Journal of Applied Fluid Mechanics, Vol. 11, No. 5, pp. 1407-1419, 2018.
Available online at www.jafmonline.net, ISSN 1735-3572, EISSN 1735-3645.
DOI: 10.29252/jafm.11.05.28823

One-Dimensional Mathematical Model for Solar Drying of Beds of Sludge

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(Received January 25, 2018; accepted February 19, 2018)

ABSTRACT

A one-dimensional mathematical model for the description of solar drying of beds of sewage sludge is developed and implemented with the software Engineering-Equation-Solver. After the discussion of relevant literature, the assumptions and general conditions of the model are explained. Governing differential equations for heat and mass transport inside the sludge are derived, followed by a discussion about correlations used for the determination of surface transport phenomena and for the description of thermophysical properties. Numerical discretization is achieved locally through the Finite Difference Method and temporary through the Runge-Kutta-Method. Finally, a parametric analysis on the sludge drying process is carried out. The obtained results demonstrate the effect of ambient parameters such as solar radiation, airflow rate, gas temperature, geometric aspects etc. on the drying process. The developed model may be used for further prediction and estimation of drying characteristics under several conditions.

Keywords: Heat and mass transfer; Dryer performance; Mathematical model, Simulation; Porous material.

NOMENCLATURE

A	antoine parameter
b	width of sludge
B	antoine parameter
C	antoine parameter
c_p	isobaric heat capacity
D	diffusion coefficient
F_1	mass transfer factor
h	height
h_q	heat transfer coefficient
k_x	mass transfer coefficient (moisture)
l	length
m	mass
M	molar mass
Nu	Nusselt number
P	pressure
Pr	Prandtl number
$\dot{q}$	heat flux
R	gas constant
Re	Reynolds number
Sc	Schmidt number
Sh	Sherwood number
T	temperature
V	volume
w	velocity
X	moisture
Δ	difference
ε	emission coefficient
λ	thermal conductivity
υ	kinematic viscosity
ρ	density
σ	stefan-boltzmann constant
φ	relative humidity

Subscript

0	initial
conv	convective
D	dry
eff	effective
eva	evaporation
g	gas
H_2O	water
LV	liquid-vapor
sat	saturation

1. INTRODUCTION

During wastewater treatment processes, large amounts of sewage sludge are produced all over the world. Due to its high level of organic matter and richness in nutrients, valorization of sludge can be achieved through agricultural appliance or incineration in order to generate energy. Reduction of the moisture content of fresh sewage sludge is an important part of sludge processing, as it increases transport and storage efficiency as well as caloric value. How moisture contents achievable through industrial dewatering techniques are not sufficient for most following appliances, drying has become a necessary step for valorization of the biomass (Ficza 2010, Mujumdar 2014, Hassine, Chesneau and Laatar, 2017, Bennamoun, 2012).

Sewage sludge presents a real problem with the urban and industrial expanding. So, the drying technique is indispensable in the sludge treatment process to minimize its volume and its revalorization. For cost and environmental reasons, the solar drying is becoming increasingly attractive for small and medium wastewater treatment plants.

With conventional convective drying being highly demanding in energy, solar drying processes represent energy efficient alternatives. Mathematical models are valuable tool in evaluating the feasibility of solar drying of sewage sludge, as their efficiency highly depend on location dependent variables, such as solar irradiance and ambient air conditions.

Modeling and simulation is predominantly an essential tool for solving complex problems (Shrivastava, Kumar and Untawale, 2017). There are numerous advantages of using these techniques. Some of them are mentioned according to Shrivastava, Kumar and Untawale (2017):

a) Cost effective and consumes least time.

b) Complex and critical phenomenon can be analyzed easily.

c) Resources may be deployed to more critical parameters, so that effective and optimum use may be ensured.

d) Facilitates iteration through interpolation.

e) Deficiencies and discrepancies in construction may be identified.

In this paper a mathematical and numerical model of solar dryer of sludge was developed. The goal of this paper is the design of an approximate one-dimensional model of a solar drying process of sludge, in order to allow simulation and evaluation of the dynamics of solar drying at different locations in Brazil (and/or other places) in future works, through implementation of the model with the software Engineering-Equation-Solver - EES (Dash, 2014) and TRNSYS Simulation Studio program.

2. Model Design And Assumptions

The model derived in this report is based on a previous work by Ficza (2010). The existing models for numerical simulation of one-dimensional drying processes in porous substances has been well documented several times in the relevant literature (Mujumdar 2014). Crucial for the validity of the used model is the correct assumption of transport phenomena and characteristics of the material. Mainly the diffusive mass transport inside, and the convective mass transport at the surface, are challenging to simulate, due to the complex structure of the porous materials, which itself is dependent on the moisture content and the dynamics of the drying process at hand (Jurena, 2013). While drying takes place, moisture migration inside the material may occur by capillary forces as well as liquid and vapor diffusion (Font, Gomez-Rico and Fullana, 2011).

In recent years several papers concerning the convective drying kinetics of wastewater sewage sludge and porous material in general have been published (Reyes, Eckholt, Troncoso and Efremov, 2004; Font, Gomez-Rico and Fullana, 2011; Bennamoun, Crine and Léonard, 2013; Haghighi, Shahraeeni, Lehmann and Or, 2013; Bennamoun, Fraikin and Léonard, 2014; Huang, Chen and Jia, 2016). While some papers conclude that the moisture content of the drying material only affects the migration of moisture inside, newer studies link a decreased moisture content at the surface with a higher resistance for the mass transport from the surface to the atmosphere (Font, Gomez-Rico and Fullana, 2011; Bennamoun, Crine and Léonard, 2013; Haghighi, Shahraeeni, Lehmann and Or, 2013; Huang, Chen and Jia, 2016). Considering the complex and diverse structure of sewage sludge, the determination of universal empirical correlations for mass transfer, heat transfer and physical properties is difficult (Bennamoun, Crine and Léonard, 2013)

Bennamoun, Crine and Léonard (2013) conducted drying experiments for sewage sludge in a convective belt dryer and determined transport coefficients by applying a mathematical model. Even though the used sewage sludge stemmed from the same wastewater plant, they showed different behavior during the drying process. Similar experiments were conducted through by Reyes, Eckholt, Troncoso and Efremov (2004) for convective drying in a wind tunnel and by Font, Gomez-Rico and Fullana (2011), under similar conditions. The knowledge about the drying kinetics gained through these studies may be applied for mathematical modelling of solar sludge drying processes.

In any event, the applicability of determined parameters in literature for the model at hand has to be carefully evaluated, as the conditions under which the determinations were conducted differ greatly. Particularly relevant to this work are the determined transport coefficients by Bennamoun, Crine and Léonard (2013) as well as from Font, Gomez-Rico and Fullana (2011), which will be discussed further on in this paper.

A certain mass of wet sludge m_{sludge} is spread rectangular on a floor. The bed height h is assumed to be constant over the whole area A, which is defined by the width b and the length l of the bed. The sludge is considered to be a homogeneous material. The sludge itself consist of porous solid phase with the mass m_{solid} and water with the mass m_{H_2O}. While evaporation takes place, it is assumed that the mass of the solid phase, the total Volume of the sludge V, and therefore the bed height h remain constant.

