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Numerical Investigations on Unsteady Flow past Two Identical Inline Square Cylinders Oscillating Transversely with Phase Difference

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ABSTRACT

Two-dimensional numerical investigations have been carried out to study the temporal wake characteristics of laminar flow past two identical inline square cylinders performing transverse oscillations. Both the cylinders are forced to perform harmonic oscillations of same frequency and amplitude but with a phase difference. Computations are carried out using commercial software ANSYS Fluent 16.1 on a dynamically sliding mesh for fixed Reynolds number equal to 100. The oscillation frequency is varied from 0.4 to 1.6 times the frequency of vortex shedding behind a single stationary square cylinder. The amplitude of transverse oscillation is kept fixed equal to 0.4D (D = side of the cylinder). In addition, the effect of variation of inter-cylinder spacing has been investigated on wake interference which influences the modes of vortex shedding and resulting dynamic effects on the cylinders. Temporal signals as well as mean characteristics of lift and drag coefficients have been presented for different values of inter-cylinder spacing, phase difference between the two cylinders and frequency of oscillation.

Keywords: Transversely oscillating square cylinders; Phase difference; Wake characteristics; Wake interference; Modes of vortex shedding

1. INTRODUCTION

Study of flow past square cylinders is of much importance in engineering applications, such as flow past automobiles to improve drag and lift characteristics, design of sky scraper buildings to handle wind loads, etc. The square cross-section differs from circular cross-section in many ways. It has fixed separation points, possess wider wake and is relatively bluffer than circular. Moreover, the Karman vortex formation region is significantly longer and broader for square cylinder than for circular. Flow past rectangular cylinder has been investigated by many researchers, such as Parkinson and Brooks (1961), Scruton (1963), Vickery (1966), Nakaguchi *et al.* (1968), Bearman and Trueman (1972), Otsuki (1974), Nakamura and Mizota (1975) and Rockwell (1977), to name a few. Okajima (1982), Davis *et al.* (1984) and Suzuki *et al.* (1993) studied the effect of wall confinement on flow past square cylinder.

A wide range of experimental as well as numerical investigations on flow past a single cylinder (square/rectangular/circular) have been carried out by many researchers. Nowadays, flow past systems of two or more cylinders in various arrangements has capture the attention of researchers due to the complex nature of wake patterns and physical importance in engineering applications. Study of fluid flow behavior on a pair of cylinders become more complex when inter-cylinder spacing changes.

Oscillation of the cylinder makes the flow more complex with onset of nonlinearities in the wake zone. The study of interaction between the wake and the body becomes extremely important from structural point of view and is commonly referred to as ‘vortex induced vibration (VIV)’. A simple approach that is used extensively to study VIV in literature is to force the body to perform harmonic oscillations and study response of the wake. Many studies in literature confirm that the wake characteristics obtained from forced oscillation of cylinder show good agreement with the results for spring mounted cylinder. Leontini *et al.* (2004) conducted numerical investigations to study flow past elastically-mounted and pure tone driven circular cylinder at Reynolds number (Re) of 200. Carberry *et al.* (2005) considered forced transverse oscillations of a circular cylinder. They compared the behavior of near wake and variation of lift forces against the results of Govardhan and

Williamson (2000) for elastically mounted cylinder and found some striking similarities between them. In both the cases, the transition of near wake and lift from low-frequency to high-frequency state occurs at frequency ratio close to unity ($f_r = \frac{f_e}{f_o} \approx 1$, where f_e is excitation frequency and f_o is vortex shedding frequency for single stationary square cylinder). Most of the forced oscillation studies focused on the fluid forces and wake modes. This is because of the fact that near the lock-in region ($\frac{f_e}{f_o} \approx 1$) the cylinder motion and fluid forces can be well represented by sinusoidal functions as reported by Govardhan and Williamson (2000) as well as Khalak and Williamson (1999). Hence, the motion of the body can be expressed in the form given below.

$$y(t) = A\sin(2\pi ft) \tag{1}$$

$$F_y(t) = F_y \sin(2\pi ft) \tag{2}$$

where, A is the amplitude and f is the frequency of oscillation. Here F_y is the amplitude of the harmonic transverse lift force.

There have been observations related to situations where an unsteady flow becomes steady in the presence of another body and vice versa. The fluctuating forces generated due to the alternate vortex shedding are capable of forcing the body to oscillate. The oscillation of the body is another cause that makes the flow even more complex for which analysis invites more challenges. Oscillation of pair of cylinders induces nonlinearities due to oscillation and interference effect of both cylinders that increases the flow complexity by many folds. There are not many of the previous studies available dealing with oscillation of multiple cylinders. The oscillation of one cylinder may affect the wake or lock-in characteristic of the other. Li *et al.* (1992) did numerical simulation to study the response of an oscillating cylinder in the wake of an upstream cylinder. They found that the response of the downstream cylinder is strongly depended on the spacing between the cylinders. They observed a large lock-in zone in the vortex suppression regime. Frequency of vortex shedding from the two circular cylinders of different diameters with large upstream and small downstream one, were measured for different diameter ratio, staggered angle and spacing by Sayers and Saban (1994). They found that the base pressure increases with increase in diameter ratio until lock-in and within lock-in range it remains constant. Price *et al.* (2007) studied experimentally the flow past staggered circular cylinders with upstream cylinder vibrating harmonically in transverse direction. Their study shows that the oscillation of the upstream cylinder causes considerable modification to the flow patterns around the cylinders. In addition to the usual fundamental lock-in, sub and super harmonic resonances are also obtained. A similar study was carried out using immersed boundary method at Re = 100 for tandem cylinders of equal diameter by Yang and Zheng (2010). Tandem circular cylinders with both the cylinders vibrating transversely were studied by Mahir and Rockwell (1996) and Papaioannou *et al.* (2006). Mahir and Rockwell (1996) conducted experiments on flow past two vibrating circular cylinders in tandem arrangement. They observed that for tandem cylinders with small spacing between them, substantially wider lock-in range exists than that for single cylinder. They also reported that for small spacing the modulated patterns of wake formation can be brought to locked patterns by varying the phase of oscillation of the cylinders. Papaioannou *et al.* (2006) conducted numerical study on flow past transversely vibrating circular cylinders and found the existence of quasi-periodicity within the periodic regime which they have referred to as 'hole in the Arnold tongue'.

A detailed study on two cylinders oscillating in-phase has been reported in Mithun and Tiwari (2014). Present study considers effect of phase difference which is more general physical condition for cylinder oscillation. In order to mimic this situation, study in present research is focused on the effect of phase difference between the two oscillating tandem cylinders. The phase difference between the cylinders is varied from 0° to 180°. The amplitude and frequency of oscillation of both the cylinders are assumed to be same.

