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The Effects of Target Plate Roughness on the Parameters of Circular Hydraulic Jumps: An Experimental Investigation

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ABSTRACT

When a vertical liquid jet impinges on a horizontal flat plate, at a certain distance from the location of impingement, the depth and velocity of the liquid change and a circular hydraulic jump is formed. The importance of this phenomenon in certain industries has motivated continued quest for more thorough knowledge of the parameters affecting it. Previous research has shown that physical parameters, such as flow rate, jet diameter and geometry of target plate, significantly affect the size and shape of hydraulic jumps. In this study, the effect of target plate roughness on the parameters of circular hydraulic jumps is experimentally investigated. The results show that adding roughness to the target plate leads to an increase in hydraulic jump radius. Furthermore, utilizing the results obtained from the experiments, an empirical law is proposed which determines the hydraulic jump radius and fluid height downstream of the jump position for a given surface roughness of target plate. One of the best-known models for the characterization of the behavior of circular hydraulic jumps is the Bush and Aristoff's model, which is presented as a curve for smooth surfaces. Since the effect of roughness of target plate surface is ignored in the Bush and Arisstof's model, the results obtained in this investigation are further used, for the first time, to improve this model for different degrees of surface roughness.

Keywords: Circular hydraulic jump; Surface roughness; Bush and Aristoff model; Taguchi method.

NOMENCLATURE

a	nozzle radius	K_s	average particle diameter of sand papers
Bo	Bond number	q	volumetric flow rate
d	liquid jet diameter	r_0	radius at which the boundary layer thickness reaches that of free surface
Fr	Froude number	R_j	Radius of hydraulic jump
g	acceleration of gravity	σ	Surface tension
ΔH	height of the jump	ϑ	Kinematic viscosity
h	height of fluid	ρ	Density
H_∞	fluid height downstream of the jump position		

1. INTRODUCTION

After a vertical liquid jet impinges on a horizontal plate, a thin layer of the liquid spreads out radially and symmetrically over the plate. If the flow Froude number is greater than 1, the flow regime changes from supercritical to subcritical at a certain distance from the location where the jet impinges on the plate. This phenomenon, which involves an abrupt increase in the depth and decrease in the velocity of the liquid flow, is called a circular hydraulic jump (Khavari 2010).

The thin fluid film that spreads out radially on the flat plate constitutes a region of low thickness and high velocity. Therefore, it is commonly used as a region with an appropriate heat transfer rate from the plate. This heat transfer characteristic of the region of hydraulic jump underlies its use of equipment cooling and other industrial applications such as glass production, fuel injection, etc. (Bunker 2007; Hosain *et al.* 2016). Therefore, to have a correct estimate of the amount of convective heat transfer from the surface, the radius of the circular hydraulic jump has to be accurately determined and the parameters affecting it have to be identified.

Since the first experimental studies by Giorgio Bidone (1820), many scientists have studied this phenomenon. One of the first methods of the theory of the circular hydraulic jump was presented in 1949 by Tani (1949). Since Tani's theroy was based on some assumptions, the radius of circular hydraulic jump obtained from his model did not agree with the experimental results.

The first theory for explaining the circular hydraulic jump was proposed by Birkhoff and Zarantonello (1957). They conducted a detailed examination of the hydraulic jumps caused by the impingement of a fluid jet on a horizontal plate and proposed their non-viscous circular hydraulic jump theory.

Watson (1964) was the first who considered the influence of viscosity on circular hydraulic jumps. Watson showed that the radius of circular hydraulic jump depends on parameters such as the flow rate, fluid height downstream of the jump position, gravitational acceleration and jet diameter. A description of Watson's theory is presented in Section 4. Watson's relations are frequently used as a reference for researchers to compare and validate their results against it.

Craik *et al.* (1981) showed that if the hydraulic jump radius is ten times greater than the depth of the supercritical region, Watson's theory has a high degree of accuracy. However, for smaller radii, there is less agreement between experimental results and Watson's theory. A major assumption in Watson's theory is that it ignores the effect of surface tension in the formulation of its equations. This important assumption causes inaccuracy in its results, in particular for small radii, where the effect of surface tension is considerable. Bush and Aristoff (2003) managed to overcome this shortcoming by adding a surface tension parameter to Watson's equations, by means of a dimensionless Bond number.

Based on the results of their experimental work, Liu and Lienhard (1993) showed that when the ratio of the flow downstream to upstream of a jump is too large or the Froude number upstream of a jump is too high, Watson's model is less accurate. They also demonstrated that since Watson's model does not consider the effects of increased hydrostatic pressure and consequently does not predict the separation of flow off the surface.

After Watson, many researchers have sought out a simpler relation between hydraulic jump radius and parameters such as flow rate, gravitational acceleration, fluid viscosity and fluid jet diameter. Bohr *et al.* (1993) used shallow-water theory to obtain a scaling relation for position of the circular hydraulic jump. Their scaling relation introduced hydraulic jump radius as a function of volumetric flow rate, fluid kinematic viscosity and gravitational acceleration.

Brechet and Neda (1999) put developed a simple model for the prediction of hydraulic jump radius and compared the results of their model with experimental data. They proposed the scaling law for estimating the hydraulic jump radius as a function of flow rate, nozzle height and fluid viscosity.