At the surface of the bed a gas flow of air is streaming perpendicular over the bed with a velocity $v_{g,\infty}$ and a temperature $T_{g,\infty}$ as well as a relative humidity $\varphi_{g,\infty}$. The air is assumed to be an ideal gas. The composition of the air is constant.

The dynamics of the drying process as well as the unknown exact structure of the sludge complicate the determination of an accurate physical model for the occurring transport phenomena. If the water content of the sludge is high at the beginning of the drying process, it can be assumed that only a marginal amount of gas will be inside the porous solid material, as the inter-particle void volume will be occupied by water. As the drying process starts, water is evaporated through convective mass transfer at the surface of the sludge. But as the drying process progresses, the evaporating water leaves empty space in the porous material, which will then be filled by the surrounding gas.

Apart from diffusive mass transport of water in the sludge alongside the height of the bed, caused by the reduced moisture through evaporation at the surface, diffusive transport of water vapor in the gas phase has to be considered as well. Ficza (2010) proposes a two-phase model, in which governing equations for the sludge (containing solid and liquid water) as well as for the gas phase inside the sludge are formulated. Ficza (2010) assumes a static gas phase inside the sludge throughout the whole drying process, dependent on the porosity of the sludge. In this work, a simpler model will be developed, in which the gas phase inside the sludge is not included in the mathematical model. Through usage of an experimental determined effective diffusivity for calculations of moisture movement inside, the effects of vapor diffusion can be taken into consideration.

The concept of effective diffusivity is generally accepted to describe the moisture movement inside porous materials while drying (Reyes, Eckholt, Troncoso and Efremov, 2004). It is assumed that evaporation is only occurring at the surface ($\dot{m}^{eva}$) and that the only mass transfer inside the sludge is liquid water diffusion ($\dot{m}^{diff}$), following the gradient of moisture alongside the bed height due to evaporation at the surface. At the surface convective heat transfer with the surrounding gas ($\dot{q}^{cond}$), loss of heat due to evaporation ($\dot{q}^{eva}$), heat transfer due to thermal radiation ($\dot{q}^{rad}$) as well as heat transfer through solar radiation ($\dot{q}^{solar}$) is assumed. Inside the sludge only conductive heat transfer is occurring ($\dot{q}^{cond}$).

Additionally, a convective heat transfer at the bottom of the bed is considered ($\dot{q}^{conv,0}$). The exact nature of this heat transfer is yet to be defined. Heating could be implemented in different ways, for example through warm water running in steel pipes beneath the ground. The drying process with all occurring mass and heat transfers is depicted in Fig. 1.

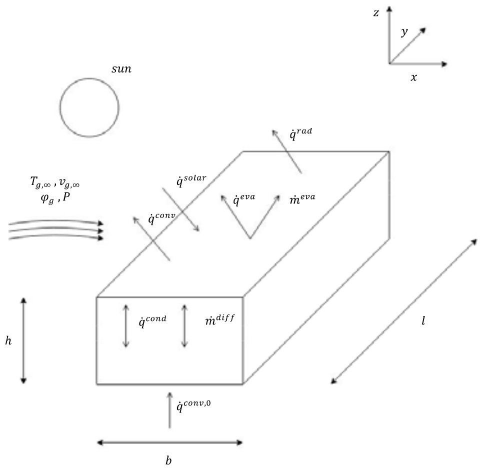

Fig. 1. Schematic diagram of the drying process.

3. MATHEMATICAL MODEL

Based on the conservation equations of mass and heat for the sludge a one-dimensional mathematical model is developed in this section. After presentation of the resulting differential equations, relevant correlations and equations for the determination of physical properties and transport coefficients are introduced and discussed.

3.1 Mass and energy balance

The conservation equations for mass and energy are determined for an infinitesimal control volume inside the sludge (as seen in Fig. 2), where only one-dimensional conductive heat transfer as well as diffusive mass transfer of liquid water are assumed. In the following sections the resulting differential equations will be discretized over the height of the bed. The heat and mass transfer at the surface and the heat transfer at the bottom of the bed will later be implemented in form of boundary conditions for the resulting differential equations.

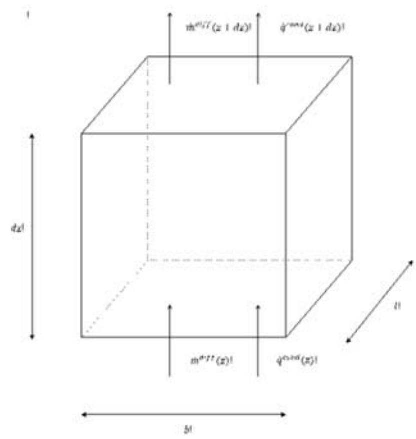

Fig. 2. Schematic diagram of control volume inside the sludge.

It is important to note that the control-volume considered in this model is assumed to contain both the solid porous sludge particles as well as the liquid water. It is assumed that the only occurring transfers of mass are the following:

i. Loss of water due to evaporation at the surface.

ii. Diffusion of water in the sludge caused by a concentration gradient alongside the height of the bed.

With Fick's law of diffusion applied for the description of the one-dimensional diffusive mass transfer, while using X, the moisture content of the sludge as the driving gradient for the diffusion and considering the density of the solid particles ρ_{solid}, the change in concentration of the water inside the control-volume can be expressed as several existing studies (Reyes, Eckholt, Troncoso and Efremov, 2004; Font, Gomez-Rico and Fullana, 2011, Mujumdar, 2014; Tzempelikos *et al.*, 2015; and Agrawal and Methekar, 2017):

$$\frac{\partial \rho_{solid} X}{\partial t} = \frac{\partial}{\partial z}\left(D_{eff}\,\rho_{solid}\,\frac{\partial X}{\partial z}\right) \quad (1)$$

With:

$$X = \frac{m_{H_2O}}{m_{solid}} \quad (2)$$

$$m_{sludge} = m_{H_2O} + m_{solid} \quad (3)$$

The density of the solid is calculated with the initial moisture content X_0:

$$\rho_{solid} = \frac{m_{sludge}}{(1+X_0)V} \quad (4)$$

And D_{eff} being the efficient diffusion coefficient for water in the sludge. The volume V is considered to be a constant. As the density of the solid is considered constant (no change in Volume and no mass diffusion of solids) the equation can be rearranged as:

$$\frac{\partial X}{\partial t} = \frac{1}{\rho_{solid}}\frac{\partial}{\partial z}\left(\rho_{solid} D_{eff}\,\frac{\partial X}{\partial z}\right) \quad (5)$$

For heat transfer, respectively:

i. Conductive heat transport through the sludge, dependent on the temperature gradient alongside the height;

ii. Convective heat transfer between the gas phase and the solid phase;

iii. Convective heat transfer at the bottom of the bed;

iv. Loss of energy caused by the evaporating water;

v. Loss of energy through thermal radiation;

vi. Heat influx through solar radiation.

A constant pressure is assumed throughout the drying process. For solids, the pV-term of the internal energy U_{Sludge} is marginal in comparison to the enthalpy term H_{Sludge} and is neglected in this model. With Fourier's law of thermal conduction this leads to the following energy balance, considering the convective and radiation terms as boundary conditions. More information can be found in Mujumdar (2014), Tzempelikos *et al.* (2015), Agrawal and Methekar (2017):

$$\frac{\partial U_{sludge}}{\partial t} = \frac{\partial (H-pV)_{sludge}}{\partial t} \approx \frac{\partial H_{sludge}}{\partial t}$$

$$\frac{\partial\left((\rho_{solid}c_{p,solid}+\rho_{solid}c_{p,H_2O}X)\,T\right)}{\partial t} = \frac{\partial}{\partial z}\left(\lambda_{eff}\,\frac{\partial T}{\partial z}\right) \quad (6)$$

With λ_{eff} being the effective thermal conductivity of the sludge, which will be defined later.