2. PROBLEM DEFINITION

Schematic of computational domain used for present study is shown in Fig. 1. The domain has a length (*L*) of 26*D* (*D* = side of the cylinder) and width (*H*) of *16D*. Air at standard atmospheric conditions has been used as working fluid that enters at uniform velocity (characteristic velocity) corresponding to Re = 100. The phase difference (ϕ) between oscillating cylinders has been varied from 0° to 180°. Both cylinders are oriented such that the upstream cylinder is placed at 6*D* downstream from the inlet to avoid any influence of inlet boundary condition. The position of the downstream cylinder is varied from 2*D* to 5*D* (in steps of 1*D*), keeping the upstream cylinder fixed. Both cylinders are of equal dimensions. The cylinders are forced to oscillate transversely to the incoming flow according to $y(t) = A\sin(\omega t + \phi)$, where A is the amplitude, ω is angular frequency $\omega = 2\pi f_e$ and ϕ is the phase difference between the displacements of the cylinders. The amplitude of the cylinders is kept fixed at a non-dimensional ratio of $A/D = 0.4$ with the frequency ratio of oscillation varying from 0.4 to 1.6.

3. GOVERNING EQUATIONS AND BOUNDARY CONDITION

3.1 Governing Equations

Fluid flow is governed by set of conservation equation for mass and momentum. The conservation equations for two-dimensional, unsteady flow with laminar and incompressible assumptions can be represented in non-dimensional form as

$$\frac{\partial u_i}{\partial x_i}=0 \tag{3}$$

$$\frac{\partial u_i}{\partial t}+\frac{\partial (u_i u_j)}{\partial x_j}=-\frac{\partial p}{\partial x_i}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_i}{\partial x_j \partial x_j}\right) \tag{4}$$

where u_i is Cartesian velocity component along x_i direction with u and v as velocity component along x and y direction respectively. For a two-dimensional coordinate system Eq. (4) represents x and y component of momentum equation for i equal to 1 and 2 respectively. Velocities are non-dimensionalized based on uniform inlet velocity (U_∞), while side of square cylinder (D) is used as characteristic length for non-dimensionalization of coordinate direction. Non- dimensionalized parameters are defined as:

Reynolds number:

$$\mathrm{Re}=\frac{U_\infty D}{\nu} \tag{5}$$

Static pressure:

$$p=\frac{P}{\rho U_\infty^2} \tag{6}$$

where P is dimensional form of static pressure at any location and ρ and ν are density and kinematic viscosity of the fluid.

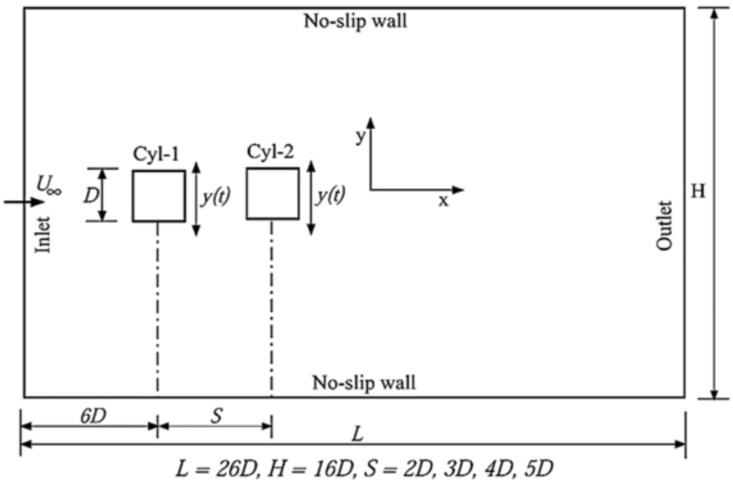

Fig. 1. Schematic of computational domain

3.2 Boundary Conditions

Boundary conditions on cylinder surfaces and channel walls are summarized below in Table 1.

Table 1 Boundary conditions

Boundary	Imposed boundary conditions
Inlet	Uniform velocity $u=U_\infty$
Outlet	Pressure outlet ($p=p_\infty$)
Top and bottom walls	No-slip boundaries, $u=0$ and $v=0$
Cylinder surfaces	No-slip boundaries, $u=0;\quad v=v_s=\omega A\cos(\omega t+\phi)$

4. NUMERICAL TECHNIQUE, GRID AND TIME INDEPENDENCE STUDY

4.1 Numerical Methodology

A finite volume based commercial software ANSYS Fluent 16.1 has been used to solve the governing equations for the considered two-dimensional flow. A hybrid mesh with structured quadrilateral elements near the cylinders and unstructured triangular elements elsewhere in the domain are generated using ANSYS Gambit 2.4. The structured quadrilateral elements offer better numerical accuracy near the cylinder where aerodynamic forces are calculated. The unstructured triangular mesh around it offers better adaptability to accommodate the grid motion during cylinder oscillation. The transverse oscillation of the cylinders with phase difference is achieved with the help of a User Defined Function (UDF) implemented separately and integrated with the solver. For the incompressible flow, a pressure based solver with SIMPLE (Semi Implicit Method for Pressure Linked Equations) algorithm for linking pressure and velocity is employed for the computations. Unsteady term in momentum equation is discretized using first order implicit scheme while convective terms are discretized using second order upwind scheme.

4.2 Grid Independence Study

In order to arrive at a numerical solution that is independent of the mesh size, a thorough grid independence study has been carried out for a configuration with two cylinders placed 5*D* apart. The results from this study are presented in Table 2. It can be clearly seen that 160 grid points on the cylinder surface give fairly accurate results and hence the same is used for further computations.

Table 2 Comparison of mean drag and RMS lift coefficient for different grids

No. of grid points on cylinders	$C_{l,rms}$ Cyl-1	$C_{l,rms}$ Cyl-2	$C_{d,mean}$ Cyl-1	$C_{d,mean}$ Cyl-2
80	0.2980	1.372	1.625	1.388
120	0.2857	1.338	1.603	1.365
144	0.2814	1.318	1.596	1.353
160	0.2815	1.2948	1.596	1.352
184	0.2815	1.295	1.595	1.352
200	0.2814	1.2953	1.5939	1.352

4.3 Temporal Independence

Temporal independence study has been carried out for single stationary cylinder for time-step size varying from 10 ms to 0.1 ms and corresponding RMS (root mean square) value of lift coefficient and mean drag coefficient are reported in Table 3. It is observed that with a change in time-step size from 1 ms to 0.1 ms, $C_{l,rms}$ shows a maximum of 4% change, while $C_{d,mean}$ shows a maximum of 3% change. However, the computational time increases almost 4-5 times with 0.1 ms as compared to 1 ms. Force coefficients obtained are in good agreement with values reported in literature at both time-steps. Considering accuracy and computational cost, a time-step size of 1 ms has been chosen for all the computations in present study.