Following the Volume-of-Fluid (VOF) method, Passandideh-Fard *et al.* (2011) numerically solved continuity and momentum equations to model the circular hydraulic jumps on smooth surfaces. They also investigated the effects of flow rate, downstream height of the jump position, kinematic viscosity and gravitational acceleration on the hydraulic jump radius. They used experimental data to show that their proposed method successfully predicts the location where the circular hydraulic jump occurs.

Rojas *et al.* (2013) presented the theoretical and numerical results on the circular and non-circular hydraulic jumps in the framework of inertial lubrication theory (Rojas *et al.* 2010). Also, they obtained a scaling law for the circular hydraulic jump.

Ellegard *et al.* (1998) used ethylene glycol instead of water to achieve more stable circular hydraulic jumps and unexpectedly observed that in certain conditions, the circular shape of jumps disappears and jumps take a polygonal shape. In another work (Ellegard *et al.* 1999), they studied this type of hydraulic jumps in detail and found that the number of sides of a polygonal jump depends on the fluid height downstream of the jump position, flow rate, and the liquid jet diameter.

Kasimov (2008) showed that if the surface tension is above a certain critical value, the steady circular hydraulic jump does not exist. The critical surface tension depends on fluid viscosity, flow rate, and other parameters. He showed that the critical surface tension decreases with increase in the viscosity of fluid. This might explain why polygonal hydraulic jumps are observed only in highly viscous fluids.

Martens *et al.* (2012) presented a phenomenological model for the polygonal hydraulic jumps. This model consists of radial force and mass conservation balance between hydrostatic pressure and viscous stresses on the roller surface.

Mokhlesi (2013) carried out experimental investigation the effect of different parameters such as flow rate, downstream fluid height, and jet diameter on the parameters of circular and polygonal hydraulic jumps. He presented the region of stability in terms of Re and We numbers.

Rojas and Tirapegui (2015) presented numerical and theoretical results of the circular and polygon hydraulic jumps derived from the inertial lubrication theory. Their results are in good agreement with the experimental results of Ellegard *et al.* (1998).

In their experiments, Teymourtash and Mokhlesi

(2015) found for the first time that at the certain Reynolds and Weber number values, polygonal hydraulic jumps take a rotational structure. They investigated how the flow rate, the fluid jet diameter and obstacle height influence the formation and angular velocity of such rotational polygonal jumps.

Since Onda *et al*. (1996) first demonstrated artificial superhydrophobic surfaces in 1996, many studies have been reported on the production of superhydrophobic surfaces, their applications and properties (Shirtcliffe *et al*. 2010; Ma and Hill 2006).

Prince *et al*. (2012) investigated the influence of superhydrophobic surfaces on the physics of laminar jet impingement on a flat horizontal surface. Their results indicated that the formation of hydraulic jumps on superhydrophobic surfaces, reduced the growth of the boundary layer, fluid height and friction coefficient, and thus increases the hydraulic jump radius.

Johnson *et al*. (2014) reports experimental results of circular hydraulic jumps on patterned surfaces with alternating microscale-ribs and cavities. They illustrated that according to the pattern of ribs and cavities, the hydraulic jump formed is asymmetrical and elliptical. Their results showed, when water wets the cavity regions compared to the non-wetting scenario, the area of the supercritical thin-film region internal to the hydraulic jump is smaller for patterned surfaces.

Dressaire *et al*. (2009) changed circular hydraulic jumps to polygonal jumps by placing micro-scale cylindrical obstacles on the target plate. They managed to change the characteristics and shape of the hydraulic jumps by changing flow rate, the height and diameter of the cylindrical obstacles and the obstacles' spacing.

Kuraan *et al*. (2017) investigated the nozzle distance to the target plate on parameters of water jet strike with the plate. According to their experimental results, they developed relationships for the stagnation Nusselt number, pressure and hydraulic jet diameter as a function of the nozzle height to the plate.

A survey of the research conducted in the field of the circular hydraulic jumps reveals that although the phenomenon has been widely studied, the research in this field is mainly focused on flows on smooth plates and yet there is no comprehensive published study on the examination of the effects of target plate surface roughness on the characteristics of circular hydraulic jumps. Following the previous researches of one of the authors (Passandideh-fard *et al*. 2011; Teymourtash and Mokhlesi 2015), the present study aims to investigate experimentally the effect of surface roughness on the circular hydraulic jump radius and downstream fluid height. Also, utilizing the results of the conducted experiments, Bush & Aristoff model (Bush and Aristoff 2003) is modified for plates of varying degrees of surface roughness, and the useful correlations are proposed which estimates circular hydraulic jump radius and fluid height downstream of the jump position.

2. EXPERIMENTAL APPARATUS

The schematic diagram of the apparatus used for the creation and measurement of circular hydraulic jump parameters is shown in Fig. 1. The setup comprises a frame, centrifugal pump, flow meter, storage tank, overflow tank, height measurement mechanism (including digital caliper with the accuracy of 0.01 mm), glass target plate, mechanism for horizontal leveling of the target plate, thermometer, glass nozzle, lighting and imaging equipment.