3.2 Boundary condition heat and mass transfer

In this section, the equations concerning the convective heat and mass transfer as well as the radiative heat transfer at the surface will be defined. After introducing the general nature of the boundary conditions the exact calculation of transport coefficients will be discussed.

The mass transfer flux due to evaporation at the surface can be described in a general form as the product of a mass transfer coefficient k and a gradient of chemical potential, which may be expressed through a difference in concentration, pressure or moisture. For description of the convective mass transfer through the boundary layer the same model as in the paper from Font, Gomez-Rico and Fullana (2011) is used. The evaporation rate depends on the mass transfer coefficient k_x and the gradient in gas-phase humidity at the surface of the sludge $X_g\big|_{z=h}$ and humidity of the surrounding air $X_{g,\infty}$. The moisture migrating to the surface of the bed through diffusion is transferred to the surrounding gas phase. This can be expressed as the following boundary condition. More information can be found in Mujumdar (2014), Tzempelikos *et al.* (2015) and Agrawal and Methekar (2017):

$$\rho_{solid}\, D_{eff}\left.\frac{\partial X}{\partial z}\right|_{z=h} = -\dot{m}^{eva}_{H_2O} = -k_x\left(X_g\big|_{z=h} - X_{g,\infty}\right) \quad (7)$$

The boundary condition for the occurring heat transfers at the surface can be generally written as:

$$\lambda_{eff}\left.\frac{\partial T}{\partial z}\right|_{z=h} = \dot{q}^{conv} + \dot{q}^{solar} - \dot{q}^{rad} - \dot{q}^{eva} \quad (8)$$

Sorption heat is neglected in this model. The heat influx through solar radiation $\dot{q}^{solar}$ is assumed to be a constant user input in the later simulation. Analogously to the convective mass transfer flux the convective heat transfer flux can be expressed through a heat transfer coefficient h_q, and the temperature gradient at the surface, defined by the temperature of the sludge $T|_{z=h}$ at the surface and the temperature $T_{g,\infty}$ of the overflowing air:

$$\dot{q}^{conv} = h_q(T_{g,\infty} - T|_{z=h}) \quad (9)$$

The loss of energy due to evaporation can be expressed as:

$$\dot{q}^{eva} = \dot{m}^{eva}_{H_2O}\,\Delta h^{LV}_{H_2O} = k_x\left(X_g\big|_{z=h} - X_{g,\infty}\right)\Delta h^{LV}_{H_2O} \quad (10)$$

With $\Delta h^{LV}_{H_2O}$ being the specific enthalpy of evaporation for water.

Heat loss flux through thermal radiation can be calculated with the Stefan-Boltzmann law. Thermal radiation of the surrounding air is neglected.

$$\dot{q}^{rad} = \varepsilon_{sludge}\,\sigma\,(T|_{z=h}{}^4 - T_{g,\infty}{}^4) \tag{11}$$

Defining parameters are the Stefan-Boltzmann constant σ and the emissivity coefficient of the sludge ε.

At the bottom of the bed no mass transfer is occurring in this model. In terms of a boundary condition this can be expressed as:

$$\rho_{solid}\,D_{eff}\frac{\partial X}{\partial z}\Big|_{z=0} = 0 \tag{12}$$

The heat transfer at the bottom of the bed will for now be assumed to be a constant heat flux defined by the user. The corresponding boundary condition:

$$\lambda_{eff}\frac{\partial T}{\partial z}\Big|_{z=0} = -\dot{q}^{conv,0} \tag{13}$$

3.3 Transport coefficients and physical properties

In general, heat and mass transfer coefficients are dependent on the thermo-physical properties of the components involved, the flow characteristics as well as the geometry of the system. In practice, the transfer coefficients can be obtained by theoretical calculations or empirical correlations. For comparison of systems with similar characteristics groups of dimensionless numbers are used. In order to yield satisfying results through the calculation of transfer coefficients with dimensionless numbers, the comparability of the system at hand and the system for which the values or correlations where obtained has to be given.

In any case, the applicability of determined parameters in literature for the model at hand has to be carefully evaluated, as the conditions under which the determinations were conducted differ greatly. For example, the correlation used by Ficza (2010) for calculation of the mass transfer coefficient describing the evaporation at the surface stems from a table given in Mujumdar (2014). It is stated that the correlation was obtained for a solid droplet spray-drying system. The use of the correlation for the determination of the Sherwood number is therefore questionable, as the solid-bed drying model of sludge in this work differs greatly to that of a spray dryer, in which small droplets of solid are dried through turbulent gas flow (Bennamoun, Fraikin and Léonard, 2014).

The product of effective diffusivity and solid density inside the sludge will be calculated with a correlation from Font, Gomez-Rico and Fullana, (2011). Other correlations exist, but often they are determined at high temperatures, while this correlation was obtained through convective drying experiments in the temperature range of 30 – 60 °C with two different types of sewage sludge. The correlation considers the temperature of the sludge as well as the moisture content, as the diffusivity changes in dependence of the amounts of gas and water inside the porous material:

$$\rho_{solid}\,D_{eff} = \left(0.140 - 0.0946\frac{X}{X_0}\right)e^{\left(-\frac{3245}{T}\right)} \tag{14}$$

With X_0 being the initial moisture content.

As recent research suggests the evaporation and heat transfer rate at the surface seem to not only be functions of the temperature and the properties of the drying air, but of the moisture content at the surface (Font, Gomez-Rico and Fullana, 2011; Bennamoun, Crine and Léonard, 2013; Haghighi, Shahraeeni, Lehmann, and Or, 2013; Huang, Chen and Jia, 2016).

Derived from the same experiments discussed in the last paragraph, Font, Gomez-Rico and Fullana (2011) formulate a correlation for the description of the evaporative mass transfer at the surface as well. While the mass transfer coefficient is calculated through the well-established means of the dimensionless numbers Sh, Re and Sc, two correcting factors F_1 and F_2 are introduced, which consider the deviation from the calculated coefficient to the obtained experimental data. F_1 considers the fact that the outer surface dries out during the process, which causes the formation of a solid skin, resulting in a decreased evaporation rate. The factor F_2 amounts for shrinkage and a decrease in transfer Area. As shrinkage is neglected in this model, the factor F_2 will not be considered.

$$k_c = \frac{Sh\,D_{H_2O,g}}{l}\,F_1 \tag{15}$$

The mass transfer coefficient k_c describes the occurring mass transfer as a function of a concentration gradient. Assuming ideal gas behavior, the coefficient can be related to the gradient in humidity in the following way (Font, Gomez-Rico and Fullana, 2011). Determining factors are the Sherwood number Sh, the diffusivity of water vapor in air $D_{H_2O,g}$, the total pressure P, the molar weight of the humid air M_g, the characteristic length of the surface b as well as the temperature at which the mass transfer is occurring. It is assumed, that this temperature can be described as the arithmetic mean value of the sludge surface temperature and the temperature of the surrounding gas $\bar{T}_g$.

$$k_x = \frac{D_{H_2O,g}\,Sh\,M_g\,P}{b\,R\,\bar{T}_g}\,F_1 \tag{16}$$

$$\bar{T}_g = \frac{T_{g,\infty} + T|_{z=h}}{2} \tag{17}$$

The factor F_1 is dependent on the moisture content at the surface $X|_{z=h}$ the initial moisture content $X_0|_{z=h}$, the equilibrium moisture content X_e and a factor n1.

$$F_1 = \left(\frac{X|_{z=h} - X_e}{X_0|_{z=h} - X_e}\right)^{n1} \tag{18}$$

Font, Gomez-Rico and Fullana (2011) determined the factor n1 according to their experimental data to be:

a) For a sludge sample where only slight skin forming was observed: n1 = 0.3.

b) For a sample where the formation of strong skin was observed: $n1 = 1.3$.