5. VALIDATION OF COMPUTATIONS

Comprehensive validation of the computational approach has been presented in Mithun and Tiwari (2014). However, for completeness of the present study, a part of validation is being reproduced here. Table 4 shows the comparison of RMS (root mean square) value of lift coefficient, mean value of drag coefficient and wake Strouhal number from present computations against those reported in literature. Figure 2 presents variation of normalized vortex shedding frequency ($f_{ns} = \frac{f_e}{f_s}$, where f_s is vortex shedding frequency of oscillating cylinder) for single oscillating cylinder with change in frequency ratio ($f_r = \frac{f_e}{f_o} \approx 1$, where f_e is excitation frequency and f_o is the vortex shedding frequency in the wake of single stationary square cylinder). Results are compared with those of Tanida *et al.* (1973) and Singh *et al.* (2009). The results show good agreement in capturing qualitative behavior of lock-in band even though computed values deviate little more at certain values of frequency ratio (say, 1.2 and 3.0). This difference can certainly be reduced by considering smaller time step and more fined grid which will be at the cost of much increased computational time.

Table 3 Mean drag and RMS lift coefficient for different time step size

Time step size (ms)	$C_{l,rms}$	$C_{d,mean}$
10	0.1286	1.5463
6	0.1367	1.561
1	0.171	1.59
0.1	0.178	1.635

Table 4 Validation of lift and drag coefficients and Strouhal number for single stationary square cylinder at Re = 100 (Mithun and Tiwari, 2014)

Author	Blockage ratio	$C_{l,rms}$	$C_{d,mean}$	*Strouhal Number*
Present	0.0625	0.171	1.59	0.150
Robichaux *et al.* (1999)	0.056	--	1.530	0.154
Sharma and Eswaran (2004)	0.050	0.192	1.494	0.1488
Singh *et al.* (2009)	0.050	0.160	1.510	0.1470
Sahu *et al.* (2009)	0.050	0.188	1.488	0.1486
Sen *et al.* (2011)	0.050	0.1928	1.5287	0.1452

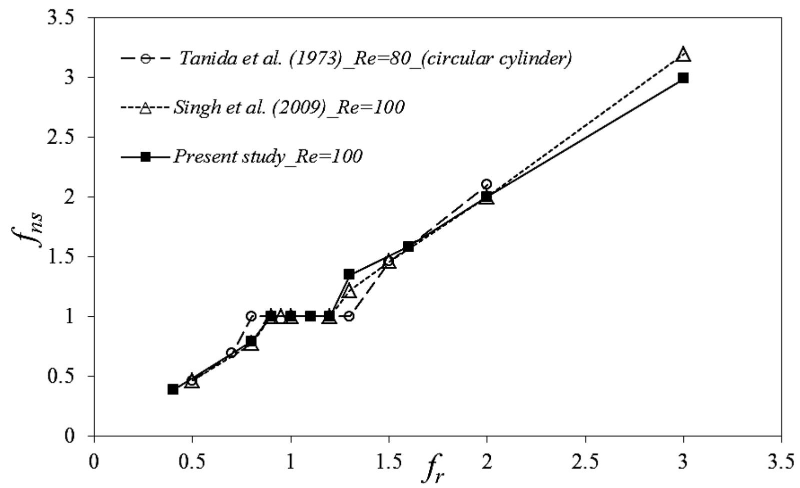

Fig. 2. Comparison of lock-in band of frequency for single cylinder oscillation (Mithun and Tiwari, 2014)

6. RESULTS AND DISCUSSIONS

Computations have been carried out for the flow past two out-of-phase vibrating square cylinders in tandem arrangement. The two cylinders are forced to oscillate at different frequency ratios ranging from 0.4 to 1.6 while their amplitude of oscillation is fixed at 0.4*D*. The cylinders are placed in a rectangular confined channel of width 16*D* where a laminar uniform flow corresponding to Reynolds number Re = 100 impinges at the inlet.

6.1 Lift and Drag Characteristics

The variations in the time history of lift and drag coefficients with change in phase between the two oscillating cylinders for sub-synchronous oscillations are shown in Figs. 3 and 4 for selected values of spacing. For $S = 2D$, the change in phase difference between the cylinders when they oscillate at sub-synchronous frequency changes the nature of variation of the lift and drag when the cylinders are at close proximity as shown in Fig. 3. The amplitude of oscillation of the drag signal of the downstream cylinder increases with increase in phase difference for $S = 2D$ as shown in Fig. 3(b). On the other hand, when the cylinders are farther apart, not much variation in the nature of the force signals is observed as shown in Figs. 4(a) and (b).

Figure 5 shows the Fourier spectra of lift signal for selected inter-cylinder spacing and phase difference between the cylinders. For $S = 2D$, the spectra clearly show that with an increase in phase difference between the cylinders vibration, the strength of harmonics also increases. The spectra of the upstream cylinder in Fig. 5 (for $S = 2D$) show that for $\phi > 0^o$ the strength of the third harmonic of excitation is higher than that of vortex shedding frequency while for the downstream cylinder it almost equals the strength of the vortex shedding frequency. A similar observation can be made for $S = 3D$ also. The increase in strength of the harmonics expected to be due to the interference between the cylinders. When the spacing between the cylinders is increased and brought to $S = 5D$, apart from the excitation and vortex shedding frequency being present in the spectra, there appears a small modulation frequency of the order of 0.02. The third harmonic present in the spectra for $S = 2D$ and $3D$ now changes to a combination of excitation and modulation frequencies.