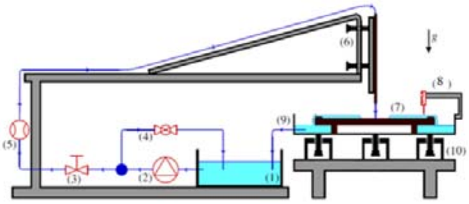

Fig. 1. Schematic diagram of the apparatus for the creation of the circular hydraulic jump

1. Storage tank, 2. Centrifugal pump, 3. Control valve, 4. Bypass valve, 5. Flow meter, 6. Vertical leveling mechanism, 7. Target plate, 8. Height measurement mechanism, 9. Overflow tank, 10. Horizontal leveling mechanism

The fluid starts flowing from the storage tank, through plastic tubes, to the nozzle glass tube by means of a centrifugal pump with a large number of blades and, after exiting the nozzle glass tube, the fluid impinges on the target plate. After striking against the target plate, the fluid jet spreads radially on it, and having formed the jump, gets directed towards the storage tank for the measurement of the volumetric flow rate. In the tank, the volumetric flow rate is determined using the time it takes the storage tank to be filled with a certain volume of fluid. After measuring the flow rate, the fluid gets pumped to the cycle again.

In order to achieve greater stability in the circular hydraulic jumps formed in all the experiments, ethylene glycol is selected as the operating fluid. The ethylene glycol used in the experiments has the properties of density ρ=1.1 gr/cm^3, kinematic viscosity ϑ=0.12 St, and surface tension σ =47.5 dyn/cm.

Nozzles of different diameters d=0.5, 0.7, 0.8, 1 cm are used to investigate the effects of jet diameter on the experimental results. The nozzle is fixed at a height of 1 cm above the target plate in all trials. Due to the small distance between the nozzle and the target plate, the fluid jet diameter is taken to be equal to the nozzle diameter. The target plate is a circular piece of glass, 6 mm thick and 45 cm in diameter. The experimental apparatus is depicted in Fig. 2.

Fig. 2. Experimental apparatus for creating the circular hydraulic jumps

The most important aspect in the study of circular hydraulic jumps is the determination of the hydraulic jump radius. In order to accurately measure this attribute, high-quality imaging followed by image processing is employed. Considering the quality of the images, circular hydraulic jump radius can be determined with an accuracy of 0.2 mm. A typical circular hydraulic jump formed in our experiments is shown in Fig. 3. The liquid used in the research is the green ethylene glycol and the target plate is made of glass. Because the liquid in the supercritical region has very low thickness, the color of this region appears brighter than the area after the jump.

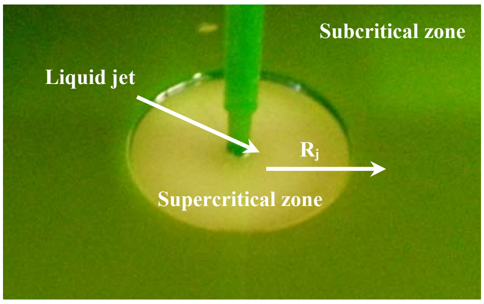

Fig. 3. Formation of a circular hydraulic jump. The color of supercritical zone is brighter than the region of the after jump

Water-proof sandpapers are used to create roughness of various degrees, on the target plate. Sandpapers used in this study, are produced according to FEPA standards (FEPA-standard43-1 2017). This standard classifies sandpapers according to certain numbers which denote the number of holes, per inch, in the screen used to sort the sandpaper particles. Several sandpapers are glued onto the glass plates to make the different degrees of roughness for the target plates. Based on the FPEA standard, the values of the particles average diameter of the sandpapers (K_s) used in present work are shown in Table 1. In this study, sandpapers of various sizes of silicon carbide particles are selected (Matador, Germany). Shen *et al.* (2016) reported the average particles size, surface roughness and coefficient of friction for this type of sandpapers.

The experiments are conducted for a variety of volumetric flow rates (22-140 ml/s), and jet diameters (d=0.5, 0.7, 0.8, 1 cm). In order to investigate the influence of surface roughness of the target plate on the results, the experiments are conducted using sandpapers of 7 grit sizes, i.e. P2000, P1500, P1000, P600, P320, P180, P100, and the results are compared with those of the smooth surface.

Table 1 Sandpapers specifications according to FEPA standard

Grit Designation	K_s (mm)
P2000	0.0103
P1500	0.0126
P1000	0.0183
P600	0.0258
P320	0.0462
P180	0.082
P100	0.162

In Fig. 4, the results of the present work are compared with those of Mokhlesi (2013) and those of Bush and Aristoff model for smooth target plate. The circular hydraulic jump radius was conducted for different flow rates and the jet diameters of d= 0.6 and 1 cm. The average error of the results of the present work relative to the results of the Bush & Aristoff model (2003) and Mokhlesi tests (2013) is about 3% and so experimental results obtained in this study are in good agreement with the results of others. Therefore, the apparatus used in this research has the suitable precision for conducting the experiments

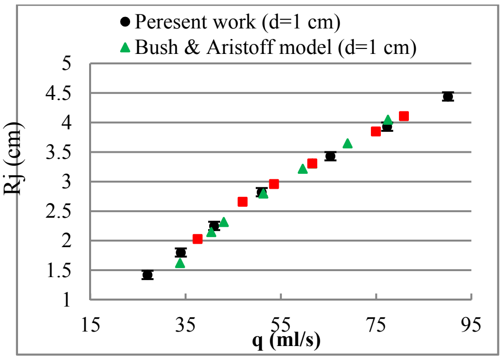

Fig. 4. Comparison of the experimental result of the circular hydraulic jump radius with Mokhlesi experiments (2013) and the Bush & Aristoff's model on a smooth surface

3. EXPERIMENTAL METHOD

In this section presents the results of experimental investigation of the effects of surface roughness on the radius of circular hydraulic jumps and the downstream height of the jump. Subsequently, an empirical equation will be proposed, which estimates circular hydraulic jump radius and downstream fluid height as a function of the diameter of roughness particles, flow rate and jet diameter.