Both values will be implemented as options in the model.

The calculation of the equilibrium moisture content is based on the correlation (derived through fitting of the experimental data by Font, Gomez-Rico and Fullana, 2011). The adopted correlation was able to reproduce experimental data of other authors with satisfactory precision:

$$X_e = 0.11 \frac{\varphi_g\, 0.84\, e^{3336\left(\frac{1}{T_{g,\infty}} - \frac{1}{303.15}\right)}\, 60.5}{\left(1 - \varphi_g e^{3336\left(\frac{1}{T_{g,\infty}} - \frac{1}{303.15}\right)}\right)\left(1 - \varphi_g e^{3336\left(\frac{1}{T_{g,\infty}} - \frac{1}{303.15}\right)}(1 - 60.5)\right)} \tag{19}$$

With φ_g being the relative humidity of the air.

The molar mass of the air can be expressed as a function of the relative air humidity, the molar mass of dry air $M_{D,g}$ and water M_{H_2O} and the vapor pressure of water at the airs temperature $P^{sat}_{H_2O}(T_{g,\infty})$, assuming ideal gas behavior and applying Dalton's law:

$$M_g = \frac{\left(P - \varphi_g P^{sat}_{H_2O}(T_{g,\infty})\right)}{P} M_{D,g} + \frac{\varphi_g P^{sat}_{H_2O}(T_{g,\infty})}{P} * M_{H_2O} \tag{20}$$

Under the same assumption the humidity in the surrounding gas-phase can be calculated:

$$X_{g,\infty} = \frac{\varphi_g P^{sat}_{H_2O}(T_{g,\infty})}{(P - \varphi_g P^{sat}_{H_2O}(T_{g,\infty}))} \frac{M_{H_2O}}{M_{D,g}} \tag{21}$$

It is assumed that the gas phase humidity at the surface of the sludge is saturated with water:

$$X_g\big|_{z=h} = \frac{P^{sat}_{H_2O}(T|_{z=h})}{(P - P^{sat}_{H_2O}(T|_{z=h}))} \frac{M_{H_2O}}{M_{D,g}} \tag{22}$$

The vapor pressure is calculated with the classic Antoine equation:

$$P^{sat}_{H_2O}\left([T_{g,\infty}, T|_{z=h}]\right) = 10^{A - \frac{B}{T+C}} \tag{23}$$

The parameters are chosen accordingly to the temperature at hand and are taken from Bridgeman, and Aldrich (1964).

The diffusivity of water vapor in air at low temperatures can be described by the following empirical correlation in the temperature Range of 0°C – 100 °C (Marrero and Mason, 1972). To describe the diffusivity in the boundary layer the arithmetic temperature average of sludge surface and gas phase will be used Marrero and Mason (1972):

$$D_{H_2O,g} = 1.97 \cdot 10^{-5} \frac{1.01325 \cdot 10^5\ Pa}{P} \left(\frac{\overline{T}_g}{256\ K}\right)^{1.685} \tag{24}$$

The mean Sherwood number is calculated by given correlations from the literature for the parallel flow over a plate, depending of the flow regime (turbulent or laminar) and the corresponded Reynolds number and Schmidt number (Çengel and Boles, 2008; Bergman, Lavine and Incropera, 2011; Kraume, 2012):

For turbulent regime:

$5 \cdot 10^5 < Re < \infty$

$0.8 < Sc < \infty$

$$Sh = \frac{0.037\, Re^{0.8}\, Sc}{1 + 2.44\, Re^{-0.1}(Sc^{2/3} - 1)} \tag{25}$$

And for laminar regime respectively:

$0 < Re \leq 5 \cdot 10^5$

$0 < Sc < \infty$

$$Sh = 0.8(Re\, Sc)^{0.1} + \frac{1.47}{[1 + (1.67\, Sc^{1/6})^2]^{1/2}} \frac{Re\, Sc}{1 + 1.30\, (Re\, Sc)^{1/2}} \tag{26}$$

With the equations of Reynolds and Schmidt number:

$$Re = \frac{w_{g,\infty}\, b}{\upsilon_g} \tag{27}$$

$$Sc = \frac{\upsilon_g}{D_{H_2O,g}} \tag{28}$$

Tsilingiris (2008) found only marginal influence of the humidity of air on the viscosity in low temperature ranges. Therefore, the viscosity of the air in the boundary layer will be calculated through linear interpolation of experimental values for dry air given in VDI Heat Atlas (2010) at atmospheric pressure in the temperature range of 0 °C – 50°C:

$$\upsilon_g = [135 + 0.904\,(\overline{T}_g - 273.15)]\, 10^{-7} \frac{m^2}{s} \tag{29}$$

The thermal conductivity of the sludge is highly dependent on the moisture content. While it is close to the conductivity of water at high moisture contents, it decreases significantly as drying progresses. For the calculation another empirical correlation from Font, Gomez-Rico and Fullana (2011) is used.

$$\lambda_{eff} = A + B\, X^2 \tag{30}$$

The parameter A represents the conductivity at equilibrium moisture, while B is the proportionality factor for the moisture content at hand. Font, Gomez-Rico and Fullana (2011) determined the parameter B to be in the range of 0.004 – 0.011 $\frac{W}{m\,K}$. For the calculation the arithmetic mean value will be used.

$$\lambda_{eff} = 0.03 + 0.0075 X^2 \frac{W}{mK} \tag{31}$$

On the other hand, correlations for the convective heat transfer at the surface dependent on the moisture content exist, they have often been determined through drying experiments at higher temperatures (Reyes, Eckholt, Troncoso and Efremov, 2004; Huang, Chen and Jia, 2016).