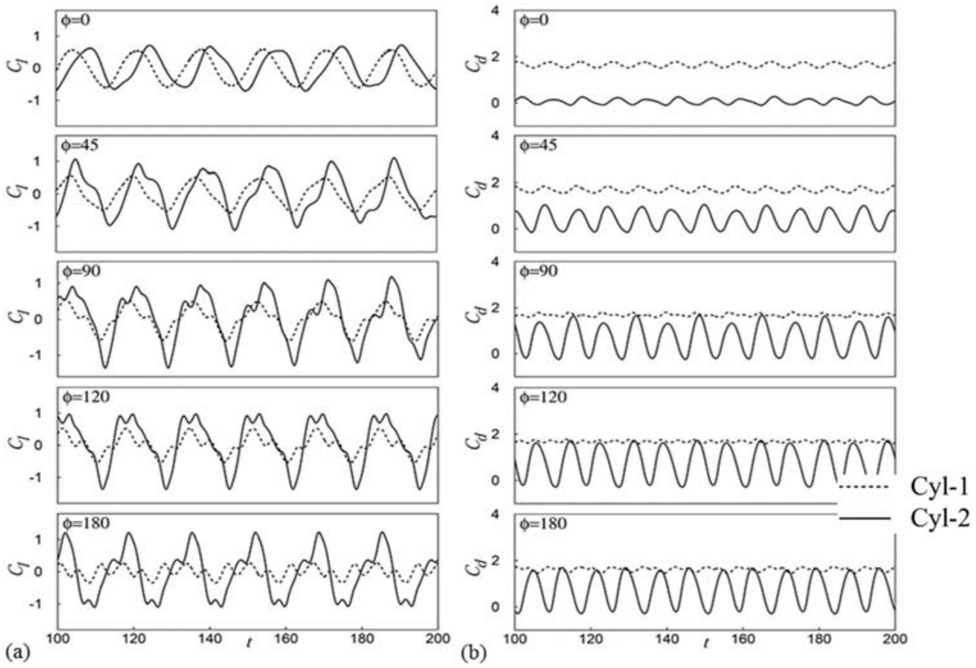

Fig. 3. Time series of lift and drag with change in ϕ for f_r = 0.4 and $S = 2D$

For the case of in-phase oscillation of the cylinders, lock-in is observed at $f_r = 0.8$ for all values of inter cylinder spacing as shown in Fig. 6 for $S = 2D$ and $S = 5D$. For $S = 5D$, when the phase difference between the cylinder oscillation increases and reaches a value greater than $\phi = 45^o$, no lock-in is observed between the wake and cylinder oscillation as seen from Fig. 6(b). In Fig. 6(b), for the cylinders placed 5*D* apart, beats appear in the signals of both upstream and downstream cylinders with their period decreasing with increase in phase difference. The amplitude of oscillation of the lift signals increases with increase in phase difference between the cylinders. This is due to enhanced interaction of the low pressure shear layer from the upstream cylinder. Similarly, a reduction in amplitude of oscillation is observed in the lift signals of upstream cylinder. It can be observed from Figs. 6 and 7 that the lift signals of upstream and downstream cylinders are almost in-phase for $S = 2D$. On the other hand, for $S = 5D$, the lift of the downstream cylinder lags behind that of the upstream cylinder and at $\phi = 90^o$ they are out-of-phase to each other. As the frequency of vibration crosses a value greater than the vortex shedding frequency behind a

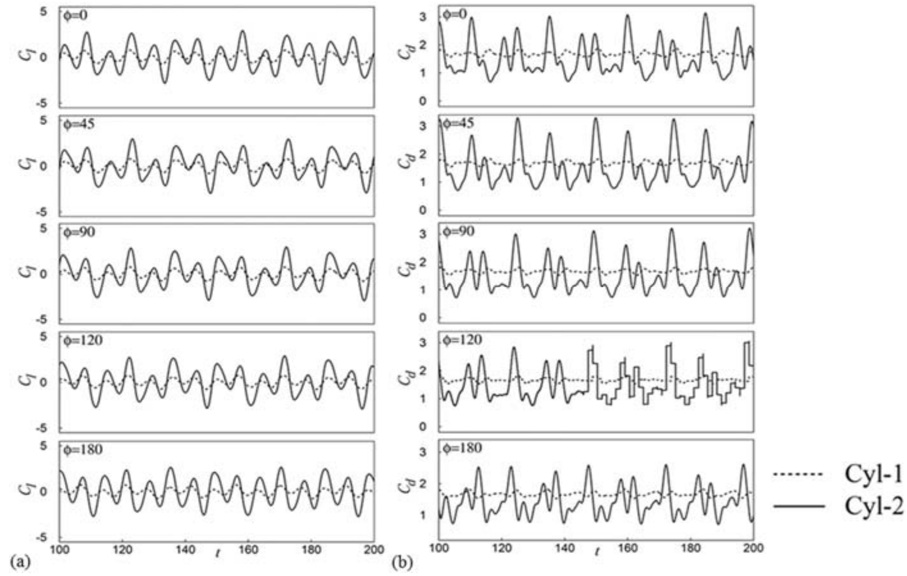

Fig. 4. Time series of lift and drag with the change in ϕ for f_r = 0.4 and *S* = 5*D*

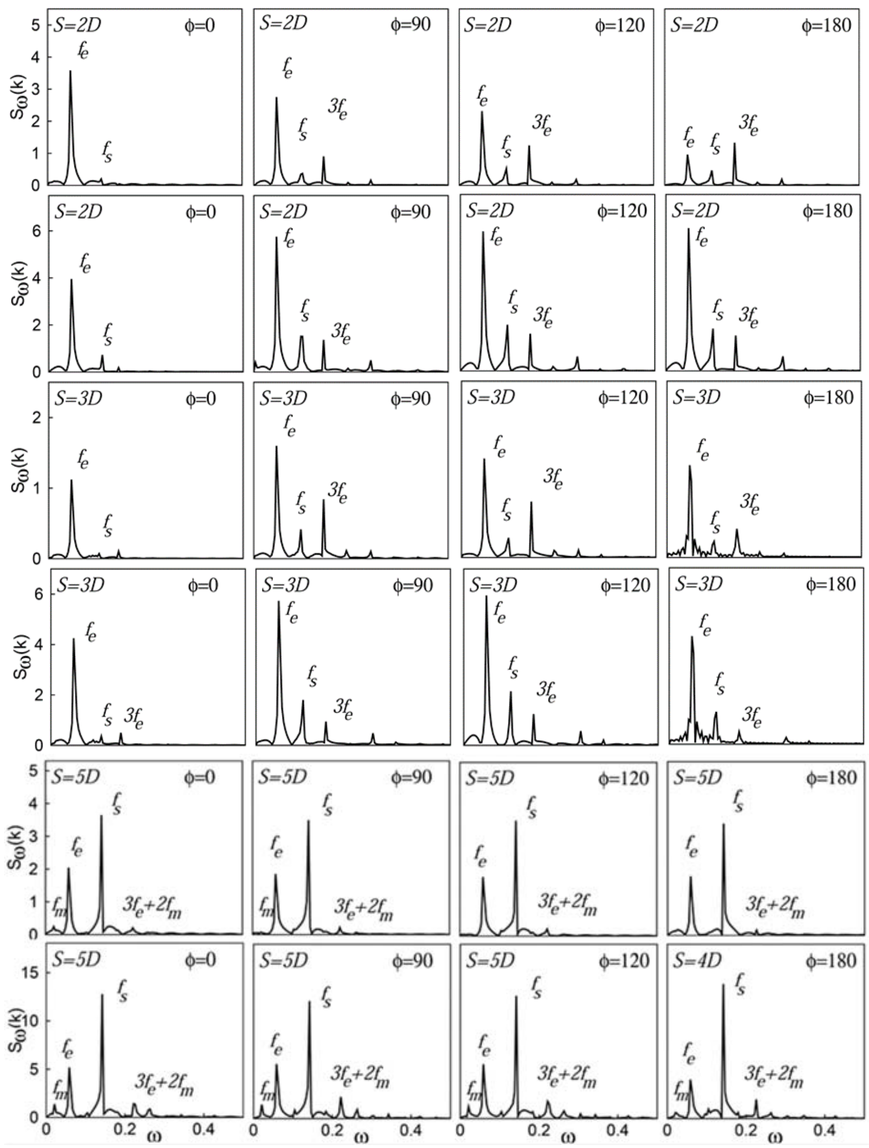

Fig. 5. Fourier spectra of lift signal for the upstream (upper row) and downstream cylinders (lower row) at f_r = 0.4 with change in phase for *S* = 2*D*, 3*D* and 5*D*