The effect of flow rate on the fluid depth downstream of the jump position, for different diameters of a fluid jet, is illustrated in Fig. 5. As can be seen, increase in the flow rate increases downstream fluid depth. Furthermore, as the fluid flow rate increases, it displays reduced effects on the downstream fluid depth. The results in Fig. 5 also reveal that the enlargement of the jet diameter leads to an increase in the downstream fluid depth.

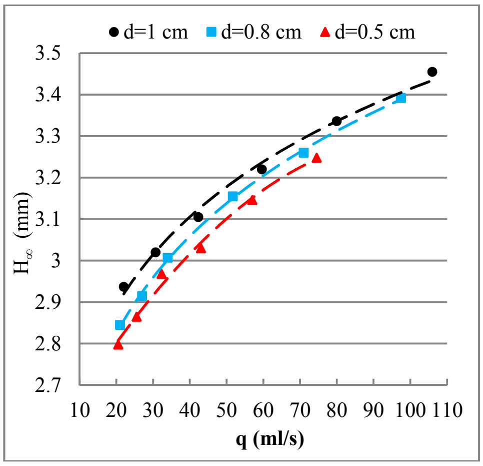

Fig. 5. Effect of the flow rate on the downstream fluid depth on a smooth surface for different jet diameters

The effect of flow rate on circular hydraulic jump radius for the jet diameters of d=0.5, 0.8, 1 cm is shown for the case of a non-rough (smooth) plate in Fig. 6. The results demonstrate that changes in the fluid flow rate considerably affect the hydraulic jump radius. As the flow rate increases, the initial value of the Froude number increases (due to the initial speed increase); therefore, the hydraulic jump radius increases. Also, by increasing the diameter of the fluid jet, the initial velocity of the fluid decreases and the hydraulic jump decreases as well.

Next, the results of investigating the effects of target plate surface roughness on the radius of the circular hydraulic jump and the downstream fluid height are presented and analyzed. The coatings of the target plate using different sandpapers create superhydrophobic surfaces (Gogte *et al.* 2005). Since superhydrophobic surfaces reduce the growth of the boundary layer and therefore reduce the shear stress and friction, it increases the radius of the hydraulic jump (Prince *et al.* 2012).

Figure 7 shows the effect of the target plate roughness on circular hydraulic jump radius at different flow rates. As demonstrated by the results, rough surfaces cause increase in hydraulic jump radius. When the jet diameter is d=1 cm, depending on the average diameter of surface roughness particles and the flow rate, the increase in the average hydraulic jump radius is found to be between 4.4 and 10.7 percent.

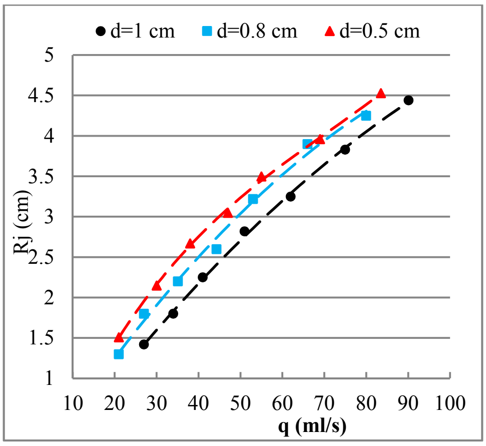

Fig. 6. Effect of the flow rate on the circular hydraulic jump radius for different jet diameters on a smooth surface

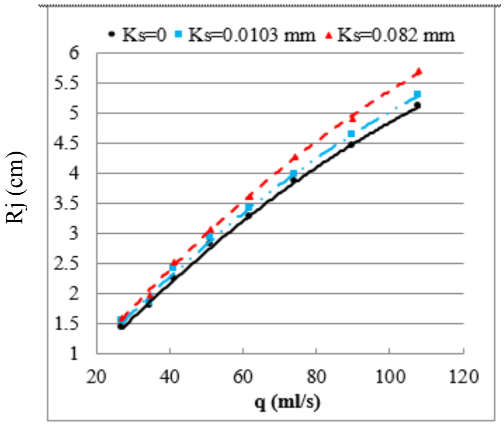

Fig. 7. Effect of surface roughness on the radius of circular hydraulic jump for d=1 cm

Figure 8 shows the effect of target plate surface roughness on the downstream fluid height of the jump position for the jet diameter of d=1 cm. The results depicted correspond to experiments with 4 different diameters for the surface roughness particles, i.e. K_s=0.0103, 0.0183, 0.0462, 0.082 mm, and they are presented in comparison with the case of a smooth plate. For the jet diameter of d=1 cm, depending on the flow rate and the degree of surface roughness, the drop in the jump downstream depth is found to be between 3.9 and 17.5 percent in comparison with the smooth plate.