Noteworthy is especially the work of Huang, Chen and Jia (2016), in which such a correlation is derived through sludge drying experiments in the 100 °C – 160 °C temperature range. But as the calculated Nusselt numbers show deviations up to more than 25% to the experimental data at considered temperatures, the applicability to the model at hand is questionable, as drying air temperatures for solar drying are considerably lower. As the heat flux for solar drying is primarily influenced by solar radiation, it is assumed that the description of the convective heat transfer through the dimensionless numbers *Nu, Re* and *Pr* for a flat plate parallel flow is sufficient for this model. The correlations are taken from VDI Heat Atlas (2010):

For turbulent regime:

$5 \cdot 10^5 < Re < 10^7$

$0.5 < Pr < 2000$

$$\mathrm{Nu} = \frac{0.037\,\mathrm{Re}^{0.8}\,\mathrm{Pr}}{1 + 2.443\,\mathrm{Re}^{-0.1}(\mathrm{Pr}^{2/3} - 1)} \tag{32}$$

And for laminar regime respectively:

$0 < Re \leq 5 \cdot 10^5$

$0.5 < Pr < \infty$

$$\mathrm{Nu} = 0.664\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3} \tag{33}$$

With the equations for the Prandtl and Nusselt number:

$$h_q = \frac{\mathrm{Nu}\,\lambda_g}{b} \tag{34}$$

$$\mathrm{Pr} = \frac{\upsilon_g\,\rho_g\,c_{p,g}}{\lambda_g} \tag{35}$$

With λ_g being the thermal conductivity, ρ_g the density of the gas phase and $c_{p,g}$ the isobaric heat capacity of the humid air in the boundary layer. As for the viscosity, the influence of humidity on the thermal conductivity at atmospheric pressure and low temperatures is marginal. Therefore, the thermal conductivity of dry air will be used, linearly interpolated from experimental data given in VDI Heat Atlas (2010) in the 0 °C - 50 °C temperature range.

$$\lambda_g = 0.02436 + 0.0000744\,(\overline{T}_g - 273.15)\,\frac{W}{m\,K} \tag{36}$$

The density of the humid air in the boundary layer can be expressed as a function of moisture content and temperature in the following way, assuming ideal gas behavior (Mujumdar, 2014):

$$\rho_g = \frac{P}{R_{D,g}\,\overline{T}_g}\,\frac{1 + \overline{X}_g}{1 + \overline{X}_g \frac{R_{H_2O}}{R_{D,g}}} \tag{37}$$

Where $\overline{X}_g$ is the arithmetic mean value of the moisture content at the surface of the sludge and in the surrounding air:

$$\overline{X}_g = \frac{X_g|_{z=h} + X_{g,\infty}}{2} \tag{38}$$

And the specific gas constants of dry air $R_{D,g}$ and water vapor R_{H_2O} (Mujumdar, 2014):

$$R_{H_2O} = 461.5\ \frac{J}{kg\,K}$$

$$R_{D,g} = 286.8\ \frac{J}{kg\,K}$$

At low temperatures (0 °C – 50 °C) the specific heat capacity of moist air can be approximated in the following way by Çengel and Boles (2008).

$$c_{p,g} = 1005 + 1820\,\overline{X}_g\ \frac{J}{kg\,K} \tag{39}$$

Vaxelaire and Puiggali (2002) suggest that the influence of temperature on the heat capacity of dried sewage sludge in the range of 50 °C – 90 °C is marginal and can be considered constant. The proposed value is similar to the determined value through simulation of experiments by Font, Gomez-Rico and Fullana (2011) at lower temperature ranges.

$$c_{p,solid} = 1350\ \frac{J}{kg\,K} \tag{40}$$

For water respectively (Font, Gomez-Rico and Fullana, 2011):

$$c_{p,H_2O} = 4180\ \frac{J}{kg\,K} \tag{41}$$

The heat of evaporation is calculated by the Regnault equation at the averaged boundary layer temperature (Font, Gomez-Rico and Fullana, 2011):

$$\Delta h^{LV}_{H_2O} = 2540000 - 2910\,(\overline{T}_g - 273.15)\,\frac{J}{kg\,K} \tag{42}$$

As emissivity coefficients for sludge are not published in the relevant literature, the emissivity coefficient of soil is used for the description of the thermal radiation heat loss instead (VDI Heat Atlas, 2010):

$$\varepsilon = 0.9 \tag{43}$$

4. NUMERICAL METHOD AND IMPLEMENTATION

In this section the numerical discretization required for the solution of the governing differential equations is discussed. The implementation of the equation system in EES is explained and input parameters for the calculation are defined.

4.1 Local discretization

For local discretization of the partial differential equations of heat and mass transfer the method of lines is applied. The computational domain is divided into equidistant elements alongside the height of the sludge and the governing equations of mass and heat transfer are applied for each element respectively. The gradients of moisture and temperature are approximated by the central difference quotient of second order for the length of one element Δz. The length of one element Δz is defined through the number of elements n and the height of the sludge:

$$\Delta z = \frac{h}{n} \tag{44}$$

The locally discretized differential equation describing the moisture of an inner element i is derived as follows:

$$\left.\frac{\partial X}{\partial t}\right|_i = \frac{1}{\rho_{solid}}\left[\frac{\partial}{\partial z}\left(\rho_{solid}D_{eff}\frac{\partial X}{\partial z}\right)\right]_i =$$

$$\frac{1}{\rho_{solid}}\frac{\rho_{solid}D_{eff}|_{i+\frac{1}{2}}\left.\frac{\partial X}{\partial z}\right|_{i+\frac{1}{2}} - \rho_{solid}D_{eff}|_{i-\frac{1}{2}}\left.\frac{\partial X}{\partial z}\right|_{i-\frac{1}{2}}}{\Delta z} =$$

$$\frac{1}{\rho_{solid}}\frac{\rho_{solid}D_{eff}|_{i+\frac{1}{2}}\frac{X|_{i+1}-X|_i}{\Delta z} - \rho_{solid}D_{eff}|_{i-\frac{1}{2}}\frac{X|_i - X|_{i-1}}{\Delta z}}{\Delta z} \tag{45}$$

As the equations are evaluated at the border of the elements, the arithmetic mean value of the product of effective diffusivity and density as well as of the thermal conductivity are used to describe heat and mass transfer over the border of the elements.

$$\rho_{solid}D_{eff}|_{i+\frac{1}{2}} = \frac{\rho_{solid}D_{eff}|_i + \rho_{solid}D_{eff}|_{i+1}}{2}$$

$$\rho_{solid}D_{eff}|_{i-\frac{1}{2}} = \frac{\rho_{solid}D_{eff}|_i + \rho_{solid}D_{eff}|_{i-1}}{2}$$

$$\left.\frac{\partial X}{\partial t}\right|_i = \frac{1}{\rho_{solid}}\frac{\frac{\rho_{solid}D_{eff}|_i + \rho_{solid}D_{eff}|_{i+1}}{2}\frac{X|_{i+1}-X|_i}{\Delta z} - \frac{\rho_{solid}D_{eff}|_i + \rho_{solid}D_{eff}|_{i-1}}{2}\frac{X|_i - X|_{i-1}}{\Delta z}}{\Delta z} \tag{46}$$

For the surface element n, boundary conditions are considered in the following way:

$$\left.\frac{\partial X}{\partial t}\right|_n =$$

$$\frac{1}{\rho_{solid}}\frac{\rho_{solid}\,D_{eff}\left.\frac{\partial X}{\partial z}\right|_{z=h} - \rho_{solid}D_{eff}|_{n-\frac{1}{2}}\left.\frac{\partial X}{\partial z}\right|_{n-\frac{1}{2}}}{\Delta z} =$$

$$\frac{1}{\rho_{solid}}\frac{-k_x\left(X_g|_{z=h} - X_{g,\infty}\right) - \frac{\rho_{solid}D_{eff}|_n + \rho_{solid}D_{eff}|_{n-1}}{2}\frac{X|_n - X|_{n-1}}{\Delta z}}{\Delta z} \tag{47}$$