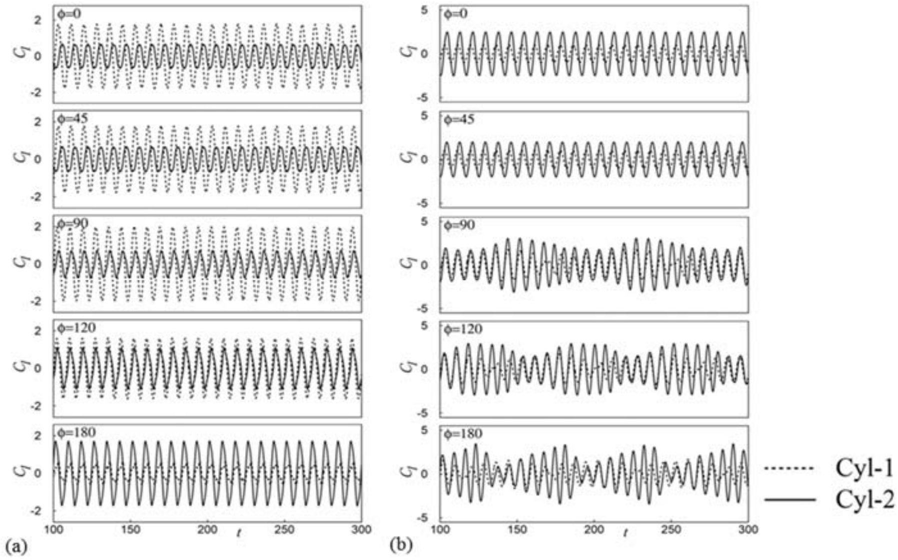

Fig. 6. Time series of lift coefficient with the change in ϕ for f_r = 0.8, (a) *S* = 2*D* (b) *S* = 5*D*

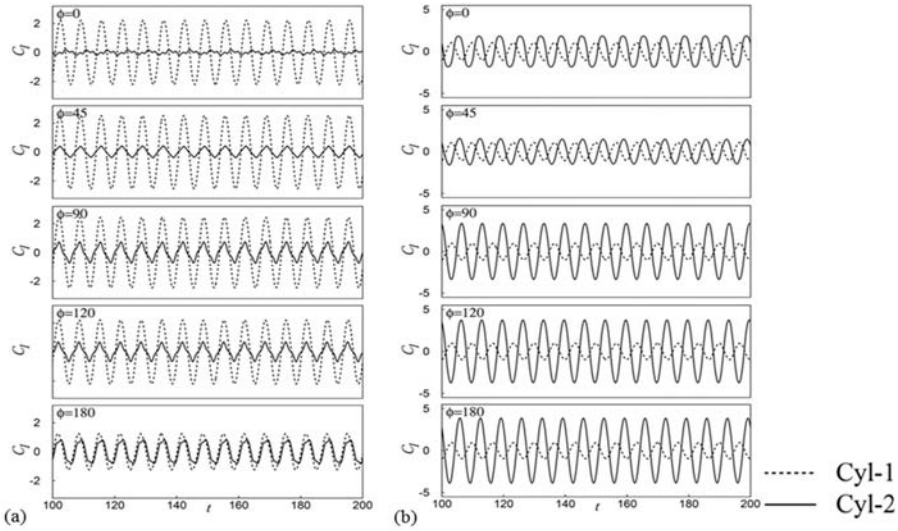

Fig. 7. Time series of lift coefficient with the change in ϕ for f_r = 1, (a) *S* = 2*D* (b) *S* = 5*D*

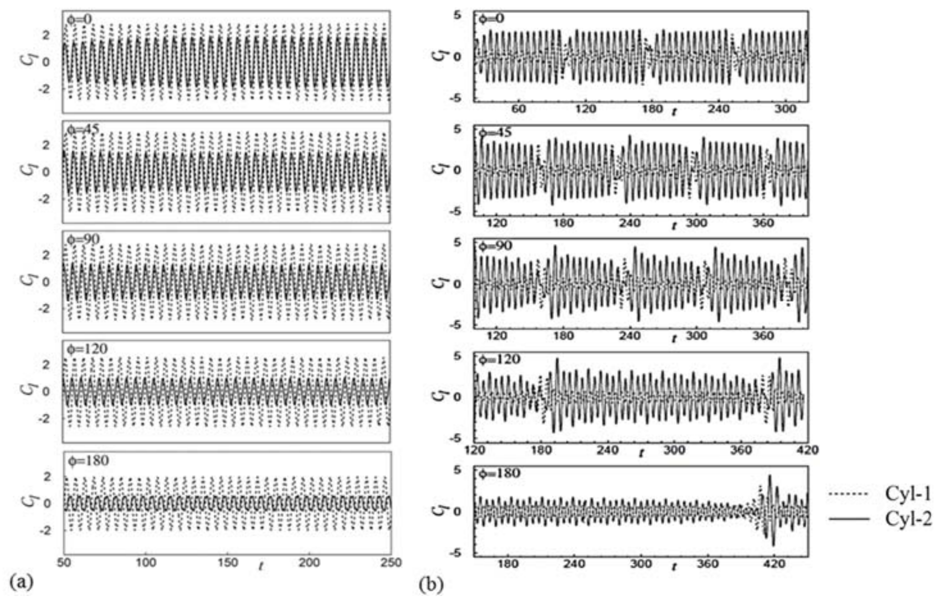

Fig. 8. Time series of lift coefficient with change in ϕ for f_r = 1.2, (a) *S* = 2*D* (b) *S* = 5*D*

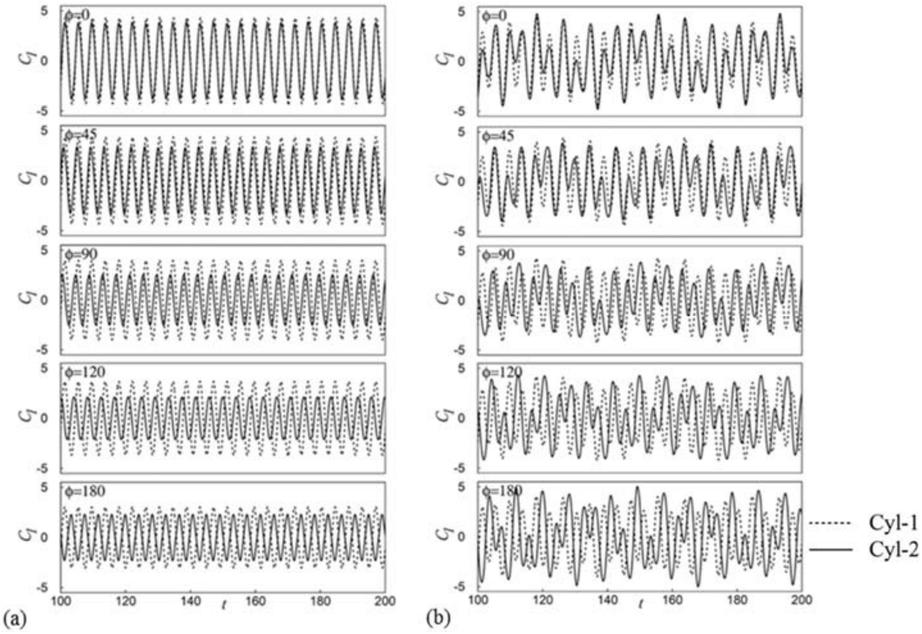

Fig. 9. Time series of lift coefficient with change in ϕ for f_r = 1.6, (a) *S* = 2*D* (b) *S* = 5*D*

stationary cylinder, i.e. as $f_r > 1$, the nature of variation of lift coefficient with increase in phase difference becomes different than that observed for $f_r \leq 1$. Up to $f_r = 1$, the amplitude of oscillation of lift signal grows with increase in phase difference, whereas at $f_r = 1.2$ it decreases with increase in phase difference as shown in Fig. 8(a). Another important observation is that the time period of beats observed in the signals of Fig. 8(b) increases with increase in ϕ .

