2011). Thermal instability of rotating nanofluid layer. *Int. J. Engineering Science* 49, 1171–1184.
- Yadav, D, G. S. Agrawal and R. Bhargava (2013b). The onset of double diffusive nanofluid convection in a layer of a saturated porous medium with thermal conductivity and viscosity variation. *J. Porous media* 16, 105–121.
- Yadav, D. and M. C. Kim (2015a). The onset of transient Soret-driven buoyancy convection in nanoparticle suspensions with particle-concentration-dependent viscosity in a porous medium. *J. Porous media* 18, 369–378.
- Yadav, D. (2014). *Hydrodynamic and Hydromagnetic Instability in Nanofluids*. Lambert Academic Publishing Germany.
- Yadav, D., D. Lee, H. H. Cho, J. Lee (2016a). The onset of double-diffusive nanofluid convection in a rotating porous medium layer with thermal conductivity and viscosity variation: a revised model. *Journal of Porous media* 19(1), 31-46.
- Yadav, D., G. S. Agrawal and J. Lee (2016b). Thermal instability in a rotating nanofluid layer: A revised model. *Ain Shams Engineering Journal* 7, 431-440.
- Yadav, D. and J. Lee (2015c). The onset of MHD nanofluid convection with Hall current effect. *The European Physical Journal Plus* 130, 162-184.
- Yadav, D. and J. Lee (2015b). The effect of local thermal non-equilibrium on the onset of Brinkman convection in a nanofluid saturated rotating porous layer. *Journal of Nanofluids* 4, 1-8.
- Yadav, D., J. Lee and H. H. Cho (2016c). Electrothermal instability in a porous medium layer saturated by a dielectric nanofluid. *Journal of Applied Fluid Mechanics* 9(5), 2123-2132.
- Yadav, D., J. Wang, R. Bhargava, J. Lee and H. H. Cho (2016d). Numerical investigation of the effect of magnetic field on the onset of nanofluid convection. *Applied Thermal Engineering* 103, 1441-1449.
- Yadav, D., R. Bhargava and G. S. Agrawal (2012). Boundary and internal heat source effects on the onset of Darcy–Brinkman convection in a porous layer saturated by nanofluid. *Int. J. Therm. Sciences* 60, 244–254.
- Yadav, D., R. Bhargava and G. S. Agrawal (2013a). Numerical solution of a thermal instability problem in a rotating nanofluid layer. *Int. J. Heat Mass Transf.* 63, 313–322.
- Yadav, D., R. Bhargava and G. S. Agrawal (2016g). Erratum to: Thermal instability in a nanofluid layer with a vertical magnetic field. *Journal of Engineering Mathematics* 100, 211-211.
- Yadav, D., R. Bhargava, G. S. Agrawal, G. S. Hwang, J. Lee, M. C. Kim (2014a). Magneto-convection in a rotating layer of nanofluid. *Asia-Pacific Journal of Chemical Engineering* 9, 663–677.
- Yadav, D., R. Bhargava, G. S. Agrawal, N. Yadav, J. Lee and M. C. Kim (2014b). Thermal instability in a rotating porous layer saturated by a non-Newtonian nanofluid with thermal conductivity and viscosity variation. *Microfluid Nanofluid* 16, 425–440.
- Yadav, D., R. A. Mohamed, H. H. Cho and J. Lee (2016e). The effect of Hall current on the onset of MHD convection in a porous medium layer saturated by a nanofluid. *Journal of Applied Fluid Mechanics* 9(5), 2379-2389.
- Yadav, D., R. A. Mohamed, J. Lee and H. H. Cho (2016f). Thermal convection in a Kuvshinski viscoelastic nanofluid saturated porous layer. *Ain Shams Engineering Journal*.