For the ground element 1 respectively:

$$\left.\frac{\partial X}{\partial t}\right|_1 = \frac{1}{\rho_{solid}}\frac{\frac{\rho_{solid}D_{eff}|_1 + \rho_{solid}D_{eff}|_2}{2}\frac{X|_2 - X|_1}{\Delta z}}{\Delta z} \tag{48}$$

Applying the same discretization for the energy balance yields for an inner element i:

$$\left.\frac{\partial\left(\left(\rho_{solid}c_{p,solid} + \rho_{solid}c_{p,H_2O}X\right)T\right)}{\partial t}\right|_i =$$

$$\frac{\frac{\lambda_{eff}|_i + \lambda_{eff}|_{i+1}}{2}\frac{T|_{i+1} - T|_i}{\Delta z} - \frac{\lambda_{eff}|_i + \lambda_{eff}|_{i-1}}{2}\frac{T|_i - T|_{i-1}}{\Delta z}}{\Delta z} \tag{49}$$

For the surface element n:

$$\left.\frac{\partial\left(\left(\rho_{solid}c_{p,solid} + \rho_{solid}c_{p,H_2O}X\right)T\right)}{\partial t}\right|_n =$$

$$\frac{\dot{q}^{conv} + \dot{q}^{solar} - \dot{q}^{rad} - \dot{q}^{eva} - \frac{\lambda_{eff}|_n + \lambda_{eff}|_{n-1}}{2}\frac{T|_n - T|_{n-1}}{\Delta z}}{\Delta z} \tag{50}$$

And for the ground element 1:

$$\left.\frac{\partial\left(\left(\rho_{solid}c_{p,solid} + \rho_{solid}c_{p,H_2O}X\right)T\right)}{\partial t}\right|_1 =$$

$$\frac{\dot{q}^{conv,0} + \frac{\lambda_{eff}|_1 + \lambda_{eff}|_2}{2}\frac{T|_2 - T|_1}{\Delta z}}{\Delta z} \tag{51}$$

4.2 Temporary discretization

The resulting ordinary differential equations through appliance of the method of lines are solvable with an adequate iterative method for temporary discretization. In EES calculation of ordinary differential equation initial value problems can be done by implemented Explicit-Euler-Method or Crank-Nicholson-Method, or with self-written functions. While implicit or semi-implicit methods like the Crank-Nicholson-Method offer higher stability and accuracy when function values are rapidly changing, the computational effort is higher, as for every calculated time step an nonlinear equation systems has to be solved.

Additionally, EES mathematical functions are somewhat limited, as the user has no influence on how the equation systems in the implemented methods are solved. It can be assumed that during the drying process rapid changes in process conditions and physical properties do not occur.

Therefore, the use of an explicit method is reasonable. For temporary discretization, the well-established Finite Difference Method and Runge-Kutta-4-Method are used. The algorithm is implemented by self-written functions in EES.

Apart from the temperature, the derived energy balance contains terms that are time dependent as well, namely the heat capacity of the sludge and therefore the moisture content itself. Considering the change of heat capacity through moisture migration in the differential equation while not considering transfer of heat through migrating moisture would lead to physical inconsistent results. As the heat transfer through migration is neglected, the heat capacity will be considered constant over evaluated time steps Δt.

The energy balance for an inner element i may then be written as:

$$\left.\frac{\partial T}{\partial t}\right|_i = \frac{1}{(\rho_{solid} c_{p,solid} + \rho_{solid} c_{p,H_2O} X)} \frac{\frac{\lambda_{eff}|_i + \lambda_{eff}|_{i+1}}{2} \frac{T|_{i+1} - T|_i}{\Delta z} - \frac{\lambda_{eff}|_i + \lambda_{eff}|_{i-1}}{2} \frac{T|_i - T|_{i-1}}{\Delta z}}{\Delta z} \quad (52)$$

4.3 Implementation

The discretized mathematical model is implemented in EES-Code. The time step for the Runge-Kutta-Method, the total time considered, as well as the initial temperatures and moistures in the sludge elements are required. The number of elements n is initially set to 20, but may be changed by the user. After calculation the values for temperature and moisture inside the cells, as well as average moisture and temperature inside the sludge are displayed in the screen. Additionally, plots of the temperature and moisture over the bed height can be accessed through implemented buttons.

4.4 Validation of the model

The model validation was accomplished through comparison of the simulated results with laboratory measurements of drying of sludge obtained from an activated sludge effluent treatment plant. Piles of the sludge were built with initial mass of (150,2 ± 0.3) kg, 2.0 m of length and 27 cm of height. The hot gas flow of (0.64 ± 0.02) $m^3.s^{-1}$ at a temperature of (105 ± 5)°C, provided by the direct coal-fired furnace was used as drying gas. The system operated during 5 hours. During the drying system operation, the sludge pile temperatures were measured at 15 points, every 30 minutes, using HIGHMED HM-600 thermometers, located at the top, middle and bottom of the pile. The moisture content was monitored using a moisture analyzer OHAUS MB45 at 15 points, every 60 minutes. Simulation results were compared with experimental data for temperature and moisture content profiles, as shown in Figs. 3 and 4.

Both simulated temperature and moisture were in agreement with experimental data, even though the experimental temperature values were a little scattered. This comparison indicates that the model Figures 5 to 16 show results of the parametric simulation program for the following reference conditions: 1) physical conditions of the sludge bed: wet sludge mass m_{sludge} = 40,000 kg, initial sludge moisture X = 5, initial sludge temperature T = 290 K; 2) geometric conditions: bed height h = 0.5 m; bed width b = 20 m, bed length l = 20 m; 3) climatic conditions: gas velocity $v_{g,\infty} = 1$ m/s, gas temperature $T_{g,\infty}$ = 290 K, relative humidity $\varphi_{g,\infty}$ = 0.8, solar radiation $\dot{q}^{solar}$ = 150 W/m^2; 4) simulation conditions: time step Δt = 60 s.

The numerical calculations were found to be stable and required relatively small computational effort.. It is clear the sludge moisture decrease with increasing time of drying. A number of general observations can be made from Figs. 5 to 16.

Figures 5, 6 and 7 show the influence of the solar energy on the drying process. As expected, higher drying efficiency is achieved at greater levels of solar radiation, caused by the corresponding increasing water evaporation rates at higher surface temperatures.

The influence of the heated gas temperature on the moisture content is represented in Fig. 8. Similar to the effect of solar radiation, it can be noticed that an increase in gas temperatures causes an acceleration of the drying process due to higher evaporation

rates. This could explained by the facts: a) higher heated gas temperatures contribute to higher vapor pressures in the sludge mass, and producing higher drying gradients; and b) increase the gas temperature decreases the heat of vaporization of water, thus lowering the amount of heat energy needed to vaporize the moisture. The effect on the equilibrium moisture content is marginal.

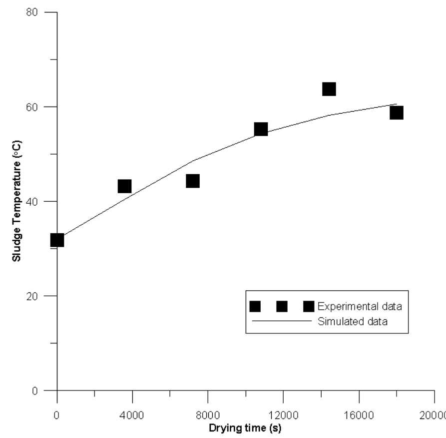

Fig. 3. Variation of the sludge temperature with respect to drying time.