At frequency ratio 1.6 which falls under the super-synchronous regime the lift signal almost shows sinusoidal behavior at smaller values of spacing. This means that the lift force acting on the cylinder is influenced mainly by a single frequency which is the excitation frequency since there is no vortex shedding in the gap between the cylinders. An observation similar to that made for $f_r = 1.2$ can also be made for $f_r = 1.6$ that the amplitude of oscillation of the lift signal decreases with increase in phase difference between the signals as depicted in Fig. 9(a).

6.2 Flow Characteristics

The change in phase of the two oscillating cylinders results in various modes of interaction between the wakes of the upstream and the downstream cylinders. Some selected cases have been considered where the change in phase results into change in the shedding mechanism. Figure 10 shows interesting changes in the vortex shedding pattern for the selected cases. At sub-synchronous frequency, with increase in phase difference, an ordered '2S' mode observed at $\phi = 0^o$ (Fig. 10(a)) changes to a different mode at $\phi = 180^o$ (Fig. 10(b)). At $\phi = 180^o$, a single vortex is shed simultaneously from the bottom and top edges of the downstream cylinder when the upstream cylinder reaches its extreme top position. Again, when the upstream cylinder reaches its bottom most position, the vortices are shed alternatively from the top and bottom edges of the downstream cylinder as shown in Fig. 10(b).

At synchronous frequency, the change in mode from '2S' to '2P' is observed for *S* = 3*D* when the phase difference is increased from $\phi = 0^o$ to 180^o as shown in Figs. 11(a) and (b). For *S* = 4*D* at f_r = 0.8, the mode change is from '2P' at $\phi = 0^o$ to '2S' at $\phi = 180^o$ at the same frequency as shown in Figs. 11(c) and (d). Same change is observed for *S* = 5*D* at $f_r = 0.8$. For $f_r = 1$ with inter-cylinder spacing of 2*D*, vortex shedding behind the downstream cylinder encounters a phase reversal between ϕ = 0^o and 180^o as shown in Figs. 11(e) and (f). Moreover, the incident vortex from upstream cylinder switches the side on which it interacts with the downstream cylinder. When the spacing between the cylinders is increased for fixed frequency ratio, say for $f_r = 1$, a reduction in longitudinal spacing between the consecutive vortices is observed when ϕ is increased from 0^o to 180^o. At $f_r = 1.2$, for *S* = 5*D*, the shed vortices resemble polar vortices at $\phi = 0^o$ as shown in Fig. 11(i). At super- synchronous frequency for smaller spacing between the cylinders, the vortex street formed behind the downstream cylinder coalesces and gives rise to an 'Anti-symmetrical mode-AI' mode of vortex shedding at $f_r = 1.6$ (Ongoren and Rockwell, 1988) as shown in Fig. 12(d).

6.3 Variation of Lift and Drag Coefficients with Phase Difference

Variation of mean drag and RMS lift coefficient with change in phase between the cylinders for different values of inter-cylinder spacing for a

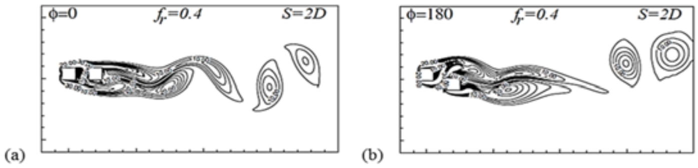

Fig. 10. Vorticity contours with change in ϕ at sub – synchronous frequency

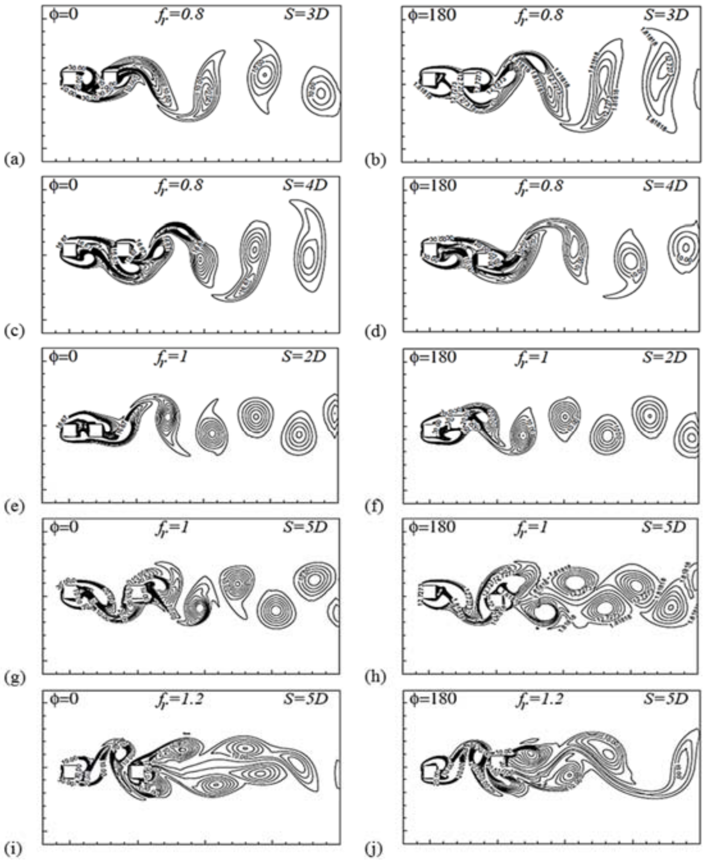

Fig. 11. Vorticity contours showing variation in shedding pattern with change in ϕ for selected spacing at synchronous frequencies