In Fig. 9, the variations in the downstream fluid height are illustrated versus of the hydraulic jump radius for different flow rates and surface roughnesses. The hydraulic jump radius is related to the fluid height downstream of the jump position (Rojas *et al.* 2013). The results of Fig. 9 show that the surface roughness causes a change in the hydraulic jump radius with fluid height downstream of the jump position. The experimental results which presented in Figs. 5 to 9 are based on the least squares method using Excel software.

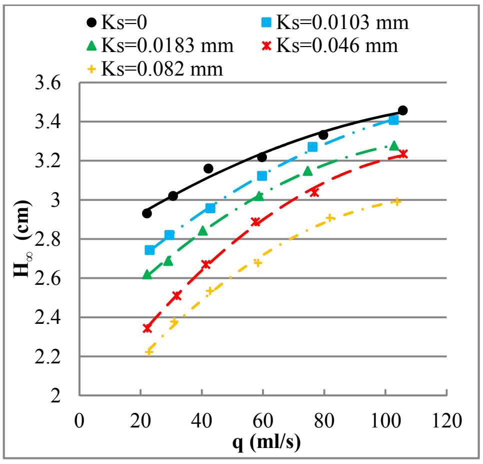

Fig. 8. Variations of the downstream fluid height as a function of flow rate for different degrees of surface roughness for d=1 cm and H₀=0

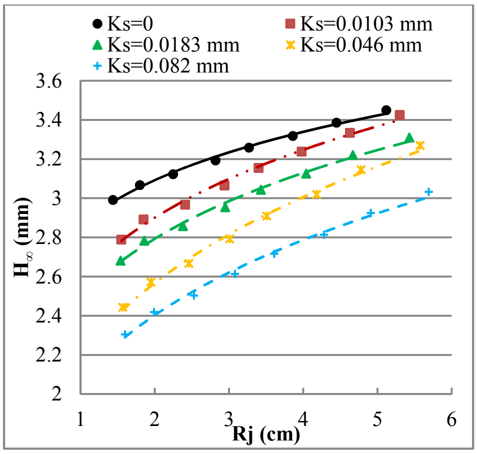

Fig. 9. Variations in the downstream fluid height as a function of the radius of circular hydraulic jump for different Kₛ and q and d=1 cm

4. THEORY

Watson (1964) was the first one to consider the effect of viscosity on circular hydraulic jumps. He proposed his analysis based on Prandtl's boundary layer theory and described the flow in terms of the Blasius sublayer expanding close to the position of impact jet and a far-field similarity solution. Watson considered the following assumptions to present his equations:

- Flow is steady.
- The fluid height downstream of the jump is much larger than fluid height upstream of the jump.
- In comparison with the viscous stresses, variations of hydrostatic pressure are not considered.
- The flow rate is considered to be uniform after the hydraulic jump.
- Surface tension is ignored.

Therefore, based on the assumptions, Watson provided the following semi-experimental equations to determine the circular hydraulic jump radius:

$$\frac{R_j H_\infty^2 g a^2}{Q^2} + \frac{a^2}{2\pi^2 R_j H_\infty} = 0.10132 - 0.1297\left(\frac{R_j}{a}\right)^{0.5} \mathrm{Re}^{-0.5}$$

$$for \qquad R_j < r_o \tag{1}$$

$$\frac{R_j H_\infty^2 g a^2}{Q^2} + \frac{a^2}{2\pi^2 R_j H_\infty} = 0.01676\left[\left(\frac{R_j}{a}\right)^3 \mathrm{Re}^{-1} + 0.1826\right]^{-1}$$

$$for \qquad R_j \geq r_o \tag{2}$$

Where R_j is the hydraulic jump radius *(cm)*, H_∞ is downstream liquid height *(cm)*, g is gravitational acceleration *(cm/s²)*, q is volumetric flow rate *(ml/s)*, a is jet radius *(cm)* and r_o is the radius at which the boundary layer thickness reaches the free surface of flow which is calculated by the following relation:

$$r_o = 0.3155\, a\, Re^{1/3} \tag{3}$$

After Watson, several researchers (Craik *et al.* 1981; Liu and Lienhard 1993) inferred the impact of surface tension on circular hydraulic jumps.

Bush and Aristoff (2003) showed that surface tension has an important effect on the parameters of circular hydraulic jumps. They introduce the effect of surface tension into Watson's theory by using of complex computational mathematics and making the several assumptions. The final equations of Bush & Aristoff's model are as follows:

$$\frac{R_j H_\infty^2 g a^2}{Q^2}\left(1 + \frac{2}{Bo}\right) + \frac{a^2}{2\pi^2 R_j H_\infty} =$$

$$0.10132 - 0.1297\left(\frac{R_j}{a}\right)^{3/2} \mathrm{Re}^{-1/2} \qquad R_j < r_o \tag{4}$$

$$\frac{R_j H_\infty^2 g a^2}{Q^2}\left(1 + \frac{2}{Bo}\right) + \frac{a^2}{2\pi^2 R_j H_\infty} =$$

$$0.01676\left[\left(\frac{R_j}{a}\right)^3 \mathrm{Re}^{-1} + 0.1826\right]^{-1} \qquad R_j \geq r_o \tag{5}$$

where Bo denotes the Bond number, which incorporates the effect of surface tension, and is defined by:

$$Bo = \frac{\rho g R_j \Delta H}{\sigma} \tag{6}$$

Since the equations introduced by Bush and Aristoff take the effects of surface tension into consideration, their model, leads to more accurate results compared to Watson's model, particularly at smaller radii. Therefore, in this study, Bush and Aristoff's model, is used to estimate circular hydraulic jump radius.