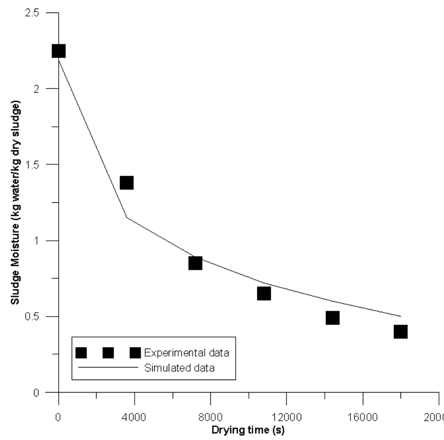

Fig. 4. Variation of the sludge moisture with respect to drying time.

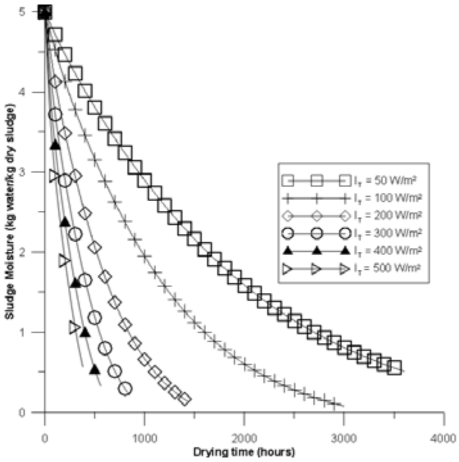

Fig. 5. Variation of the sludge moisture with solar radiation.

Figures 9 and 10 show the influence of the relative humidity of the drying air on the sludge moisture content. An increase in humidity leads to higher equilibrium moisture contents throughout the drying process, due to higher diffusion resistances and therefore inhibition of the water evaporation rate. Additionally, lower equilibrium moisture contents correspond to lower relative humidity.

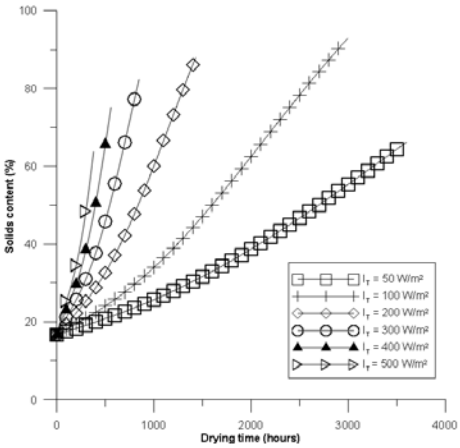

Fig. 6. Variation of the solids content with solar radiation.

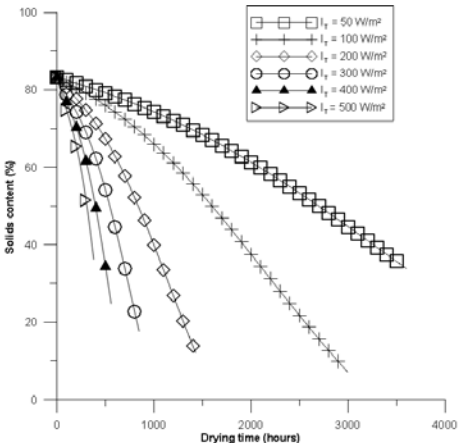

Fig. 7. Variation of the moisture content with solar radiation.

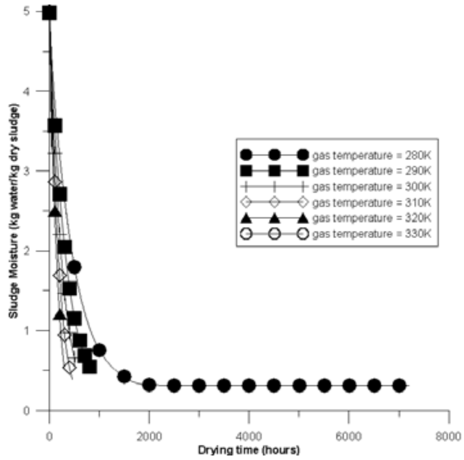

Fig. 8. Variation of the moisture content with gas temperature $T_{g,\infty}$.

Figure 11 illustrates the effect of the gas velocity combined with the relative humidity of the drying air. Considering the results, it appears that the relative humidity has a greater influence on the drying kinetics opposed to the gas velocity. The drying gas flow compared to the sludge mass is so large that any changes in the condition of the air as it passes through the sludge are not so significant. Under given conditions the equilibrium moisture content is reduced by a factor of approximately 50 % through a decrease of relative humidity from 0.8 to 0.5 and subsequently from 0.5 to 0.2. For higher gas velocities drying times are smaller than 2,000 hours.

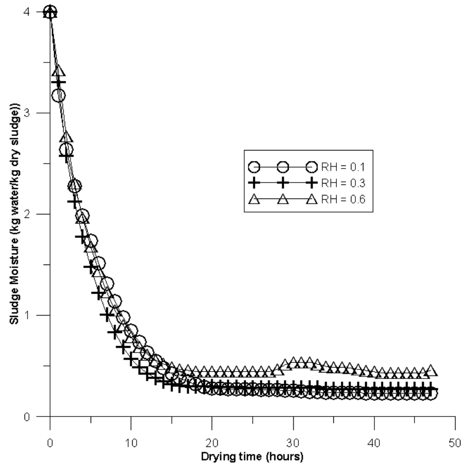

Fig. 9. Variation of the sludge moisture with of the relative humidity of the air drying.

The simulated results are in agreement to previous works on sludge drying and other products drying (Ilic and Turner, 1986; Bennamoun, Belhamri, 2003, Font, Gomez-Rico and Fullana, 2011; Oueslati, Mabrouk and Mami, 2012; Hassine, Chesneau and Laatar, 2017; Younis, Abdelkarim and El-Abdein, 2017).

Figures 12 and 13 show the influence of the height of the sludge bed on the sludge moisture at the drying process. Increasing the height of the sludge bed leads to more resistance for diffusion and the evaporation of the water will be more difficult. This result was already expected, since the increase in the thickness / height of the sludge layer implies an the increase of the internal diffusional distance within the sludge, which in turn corresponds to an increase in the mass transfer resistance by molecular diffusion, as defined by the Law of Fick (see Fig. 12), as well as in the thermal resistance corresponding to the process of conduction of heat along the thickness of the sludge layer, according to the Fourier Law (see Fig. 13).

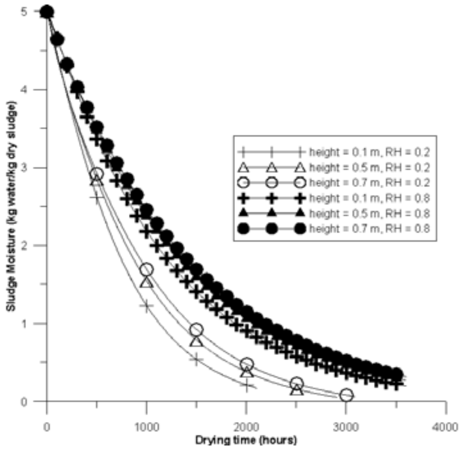

Fig. 10. Variation of the sludge moisture with of the relative humidity of the air drying and bed height sludge.