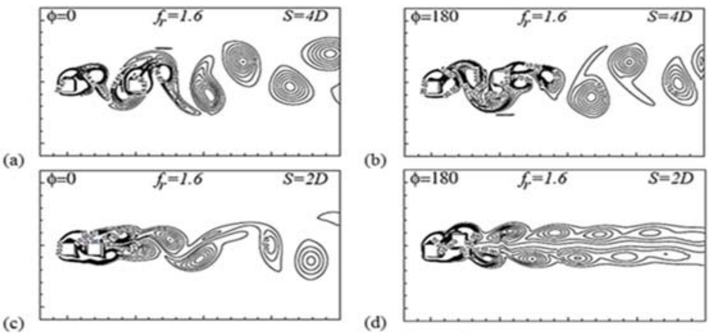

Fig. 12. Vorticity contours showing variation in shedding pattern with change in ϕ for selected spacing at super - synchronous frequency

particular frequency ratio is presented in Figs. 13 to 17. At sub-synchronous frequency (say f_r = 0.4), the mean drag and RMS lift coefficients of the upstream cylinder remain almost unchanged with change in phase difference for all the values of inter-cylinder spacing except S = 2D as shown in Figs. 13(a) and (c). On the other hand, the downstream cylinder shows variation in drag and lift values for all the inter-cylinder spacing except at S = 5D. It increases from minimum value at ϕ = 0^{o} (in-phase) and reaches a maximum at some highervalue of phase shift as shown in Figs 13(b) and (d). The value of phase shift at which the maximum drag and lift occur on the downstream cylinder varies with change of the inter-cylinder spacing. As an effect of increasing inter-cylinder spacing, a drastic increase in the value of drag and lift takes place at a particular spacing with little

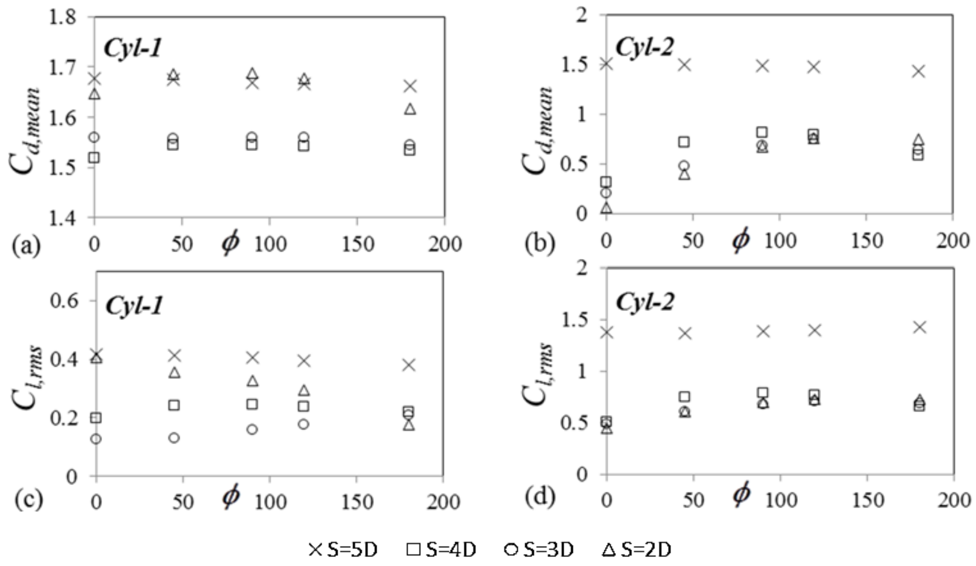

Fig. 13. Variation of lift and drag coefficients with phase difference for different *S* at f_r = 0.4

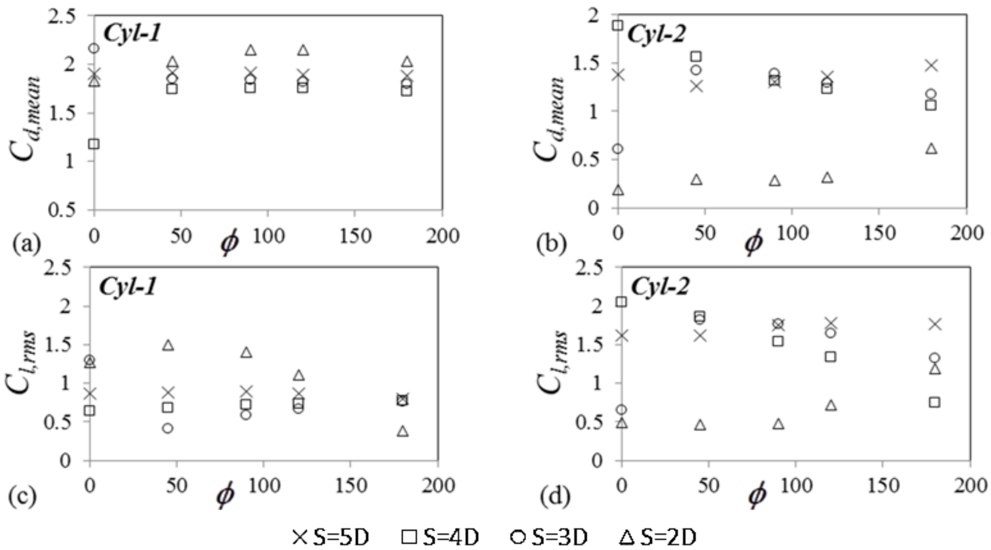

Fig. 14. Variation of lift and drag coefficients with phase difference for different *S* at f_r = 0.8

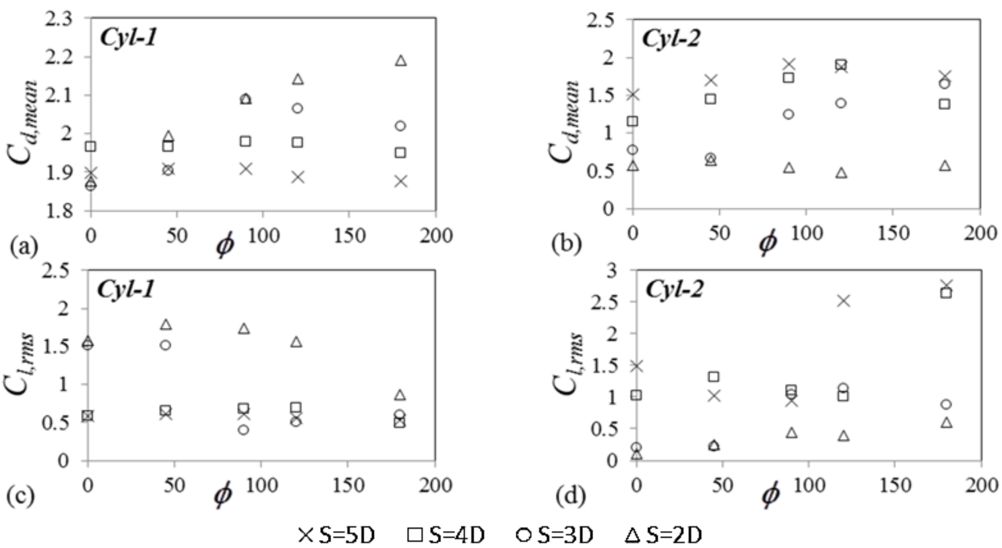

Fig. 15. Variation of lift and drag coefficients with phase difference for different *S* at f_r = 1