The results of the present work are compared with the results provided by the Bush & Aristoff's model in Fig. 10. The curve representing the Bush & Aristoff's model is drawn following the method used by several researchers such as Bush and Aristoff (2003) and Passandideh-fard *et al.* (2011). In Fig. 10, the horizontal axis denotes the term on the left-hand side of the modified equation and the vertical axis denotes its right-hand-side terms.

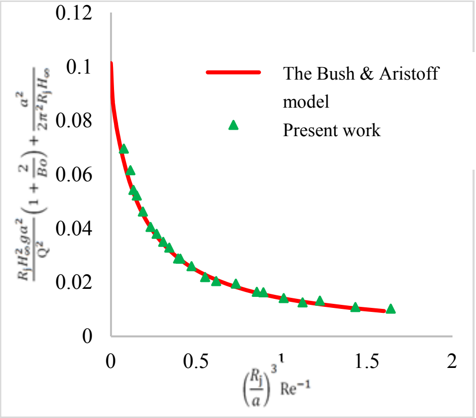

Fig. 10. Comparison of the experimental results with those of the Bush & Aristoff model for smooth target plate. The triangular points represent the experimental results for q= 15-120 ml/s and d= 0.5, 0.7, 0.8 and 1 cm

As displayed in Fig. 10, the experimental results obtained in this study match the results given by Bush and Aristoff model (2003). A comparison of the results shows an average difference of approximately 3 percent between the results.

Since the Bush & Aristoff model was proposed for smooth plates and the results obtained suggest that surface roughness affects parameters of the circular hydraulic jump, therefore, the Bush & Aristoff model is extended to include rough plates. The experiments described have been conducted for different values of flow rate and jet diameter.

The present experimental study has been investigated the effects of surface roughness on the parameters of the circular hydraulic jump; The Bush & Aristoff model has been presented for the smooth target plate. Figure 11 compares the results of the present experimental study for various values of surface roughness with the results of Bush & Aristoff model. As seen, the experimental results are in good agreement with Bush & Aristoff model when the surface of target plate is smooth, however, with increasing the surface roughness, the Bush & Aristoff model underestimates the parameters of circular hydraulic jump and this model should be modified on the rough target plate.

A correlation can be suggested to estimate the circular hydraulic jump radius where the target plate is not smooth. The correlation can be written in the general form as:

$$R_{rough} = R_{smooth} + a\, q^b\, d^c\, K_s{}^e \qquad (7)$$

where R_{rough} (cm) is circular jump radius on rough surfaces, R_{smooth} (cm) is hydraulic jump radius on smooth surfaces, q (ml/s) is the volumetric flow rate, d (cm) is the fluid jet diameter, K_s (cm) is the diameter of the sandpaper particles, and *a, b, c,* and *e* are empirical constants can be obtained.

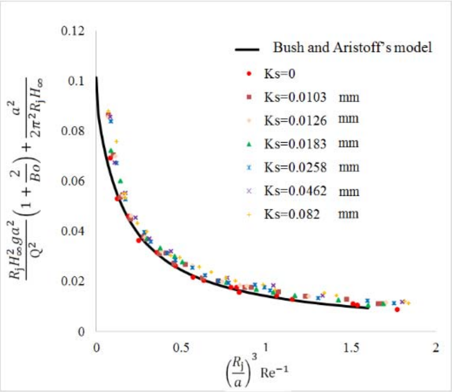

Fig. 11. Results of the Bush & Aristoff model as a function of different degrees of roughness of the target plate for q= 13-122 ml/s, d= 0.5, 0.7, 0.8 and 1 cm

Using the experimental results and the least squares method, the following relation will result for the different settings:

$$R_{rough} = R_{smooth} + 0.0495\, q^{-0.021}\, d^{-1.08}\, K_s^{-0.18} \qquad (8)$$

The unit of a is $\mathrm{cm}^{2.323}/\mathrm{s}^{0.021}$. Equation (8) was obtained for the parameter values in the ranges of d = 0.5-10 cm, q = 20 – 105 ml/s, K_s = 0 – 0.082 mm. One can determine the circular hydraulic jump radius for rough plates using Eq. (8) when the hydraulic jump radius for smooth surfaces is experimentally or theoretically known. Figure 12 provides a comparison of the results provided by the proposed relationship with the experimental results as a function of the different parameters. In this Fig., on the solid line, the results of the presented equation and the experimental results are equal. The marked points are experimental results and the area between the two dash lines has a 5% error. The results show that the proposed equation predicts the accuracy of the circular hydraulic jump radius for different roughness target plates.