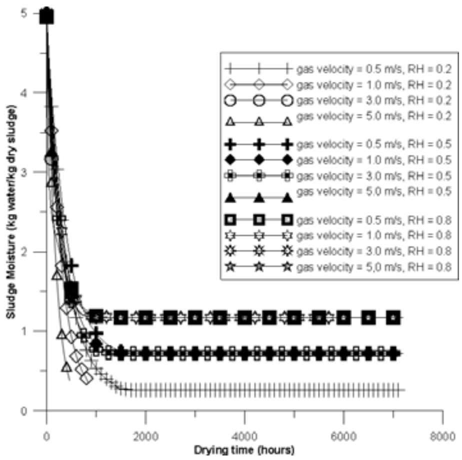

Fig. 11. Variation of the sludge moisture with gas temperature velocity and relative humidity of the air drying.

Required drying times in order to achieve equilibrium moisture increases by about 3 times with a sevenfold increase of the bed height from 0.1 m to 0.7 m. Therefore, when designing a drying process, the thickness of the sludge layer represents an important factor to be considered. While thin layers allow for maximum drying efficiency, the corresponding demand in area affects the cost of the

operation. The exact dryer geometries should be the moisture content in respect to the drying time, respectively, for different heights of the sludge layer. Obviously, these curves are in agreement

In addition, Fig. 16 plots the influence of the wet sludge mass on its moisture content during the drying process. Changing the mass of the wet sludge correlates to a change of the sludge's density. Increased density leads to a increased resistance for diffusion, resulting in longer drying times.

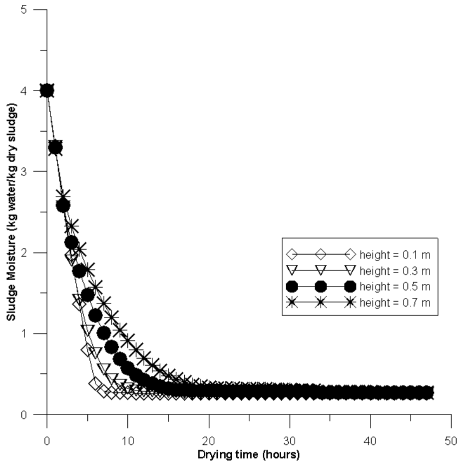

Fig. 12. Variation of the sludge moisture with drying time at different height of sludge.

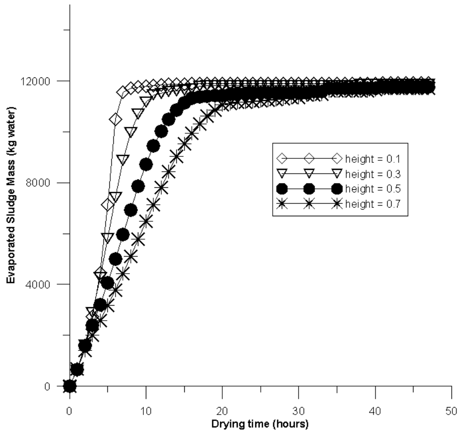

Fig. 13. Variation of the evaporated sludge mass with drying time at different height of sludge.

with those of the previous Figures and reinforce the importance of considering the effect of the diffusional distance in the drying process, as clarified by Welty *et al.* (2007); Çengel and Boles (2008); and Bergman, Lavine and Incropera (2011).

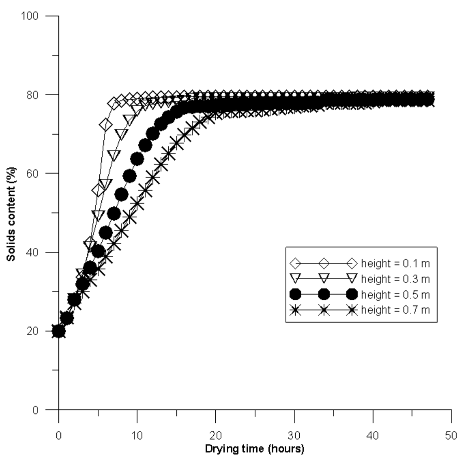

Fig. 14. Variation of the solids content with drying time at different height of sludge.

5. CONCLUSION

By formulating the established governing equations of heat and mass transfer for porous drying processes differential equations for the description of temperature and moisture contents inside the sludge were derived. Mass transfer at the surface of the sludge, as well as diffusivity, thermal conductivity and other physical properties were described through empirical correlations obtained by several authors in previous works.

Numerical discretization was achieved through the Finite Difference Method as well as the Runge-Kutta-method. Implementation of the model was done in the software Engineering-Equation-Solver.

The application of free solar energy may be an alternative solution for conventional heated drying processes, due to highly reduced operating cost. However, in solar drying processes the control of the operating conditions is severely limited. For optimal results, a low level of relative humidity in the ambient air is of most importance, next to the air velocity and temperature. The financial feasibility of such an operation has to be carefully evaluated. Other possibility is related to the use of heated gas or fluid on the ground of bed sludge.

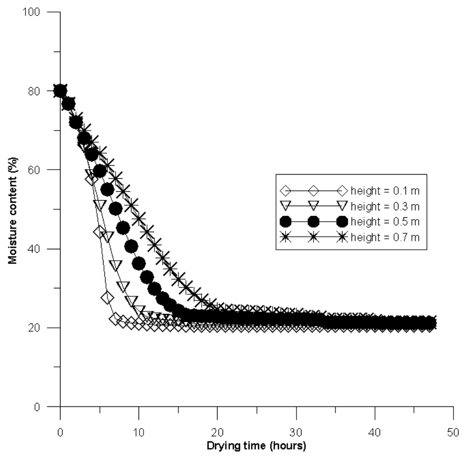

Fig. 15. Variation of the moisture content with drying time at different height of sludge.

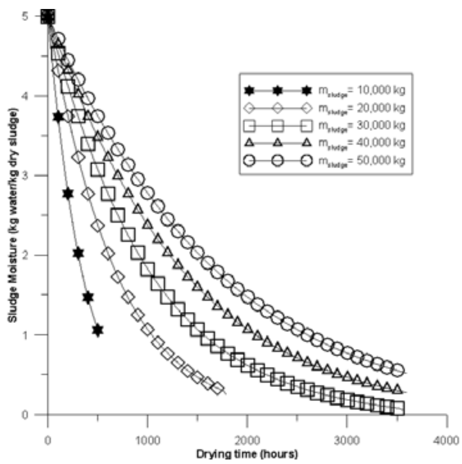

Fig. 16. Variation of the sludge moisture with the mass of sludge.

Finally, it can be emphasized that the most important parameters on the drying processes corresponding to the solar radiation, the relative humidity of the air drying, the geometric configuration of the bed sludge. The results show that gas velocity is not an so influential parameter in drying process. The mass transfer from interior layer to sludge surface controls the drying process and mass evaporation rate is mainly depended on the solar radiation, on the relative humidity of the air drying and on gas drying temperature.

ACKNOWLEDGEMENTS

The authors would like to thank all members of research team of Laboratory of Numerical Simulation and Transport Phenomena (LabSim) for their contributions. Funding support for this research was provided by Universidade Federal de Viçosa (UFV) and Foundation of Support Research of the State of Minas Gerais (FAPEMIG).

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