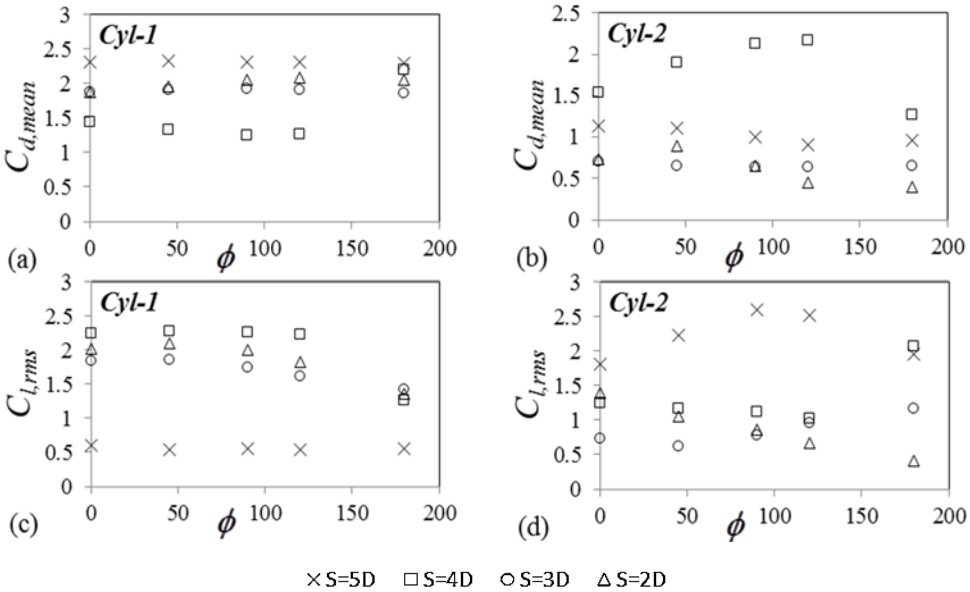

Fig. 16. Variation of lift and drag coefficients with phase difference for different *S* at f_r = 1.2

dependence on changes in phase difference. This can be attributed to the onset of vortex shedding at that particular inter-cylinder spacing which has been explained before for in-phase vibration. The upstream cylinder has higher drag and lower lift values than for the downstream cylinder for all the values of inter-cylinder spacing and phase difference.

Considerable variation in values of lift and drag coefficients is observed with change in phase angle for both the upstream and downstream cylinders in the synchronous regime (Figs. 14, 15 and 16). At f_r = 0.8, minimum lift and drag on the downstream cylinder are observed for the in-phase oscillation when *S* = 2D and 3D and for out-of-phase oscillation when *S* = 4D as shown in Figs. 14(b) and (d). A sudden increase in lift and drag on the downstream cylinder takes place between φ= 0° and 45° for cylinders at 3D separation. This is due to sudden change in the mode of vortex shedding from '2S' at φ = 0° to '2P' at φ = 45° at f_r = 0.8. At f_r = 1, the upstream cylinder also shows large deviation in lift and drag values with the increase in phase difference as shown in Figs. 15(a) and (c). A drastic increase in lift of the downstream cylinder takes place between φ = 90° and 120° associated with a change in vortex shedding mechanism for S = 4*D* and 5*D* at f_r = 1 (Fig. 15(d)). At f_r = 1.2, maximum drag on the downstream cylinder occurs at φ = 120° for *S* = 4*D* and minimum at φ = 180° for *S* = 2*D* as shown in Fig. 16(b).

At super synchronous frequency (f_r = 1.6), the drag coefficient of the upstream cylinder remains nearly invariant with phase difference as shown in Fig. 17(a). For downstream cylinder low values of drag are observed for all the phase shift values for *S* = 2*D* and 3*D* with negative drag appearing till φ = 120o for *S* = 2*D* (Fig. 17(b)). Similarly, for S = 3*D*, it shows minimum positive value till ϕ = 120° and then becomes negative at ϕ = 180°.

In general, there is no particular pattern observed in the variation of lift and drag coefficients with change in phase difference. The values of lift and drag are strongly dependent on the timing and position of the downstream cylinder when the vortices or shear layers from the upstream cylinder interact with the downstream cylinder. This interaction depends mainly on the inter-cylinder spacing, excitation frequency and the phase difference between the oscillating cylinders.

7. CONCLUSIONS

Numerical investigations on unsteady flow past two identical inline square cylinders oscillating transversely with phase difference have been carried out using finite volume based commercial software ANSYS Fluent 16.1 with the help of an inhouse developed user defined function to incorporate oscillations of the cylinders with phase difference. All the computations in the present study are performed for a fixed value of Re equal to 100. A brief summary of observations and inferences from this study are presented below.

- The width of frequency band corresponding to lock-in of oscillating tandem cylinders is more at close proximity and less when they are far apart, as compared to that for single oscillating cylinder.
- Transformation from a lock–in state to a non-lock-in state is observed when the phase difference between the cylinders is increased.
- For oscillating tandem cylinders, the lift and drag forces on both the cylinders are less than that of single oscillating cylinder for all the values of spacing and frequency ratio.

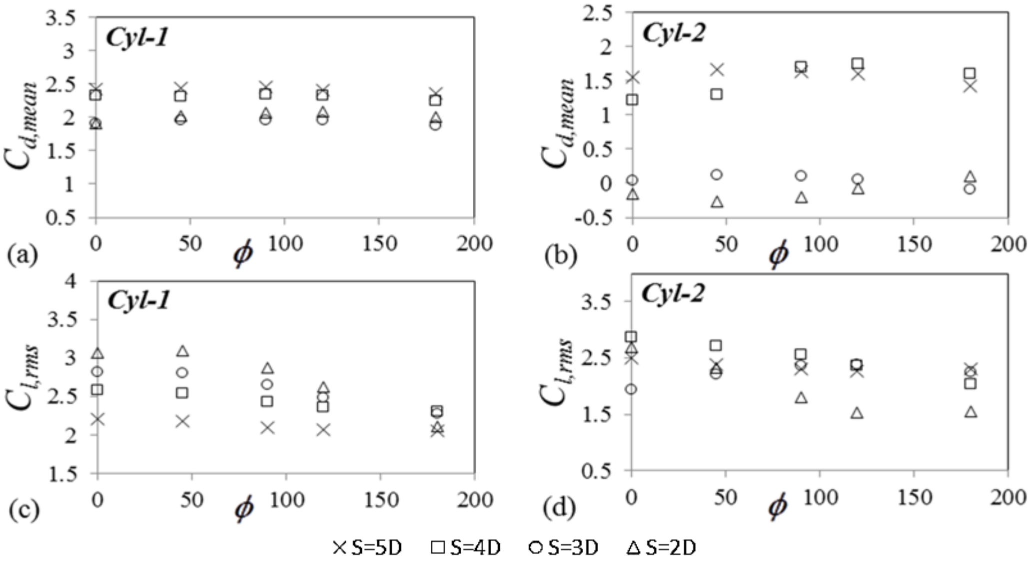

Fig. 17. Variation of lift and drag coefficients with phase difference for different *S* at f_r = 1.6

- When the oscillating cylinders are at close proximity, high frequency excitation results in negative drag on the downstream cylinder.
- With increase in inter-cylinder spacing or phase difference, change in mode of vortex shedding takes place followed by increase or decrease in lift and drag.

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