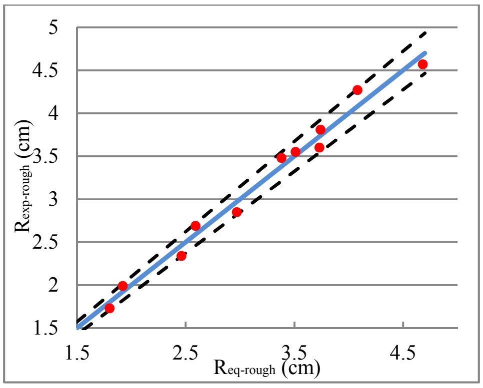

Fig. 12. Comparison of the results of Eq. (8) with the experimental results

In order to determine fluid height downstream of the jump position for different degrees of surface roughness, an empirical equation can be presented in a similar way. For this purpose, the following equation is proposed to determine the fluid height downstream of the jump position for rough plate.

$$h_{rough} = h_{smooth} - a\, q^b\, d^c\, K_s^e \qquad (9)$$

As a result, using experimental data and the method of least squares and the equation below is proposed for the estimation of the downstream fluid height.

$$h_{rough} = h_{smooth} - 0.046\, q^{0.0219}\, d^{0.6} \qquad (10)$$

The unit of a is s/cm^4 In Fig. 13, the fluid height values given by Eq. (10) for different input parameters are compared with the experimental results. The experimental results obtained in this study are in good agreement with the proposed equation.

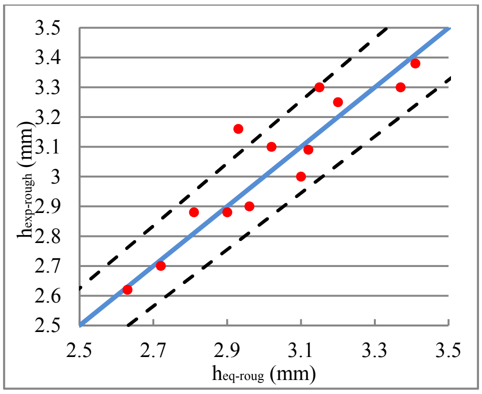

Fig. 13. Comparison of the values of downstream fluid height provided by Eq. (10) with the experimental results

5. RESULTS AND DISCUSSION

The results of the conducted experiments reveal that fluid flow rate, jet diameter, and the surface roughness of the target plate affect hydraulic jump radius and fluid depth downstream of the jump position. For a more rigorous study of the effect of the different parameters on jump radius and fluid depth downstream of the jump position, the simultaneous effects of the parameters on the actual results have to be investigated. Thus, to successfully discover the simultaneous effects of the different parameters on hydraulic jump radius and downstream fluid depth, and to do so with fewer experiments, the design of experiment (DOE) methods are required.

Design of experiments is one of the common methods for identifying effective factors and controlling these factors in order to optimize production processes and conduct more accurate experiments. DOE methods have special advantages that can be mentioned:

- Reducing the number of needed experiments to analyze the phenomenon and, consequently, reducing time and cost.
- Improving the quality of experiments and performing more accurate statistical analyzes.
- Simultaneous analysis of input variables in the output parameter.
- Determinate the factors that have the greatest impact on the output parameters.

Taguchi *et al*. (Taguchi and Wu YI 1979; Taguchi and Konishi 1987) developed a new approach for optimizing tests in order to reduce the time and cost of tests. They introduced an off-line quality improvement methodology to design a process while being aware of the presence of variation in all processes. Taguchi analysis can study each system that has independent parameters and levels and has a wide range of applications from economy to engineering. In Taguchi method, the value of S/N ratio is used as measurable value instead of the parameter of standard deviation. These changes are due to the reason that as the mean value decreases/increases; the standard deviation also decreases/increases (Karna and Sahai 2012). The value of S/N specifies how much the output of the problem is sensitive to the different input parameters and therefore if the value of S/N is larger for a factor, the output parameter is more sensitive to the variations of this factor. Signal factors are those design and process parameters that can be controlled and Noise factors are those that are uncontrollable factors.

Usually, the S/N ratio is analyzed by one of the three categories of the performance characteristics, namely larger-is-better, small-is-better or target-is-better. Therefore the results of the analysis may be utilized for the occasions when the maximum or minimum output value or the closest value to the average of the output values is desirable. The value of S/N ratio for the case of larger is better was measure by following (Montgomery 2017):

$$\frac{S}{N} = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{R_i^2}\right) \qquad (11)$$

And the S/N ratio for the case of smaller is better can be written by the following equation:

$$\frac{S}{N} = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}R_i^2\right) \qquad (12)$$

Where n is the total number of replications of each experiment, R$_i$ is responses for the selected factor and i is the test number. In this study, the two output functions are the radius of the circular hydraulic jump and fluid height downstream of the jump position. The analyses aim to find the maximum value of hydraulic jump radius (larger-is-better) and the minimum value of fluid height downstream of the jump position (small-is-better) as the output parameters. Taguchi analyses are carried out using the Minitab 16.2.2, which is a powerful software package to solve the statistical problems. This software package is used in various fields such as engineering sciences, mathematics, and economy for statistical analysis. More information can be found in Montgomery (2017) and Taguchi and Jugulum (2002).

As mentioned above, several parameters affect the hydraulic jump radius and fluid depth downstream of the jump position. Since the input parameters in Taguchi analysis have to be independent of each other and considering our experimental equipment, the three effective parameters (factors) of volumetric flow rate, surface roughness, and jet diameter are selected to be examined in terms of their effect on the output parameters. Different designs of experiment with various levels can be devised. Considering the obtained parameter values available from the Initial tests, the experiments in the present study have been designed according to Table 2.

Table 2 Selected factors and different levels in Taguchi analysis

Factors	Level 1	Level 2	Level 3	Level 4
q (ml/s)	25	40	55	70
K_s (mm)	0	0.0183	0.046	0.082
d (cm)	1	0.8	0.7	0.5

Study of the effect of different factors on the output parameter requires a large number of different experiments to be carried out. However, increase in the number of experiments would render the research costlier and more time-consuming. For decreasing the cost and time of experiments, Taguchi presented a designed method called the use of the orthogonal table to study the all of parameters with the lesser number of experiments to be conducted. Considering the number of independent parameters affecting the phenomenon under study and the number of levels selected for each parameter, to find the optimal outputs, 16 experiments are suggested by Taguchi method. The arrays of the orthogonal table of Taguchi analysis are displayed in Table 3.

Figure 14 shows the results of the Taguchi analysis while the maximum radius of hydraulic jump of interest. A greater S/N ratio at a given level indicates that that the level has a higher effect on the final resulting value (of hydraulic jump radius). The results suggest that the hydraulic jump radius enlarges with the increase in flow rate and surface roughness and decrease in jet diameter.

Base on the ranges selected for the three parameters, the analysis of the results reveals that, among the three parameters including the flow rate, jet diameter, and surface roughness, flow rate has the maximum effect and surface roughness has the minimum effect on hydraulic jump radius.

In Fig. 15, the values of S/N ratio resulted from the parameters of volumetric flow rate, jet diameter, and surface roughness are shown for the occasion when the minimum fluid height downstream of the jump position is of interest. The values of the S/N ratio in the conducted experiments indicate that reduction in flow rate and increase in surface roughness and jet diameter lead to decreased fluid depth downstream of the jump position. The results demonstrate that, considering the range of values selected for each parameter, the surface roughness of the target plate has the strongest effect and jet diameter has the weakest effect on the reduction of downstream jump height.

Table 3 Arrays of the orthogonal table of Taguchi analysis

Experiment No.	d (cm)	K_s (mm)	q (ml/s)
1	1	0	25
2	0.8	0.0183	25
3	0.7	0.046	25
4	0.5	0.082	25
5	0.8	0	40
6	1	0.0183	40
7	0.5	0.046	40
8	0.7	0.082	40
9	0.7	0	55
10	0.5	0.0183	55
11	1	0.046	55
12	0.7	0.082	55
13	0.5	0	70
14	0.7	0.0183	70
15	0.8	0.046	70
16	1	0.082	70

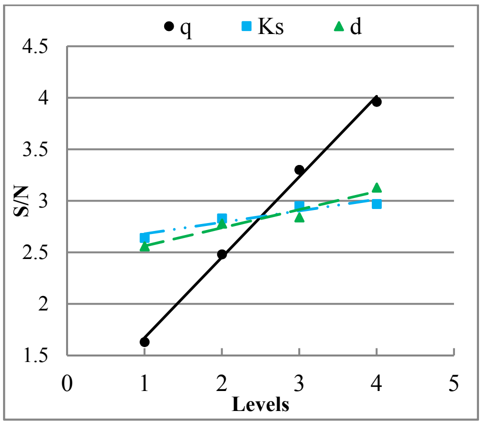

Fig. 14. Effect of different parameters levels (Table 2) on S/N ratio when the largest hydraulic jump radius is of interest (larger is better)

It is important to mention Figs. 8 and 9 show the variations of the radius of the circular hydraulic jump versus flow rate and the variations of the downstream fluid height versus flow rate at constant jet diameter and roughness surface. But in Figs. 14 and 15, the simultaneous effects of varies parameters such as flow rate, jet diameter and roughness surface are investigated on radius of the circular hydraulic jump and downstream fluid height.

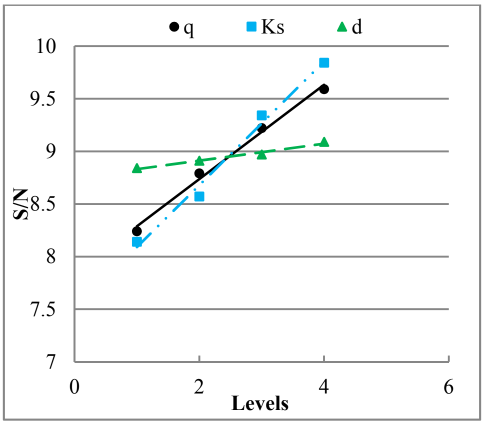

Fig. 15. Effect of different parameters levels on S/N ratio when the smallest downstream fluid height is of interest (smaller is better)

6. CONCLUSION

This study dealt with experimental investigation of the effect of surface roughness of the target plate on circular hydraulic jump radius and fluid depth downstream of the jump position. For the purpose of examining the effect of different parameters, experiments were conducted with a variety of values of volumetric flow rate and jet diameter. The results achieved in this study were utilized to the Bush & Aristoff model for different degrees of surface roughness of the target plate and to propose empirical laws for the estimation of circular jump radius on rough surfaces and fluid height downstream of the jump position. Among the findings of this study, are the following:

- Adding roughness to target plate leads to a reduction in fluid height downstream of the jump position and therefore enlarges the radius of the circular hydraulic jump.
- Because the Bush and Aristoff model is presented only for smooth plates. For smooth target plates, the results of Fig. 11 should be used, instead of using the Bush and Aristoff model.
- In the range for which the experiments have been conducted, flow rate has the strongest effect on the radius of circular hydraulic jumps and surface roughness has the strongest effect on the reduction in fluid height downstream of the jump position.

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